# Natural convection heat transfer above heated horizontal surfaces

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*Abstract:* An extensive reasoned review of the results available in the literature for free convection heat transfer from a heated flat plate facing upwards, is conducted. The review is organized in the form of a table, so as to give the reader the opportunity to compare the heat transfer data, expressed through dimensionless equations, as well as the conditions under which these data were obtained. A comparative survey of the results which may be derived at different Rayleigh numbers by the use of the heat transfer correlations presented, is also reported, showing that in some cases the discrepancies may amount to ±50%.

*Keywords*: Free convection; Horizontal surfaces; Heat transfer correlations; Comparative survey.

## 1 Introduction

Free convection heat transfer from a horizontal, upward-facing heated plate of finite size is one of the basic classic natural-convection problems, since it appears in a number of science and engineering applications, as well as in natural circumstances.

Starting from the first decades of 1900, a large body of research has been performed on this topic, for both conditions of uniform wall temperature and uniform heat flux, as witnessed by the wide variety of heat transfer correlations readily available in the literature. However, in some cases, the predictions of such correlations may also differ by ±50%, or more, depending on the investigation method, the boundary conditions, and the occurrence of more or less pronounced three-dimensional edge-effects.

In this context, it is felt the need to organize an extensive, reasoned review of the most prominent heat transfer correlations – expressed in the typical dimensionless form $Nu = C(Ra)^n$ –, with the main aim to highlight the conditions under which these correlating equations were obtained, and carry out a comparative survey of their results, so as to help the reader in applications.

## 2 Review of literature data

The review of literature data for natural convection above a heated horizontal surface is presented in Table 1, in which the heat transfer correlations, and other information useful to guide the reader through a comparative analysis, are reported.

For the sake of simplicity, the shorter side of the plate, W, is the characteristic length to be used in all dimensionless groups, which is the choice made in many studies. Whenever some authors decided to make reference to a different linear dimension, e.g., the hydraulic radius, mainly for plates whose shape was neither square nor too much slender, both the original correlations and the correlations modified by rearranging the dimensionless groups in terms of W, are reported – see notes (a) and (b) of Table 1.

In addition, since in applications is more usual, or easier, to know the plate temperature rather than the convective heat transfer rate at the plate surface, basic reference is made to situations with uniform wall temperature. However, also the data available for plates with uniform heat flux have been taken into account. Also in this case, both the correlations originally developed by the authors in terms of Nu and Ra*, and the correlations modified in terms of Nu and Ra according to the relationship $Ra^* = Ra \times Nu$, are reported – see note (c) of Table 1.

Finally, when available, also local heat transfer equations have been specified. In this case the local Nusselt and Rayleigh numbers are denoted as $Nu_x$ and $Ra_x$ or $Ra_x$*, in which the characteristic length is the distance x from the lateral edge of the plate.

## 3 Data analysis and discussion

Analysis of Table 1 shows that most of the studies were executed experimentally, by using air or water as working fluid. On the other hand, at the time of these investigations the numerical methods for the solution of the continuity, momentum and energy equations were not completely developed, while the analytical approach, based on the boundary-layer theory, could not take into account any edge-effect and flow separations from the plate.

**Table 1** – Collection of dimensionless correlating equations for upward-facing horizontal plates

| Authors | Plate geometry | Fluid | Correlations | Buoyancy strength | Investigation |
|---|---|---|---|---|---|
| Fishenden and Saunders, 1950 [1] | square | air | $Nu = 0.540Ra^{1/4}$ | $10^5 < Ra < 2\times10^7$ | experimental |
| | | | $Nu = 0.140Ra^{1/3}$ | $2\times10^7 < Ra < 3\times10^{10}$ | |
| Bosworth, 1952 [2] | square | unknown | $Nu = 0.710Ra^{1/4}$ | laminar flow | experimental |
| | | | $Nu = 0.170Ra^{1/3}$ | turbulent flow | |
| Rotem and Claassem, 1969 [3] | semi-infinite plate | $Pr = 0.72$ | $Nu = 0.639Ra^{1/5}$ | laminar flow | theoretical |
| | | $Pr = 10.0$ | $Nu = 0.726Ra^{1/5}$ | laminar flow | |
| Fujii and Imura, 1972 [4] | rectangular (L/W = 2) | water | $Nu = 0.160Ra^{1/3}$ | $7\times10^6 < Ra < 2\times10^8$ | experimental |
| | | | $Nu = 0.130Ra^{1/3}$ | $5.7\times10^8 < Ra < 6\times10^{10}$ | |
| Pera and Gebhart, 1973 [5] | semi-infinite plate | $Pr = 0.72$ | $Nu_x = 0.363Ra_x^{1/5}$ | laminar flow | theoretical |
| Goldstein et al., 1973 [6] | various shapes | $Sc = 2.5$ | $Sh = 0.960(Ra_m)^{1/6}$ | $1 < Ra_m < 10^2$ | experimental ([a]) |
| | | | $Sh = 0.590(Ra_m)^{1/4}$ | $2\times10^2 < Ra_m < 8\times10^3$ | |
| | square | | $Sh = 1.920(Ra_m)^{1/6}$ | $64 < Ra_m < 6.4\times10^3$ | ([b]) |
| | | | $Sh = 0.834(Ra_m)^{1/4}$ | $1.3\times10^4 < Ra_m < 5.4\times10^5$ | |
| | rectangular (L/W = 7) | | $Sh = 1.451(Ra_m)^{1/6}$ | $12 < Ra_m < 1.2\times10^2$ | |
| | | | $Sh = 0.725(Ra_m)^{1/4}$ | $2.4\times10^3 < Ra_m < 9.6\times10^4$ | |
| Lloyd and Moran, 1974 [7] | various shapes | $Sc = 2200$ | $Sh = 0.540(Ra_m)^{1/4}$ | $2.2\times10^4 \le Ra_m \le 8\times10^6$ | experimental ([a]) |
| | | | $Sh = 0.150(Ra_m)^{1/3}$ | $8\times10^6 \le Ra_m \le 1.6\times10^9$ | |
| | square | | $Sh = 0.764(Ra_m)^{1/4}$ | $1.4\times10^6 \le Ra_m \le 5.1\times10^8$ | ([b]) |
| | | | $Sh = 0.150(Ra_m)^{1/3}$ | $5.1\times10^8 < Ra_m \le 10^{11}$ | |
| | rectangular (L/W = 5) | | $Sh = 0.672(Ra_m)^{1/4}$ | $3\times10^5 \le Ra_m \le 1.1\times10^8$ | |
| | | | $Sh = 0.150(Ra_m)^{1/3}$ | $1.1\times10^8 < Ra_m \le 2.2\times10^{10}$ | |
| | rectangular (L/W = 10) | | $Sh = 0.657(Ra_m)^{1/4}$ | $2.3\times10^5 \le Ra_m \le 8.5\times10^7$ | |
| | | | $Sh = 0.150(Ra_m)^{1/3}$ | $8.5\times10^7 < Ra_m \le 1.7\times10^{10}$ | |

Table 1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| Al-Arabi and El-Riedy, 1976 [8] | rectangular (L/W = 1 to 4) | air | $Nu = 0.700Ra^{1/4}$<br>$Nu = 0.155Ra^{1/3}$ | $2\times10^5 \le Ra \le 4\times10^7$<br>$4\times10^7 < Ra \le 10^9$ | experimental |
| Ishiguro et al., 1978 [9] | rectangular (L/W = 1 to 4.6) | water | $Nu = 0.200Ra^{1/3}$ | $3\times10^5 < Ra < 10^{10}$ | experimental |
| Yousef et al., 1982 [10] | square | air | $Nu = 0.622Ra^{1/4}$<br>$Nu = 0.162Ra^{1/3}$ | $3\times10^6 \le Ra \le 4\times10^7$<br>$4\times10^7 < Ra \le 1.7\times10^8$ | experimental |
| Goldstein and Lau, 1983 [11] | square<br>2D infinite strip | air | $Sh = 0.746Ra^{1/5}$<br>$Nu = 0.621Ra^{1/5}$ | $10 < Ra < 4.8\times10^3$<br>$40 < Ra < 8\times10^3$ | experimental (a)<br>numerical (a) |
| | square<br>2D infinite strip | | $Sh = 1.300Ra^{1/5}$<br>$Nu = 0.819Ra^{1/5}$ | $6.4\times10^2 < Ra < 3\times10^5$<br>$3.2\times10^2 < Ra < 6.4\times10^4$ | experimental (b)<br>numerical (b) |
| Sparrow and Carlson, 1986 [12] | rectangular (L/W = 3.3) | air | $Nu = 1.070(Ra^*)^{1/6}$<br>$Nu = 1.084Ra^{1/5}$ | $3\times10^6 \le Ra^* \le 2.5\times10^7$<br>$2\times10^5 \le Ra \le 1.2\times10^6$ | experimental (c) |
| Chen et al., 1986 [13] | semi-infinite plate | $0 < Pr < \infty$ | $Nu_x = K(Ra_x/5)^{1/5}$<br>$Nu = 1.667\ K(Ra/5)^{1/5}$<br>$K = Pr^{1/2}/(0.25 + 1.6Pr^{1/2})$ | $10^3 \le Ra_x \le 10^9$<br>$10^3 \le Ra \le 10^9$ | theoretical |
| | | $Pr = 0.72$<br>$Pr = 7$ | $Nu = 0.638Ra^{1/5}$<br>$Nu = 0.713Ra^{1/5}$ | $10^3 \le Ra \le 10^9$<br>$10^3 \le Ra \le 10^9$ | |
| Kitamura and Kimura, 1995 [14] | quasi 2D geometry | air | $Nu_x = 0.660(Ra_x^*)^{1/6}$<br>$Nu_x = 0.066(Ra_x^*)^{1/3}$<br>$Nu_x = 0.700(Ra_x^*)^{1/5}$<br>$Nu_x = 0.200(Ra_x^*)^{1/4}$ | $10^2 < Ra_x^* < 10^6$<br>$10^6 < Ra_x^* < 5\times10^7$<br>$5\times10^7 < Ra_x^* < 8\times10^{10}$<br>$8\times10^{10} < Ra_x^* < 10^{14}$ | experimental |
| | | | $Nu = 1.25\ (Ra^*)^{1/6}$<br>$Nu = 0.04\ (Ra^*)^{1/3} + 9.7$<br>$Nu = (Ra^*)^{1/5} - 13.5$<br>$Nu = 0.20(Ra^*)^{1/4} + 37$ | $1.6\times10^3 < Ra^* < 1.6\times10^7$<br>$1.6\times10^7 < Ra^* < 8\times10^8$<br>$8\times10^8 < Ra^* < 1.3\times10^{12}$<br>$1.3\times10^{12} < Ra^* < 1.6\times10^{15}$ | |

Table 1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| Kitamura and Kimura, 1995 [14] (continued) | quasi 2D geometry | air | $Nu_x = 0.607Ra_x^{1/5}$<br>$Nu_x = 0.017Ra_x^{1/2}$<br>$Nu_x = 0.640Ra_x^{1/4}$<br>$Nu_x = 0.117Ra_x^{1/3}$ | $70 < Ra_x < 1.5\times10^5$<br>$1.5\times10^5 < Ra_x < 2\times10^6$<br>$2\times10^6 < Ra_x < 7.5\times10^8$<br>$7.5\times10^8 < Ra_x < 1.5\times10^{11}$ | (c) |
| | | | $Nu = 1.307Ra^{1/5}$ | $3.7\times10^2 < Ra < 8\times10^5$ | |
| Lewandowski et al., 2000 [15] | square and rectangular | water | $Nu = 0.774Ra^{1/5}$ | $4\times10^4 < Ra < 5\times10^6$ | experimental (a) |
| | square<br>rectangular (L/W = 4) | | $Nu = 1.347Ra^{1/5}$<br>$Nu = 1.116Ra^{1/5}$ | $2.5\times10^6 < Ra < 3.2\times10^8$<br>$6\times10^5 < Ra < 7.8\times10^7$ | (b) |
| Martorell et al., 2003 [16] | rectangular (L/W = 2.3 to 27.8) | air | $Nu = 1.200Ra^{0.175}$ | $2.9\times10^2 \le Ra \le 3.3\times10^5$ | experimental |
| | 2D infinite strip | | $Nu = 1.280Ra^{0.167}$ | $8\times10^2 \le Ra \le 2\times10^6$ | numerical |
| Wei et al., 2003 [17] | 2D infinite strip | air | $Nu = 0.823Ra^{0.201}$<br>$Nu_x = 0.318Nu/[0.5 - (x/L)]^{0.555}$ | $10^5 \le Ra \le 10^7$ | numerical |
| Kozanoglu and Lopez, 2007 [18] | rectangular (L/W = 2) | water | $Nu = 0.134Ra^{0.34}$<br>$Nu = 0.131Ra^{0.34}$ | $2.7\times10^4 \le Ra \le 5.9\times10^{10}$<br>$2.5\times10^5 \le Ra \le 4.2\times10^{11}$ | experimental (a)<br>(b) |

Notes:
(a) the characteristic length to be used in all dimensionless groups is the hydraulic radius defined as the ratio between the active area and its perimeter, equal to W/4 for a square plate and to W/2 for a 2D infinite strip;
(b) correlating equations modified by employing the plate width W as characteristic length in all dimensionless groups;
(c) correlating equations modified in terms of Ra, according to the relationship $Ra^* = Ra \times Nu$.

**Table 2** – Survey of Nu–values for air derived from dimensionless correlations available in the literature

| Correlation (for air) | Plate geometry | Nu | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Ra = $10^3$ | $5\times10^3$ | $10^4$ | $5\times10^4$ | $10^5$ | $5\times10^5$ | $10^6$ | $5\times10^6$ | $10^7$ |
| Fishenden–Saunders [1] | square | – | – | – | – | 9.60 | 14.36 | 17.08 | 25.53 | 30.37 |
| Goldstein et al. [6] (*) | square | 6.07 | 7.94 | 8.34 | 12.47 | 14.83 | 22.18 | – | – | – |
| Yousef et al. [10] | square | – | – | – | – | – | – | – | 29.41 | 34.98 |
| Goldstein–Lau [11] | square | 5.17 | 7.14 | 8.20 | 11.32 | 13.00 | 17.94 | – | – | – |
| Goldstein et al. [6] (*) | rectangular | 4.59 | 6.10 | 7.25 | 10.84 | 12.89 | – | – | – | – |
| Al Arabi–El Riedy [8] | rectangular | – | – | – | – | 12.44 | 18.61 | 22.13 | 33.10 | 39.36 |
| Sparrow–Carlson [12] | rectangular | – | – | – | – | 10.84 | 14.96 | 17.18 | – | – |
| Kitamura–Kimura [14] | rectangular | 5.20 | 7.18 | 8.25 | 11.38 | 13.07 | 18.03 | – | – | – |
| Martorell et al. [16] | rectangular | 4.02 | 5.33 | 6.01 | 7.97 | 9.00 | – | – | – | – |
| Goldstein–Lau [11] | 2D infinte strip | 3.26 | 4.50 | 5.17 | 7.13 | – | – | – | – | – |
| Martorell et al. [16] | 2D infinte strip | 4.06 | 5.31 | 5.96 | 7.80 | 8.75 | 11.45 | 12.86 | – | – |
| Wei et al. [17] | 2D infinte strip | – | – | – | – | 8.32 | 11.50 | 13.22 | 18.28 | 21.01 |

(*) mass transfer data for Sc = 2.5

**Table 3** – Survey of Nu–values for water derived from dimensionless correlations available in the literature

| Correlation (for water) | Plate geometry | Nu | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Ra = $10^6$ | $5\times10^6$ | $10^7$ | $5\times10^7$ | $10^8$ | $5\times10^8$ | $10^9$ | $5\times10^9$ | $10^{10}$ |
| Lewandowski et al. [15] | square | – | 29.46 | 33.83 | 46.68 | 53.62 | – | – | – | – |
| Lloyd–Moran [7] (*) | square | 24.16 | 36.13 | 42.96 | 64.24 | 76.40 | 114.24 | 150.21 | 256.88 | 323.66 |
| Fujii–Imura [4] | rectangular | – | 27.39 | 34.51 | 59.01 | 74.36 | 103.32 | 130.00 | 222.63 | 280.51 |
| Ishiguro et al. [9] | rectangular | 20.02 | 34.23 | 43.13 | 73.77 | 92.94 | 158.95 | 200.00 | 342.50 | 431.55 |
| Lloyd–Moran [7] (*) | rectangular | 21.25 | 31.78 | 37.79 | 56.51 | 67.20 | 119.21 | 150.21 | 256.88 | 323.66 |
| Kozanoglu–Lopez [18] | rectangular | 14.36 | 24.83 | 31.42 | 54.31 | 68.75 | 118.83 | 150.41 | 259.97 | 329.06 |
| Lloyd–Moran [7] (*) | quasi 2D strip | 20.77 | 31.07 | 36.94 | 55.25 | 69.71 | 119.21 | 150.21 | 256.88 | 323.66 |

(*) mass transfer data for Sc = 2200

It is interesting to notice that the results of the theoretical studies are almost the same, as it is the case of the value predicted for $Nu_x/(Ra_x)^{1/5}$ in air by Pera and Gebhart [5], i.e., 0.363, and by Chen et al. [13], i.e. 0.381, and that of the values predicted by Rotem and Claassem [3] and by Chen et al. [13] for $Nu/(Ra)^{1/5}$, i.e., 0.639 and 0.638 for Pr = 0.72, and 0.726 and 0.720 for Pr = 10, respectively.

With regard to the experimental and numerical correlations reviewed in Table 1, a comparative survey of their predictions is reported in Tables 2 and 3, in air and water, respectively. It may be seen that the numerical data show a substantially good degree of agreement, which is e.g. the case of the results obtained for a 2D infinite strip in air by Goldstein and Lau [11] and by Martorell et al. [16] in the range $10^3 \leq Ra \leq 5\times10^4$, and by Martorell et al. [16] and by Wei et al. [17] for $10^5 \leq Ra \leq 10^7$. In contrast, the experimental results may also differ by ±50%. However, as shown in Tables 2 and 3, the amount of the discrepancy decreases if the heat transfer data are distinguished according to the working fluid and, even more, the geometry of the plate. Such discrepancy among the experimental data may be ascribed mainly to: (a) the surface heating system, since the often-used single-resistor electric heating setup does not ensure at all that the condition of uniform wall temperature is actually achieved, as the heat transfer performance of the plate is not uniform; and (b) the accuracy in the evaluation of the thermal power really convected from the plate to the adjacent fluid, owing to the difficulty to calculate the heat losses through the electric cables and the support assembly to a tee.

# Nomenclature

| | |
|---|---|
| g | gravitational acceleration |
| h | coefficient of convection heat transfer |
| $h_m$ | coefficient of convection mass transfer |
| k | thermal conductivity of the fluid |
| L | longer side of the plate (length) |
| Nu | average Nusselt number, hW/k |
| $Nu_x$ | local Nusselt number, hx/k |
| Pr | Prandtl number, $\nu/\alpha$ |
| q | heat flux |
| Ra | Rayleigh number, $[g\beta(T_w - T_\infty)W^3/\nu^2]\times Pr$ |
| $Ra_m$ | Rayleigh number for mass transfer, $[g(\rho_w - \rho_\infty)W^3/\rho_\infty\nu^2]\times Sc$ |
| $Ra_x$ | local Rayleigh number, $[g\beta(T_w - T_\infty)x^3/\nu^2]\times Pr$ |
| Ra* | modified Rayleigh number, $[g\beta qW^4/k\nu^2]\times Pr$ |
| $Ra_x$* | local modified Rayleigh number, $[g\beta qx^4/k\nu^2]\times Pr$ |
| Sc | Schmidt number, $\nu/\delta$ |
| Sh | average Sherwood number, $h_m W/\delta$ |
| T | temperature |
| W | shorter side of the plate (width) |
| x | distance from the plate lateral edge along the direction parallel to its shorter side |

**Greek symbols**

| | |
|---|---|
| $\alpha$ | thermal diffusivity of the fluid |
| $\beta$ | coefficient of thermal expansion of the fluid |
| $\delta$ | coefficient of diffusion of transferred species |
| $\nu$ | kinematic viscosity of the fluid |
| $\rho$ | density of the fluid |

**Subscripts**

| | |
|---|---|
| w | referred to the plate surface |
| $\infty$ | referred to the undisturbed fluid |

*References:*


[1] M. Fishenden, O. A. Saunden, *An Introduction to Heat Transfer*, Oxford Univ. Press, London, 1950.

[2] R. L. C. Bosworth, *Heat Transfer Phenomena*, Wiley, New York, 1952.

[3] Z. Rotem, L. Claassen, Natural convection above unconfined horizontal surfaces, *J. Fluid Mech.*, Vol. 38 (1), 1969, pp. 173-192.

[4] T. Fujii, H. Imura, Natural-convection heat transfer from a plate with arbitrary inclination, *Int. J. Heat Mass Transfer*, Vol. 15, 1972, pp. 755-767.

[5] L. Pera, B. Gebhart, Natural convection boundary layer flow over horizontal and slightly inclined surfaces, *Int. J. Heat Mass Transfer*, Vol. 16, 1973, pp. 1131-1146.

[6] R. J. Goldstein, E. M. Sparrow, D. C. Jones, Natural convection mass transfer adjacent to horizontal plates, *Int. J. Heat Mass Transfer*, Vol. 16, 1973, pp. 1025-1035.

[7] J. R. Lloyd, W. R. Moran, Natural convection adjacent to horizontal surface of various planforms, *J. Heat Transfer*, Vol. 96, 1974, pp. 443-447.

[8] M. Al-Arabi, M. K. El-Riedy, Natural convection heat transfer from isothermal horizontal plates of different shapes, *Int. J. Heat Mass Transfer*, Vol. 19, 1976, pp. 1399-1404.

[9] R. Ishiguro, H. Nagase, S. Nakanishi, T. Abe, Heat transfer and flow instability of natural convection over upward-facing horizontal surfaces, *Proceedings Sixth Int. Heat Transfer Conf.*, Vol. 2, Toronto, 1978, pp. 229-234.

[10] W. W. Yousef, J. D. Tarasuk, W. J. McKeen, Free convection heat transfer from upward-facing isothermal horizontal surfaces, *J. Heat Transfer*, Vol. 104, 1982, pp. 493-500.

[11] R. J. Goldstein, K. S. Lau, Laminar natural convection from a horizontal plate and the influence of plate-edge extensions, *J. Fluid Mech.*, Vol. 129, 1983, pp. 55-75.

[12] E. M. Sparrow, C. K. Carlson, Local and average natural convection Nusselt numbers for a uniformly heated, shrouded or unshrouded horizontal plate, *Int. J. Heat Mass Transfer*, Vol. 29, 1986, pp. 369-379.

[13] T. S. Chen, H. C. Tien, B. F. Armaly, Natural convection on horizontal, inclined, and vertical plates with variable surface temperature or heat flux, *Int. J. Heat Mass Transfer*, Vol. 29, 1986, pp. 1465-1478.

[14] K. Kitamura, F. Kimura, Heat transfer and fluid flow of natural convection adjacent to upward-facing horizontal plates, *Int. J. Heat Mass Transfer*, Vol. 38, 1995, pp. 3149-3159.

[15] W. M. Lewandowski, E. Radziemska, M. Buzuk, H. Bieszk, Free convection heat transfer and fluid flow above horizontal rectangular plates, *Applied Energy*, Vol. 66, 2000, pp. 177-197.

[16] I. Martorell, J. Herrero, F. X. Grau, Natural convection from narrow horizontal plates at moderate Rayleigh numbers, *Int. J. Heat Mass Transfer*, Vol. 46, 2003, pp. 2389-2402.

[17] J. J. Wei, B. Yu, Y. Kawaguchi, Simultaneous natural-convection heat transfer above and below an isothermal horizontal thin plate, *Num. Heat Transfer*, Vol. 44, 2003, pp. 39-58.

[18] B. Kozanoglu, J. Lopez, Thermal boundary layer and the characteristic length on natural convection over a horizontal plate, *Heat Mass Transfer*, Vol. 43, 2007, pp. 333-339.