Investigation of aiding flow for natural convection around a horizontal isothermal circular cylinder by finite volume

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Abstract: - Although natural convection around a hot horizontal circular cylinder is a classical heat-transfer problem and numerous experimental and numerical studies have been reported, there are very limited studies over the issue of natural convection around a cooled horizontal circular cylinder. In the current work, finite volume method has been applied for investigating aiding flow behavior around a cold horizontal cylinder. Complete Navior-Stokes equations have been solved by finite volume scheme using SIMPLE method. As expected, Nusselt number around cooled horizontal circular cylinder has a larger value compared to hot cylinder. For aforementioned conditions a correlation of \( \text{NU} = 0.557R_{ad}^{0.251} \) for \( R_{ad} \ll 10^7 \), is extracted. Numerical results have good agreement with previous work done with previous numerical investigation[1].

Key-Words: - natural convection; circular cylinder; cooled surface; aiding flow; finite volume; SIMPLE method

1 Introduction

Natural convection flow of a viscous incompressible fluid around a horizontal circular cylinder represents an important problem, which is related to numerous engineering applications. Sparrow and Lee [2] looked at the problem of a vertical stream over a heated horizontal circular cylinder for mixed convection. They obtained a solution by expanding velocity and temperature profiles in powers of \( x \), the coordinate measuring distance from the front stagnation point on the cylinder. The exact solution of governing equations is still out of reach due to the non-linearity in the Navier–Stokes equations. It appears that Merkin [3], was the first who presented a complete solution of this problem using Blasius and Gortler series expansion methods along with an integral method and a finite-difference scheme. The problem of free convection boundary layer flow from cylinders of elliptic cross-section was also studied by Merkin [4]. Ingham [5] investigated the boundary layer flow on an isothermal horizontal cylinder. Hossain and Alim [6] have investigated natural convection–radiation interaction on boundary layer along a vertical thin cylinder. Nazar et al. [7], have considered the problem of natural convection flow from the lower stagnation point to the upper stagnation point of a horizontal circular cylinder immersed in a micropolar fluid. All mentioned works are about the issue of hot cylinder. Recently Tahavoor and Yaghoubi [1] have investigated natural cooling of horizontal cylinder numerically. They have solved governing equations in laminar flow streamline-vorticity form.

In the present study we have investigated natural convection around a cooled horizontal circle cylinder and compared the results with other available solutions and presented a correlation to calculate Nusselt number for cold horizontal cylinder.

2 Problem Formulation

A steady two-dimensional laminar natural convective flow from a cooled isothermal horizontal circular cylinder of radius \( a \) has been considered. It is assumed that the surface cylinder temperature is lesser than ambient temperate \( T_s < T_\infty \). The physical model is illustrated in Fig. 1.

In the flowing section, solution method briefly is discussed.
3 Numerical method

Complete Navier-Stocks equations, Eqs.(1-4), have been solved in finite volume with SIMPLE procedure. For computations, the physical domain from circular coordinate is converted to curve linear coordinate system. The fluid is assumed air with ideal gas properties.

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  
(1)

\[
\rho \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \nabla^2 u + \rho g
\]  
(2)

\[
\rho \left( \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \nabla^2 u - \rho g
\]  
(3)

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \nabla^2 T
\]  
(4)

Here \( u \) and \( v \) are velocity components in Cartesian coordinate, \( p \) and \( T \) are static pressure and temperature respectively, and \( \rho \) and \( \alpha \) are the density and thermal diffusivity, respectively.

3.1 Boussinesq approximation:

By using Boussenesq approximation, the inertia terms will be multiplied by the dominate term \( \rho \alpha = \) constant, whereas the leading body force term becomes \( \rho \beta g (T - T_r) \). Therefore, the momentum equation, Eq.(3) can be written as:

\[
u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{\partial^2 v}{\partial x^2} + \rho \beta (T - T_r)
\]  
(4)

Where \( \rho \), \( \beta \), \( T_r \), and \( v = \mu / \rho \alpha \) are constant. Similarly, the thermal diffusivity appearing in the energy equation, Eq.(4), \( \alpha = k / \rho \alpha c_p \) is assumed constant [8].

Eqs.(2-4) show the coupling between the temperature filed and flow filed.

The values of local and average Nusselt numbers are determined as follows:

\[ N_u_D = \frac{hD}{K}, N_U_{ave} = \frac{1}{\pi} \int_0^\pi N_u_D(\theta) \, d\theta, \]  
(6)

where \( D \) is the cylinder diameter, \( N_u_D \) is the local Nusselt number based on \( D \) as the characteristic length, \( h \) is the convective heat transfer coefficient, \( K \) is the fluid thermal conductivity and \( N_U_{ave} \) is the average Nusselt number.

3.2 Finite volume

SIMPLE procedure has been applied in the current work for two coupled momentum and energy equation. These equations have been solved simultaneously. Appropriate under relaxations are more important in cooled surfaces compared with hot surfaces.

3.2- Numerical grid and boundary condition

Physical domain with grid structures are shown in Fig. 2 and a zoom plot from cylinder surrounded grid is shown in Fig. 3.

![Fig.2: Physical model with schematic structure grids](image1)

Fig.2: Physical model with schematic structure grids in zoom area

For high accuracy, high quality structure grids have been used by stretch in zons that fluxes have extreme differentiations.

![Fig.3: Physical model with schematic structure grids](image2)
Bottom boundary is taken far from cylinder to minimize effect of this boundary on the flow field. All fluxes have zero value on the open boundaries. The convergence criteria of $10^{-6}$ are chosen for all dependent variables. For obtaining a suitable grid around the cylinder, as it is shown in Fig. 3, more small grids of O is used in closed area around the cylinder and C grid for outer space in square shown in Fig. 2. Some grid dimensions are tested to obtain optimum grid dimension. To do this, several tests are made between grid dimensions of 60x60 and 100x100. It is found that 80x80 grid dimensions are sufficiently fine to ensure a grid independent solution and for the domain below, the rectangular selected a grid of 20x15 is assumed.

4 Result and discussion

In this section, numerical results have been presented for air (Pr = 0.72) and for an aluminum cylinder at various figures and compared with some numerical and experimental results reported in [1,9]. Contours are presented for the following conditions:

$$T_s = 10 ^\circ C = 283.15 \text{ K}, T_{\infty} = 30 ^\circ C = 303.15 \text{ K},$$

$$D = 1 \text{ in}, g = -9.8 \text{ m/s}^2.$$

In this condition: $R_{ad} = 35468$.

Where $R_{ad} = g\beta/\alpha \nu (T_{\infty} - T_s)D^3$ is the Rayleigh number based on D as the characteristic length. In this relation $g$ is the acceleration of gravity, $\beta$ is the thermal expansion, $\alpha$ is the Thermal diffusivity and $\nu$ is the Kinematic viscosity.

Temperature contours around a cooled cylinder are shown in Fig.4. This figure shows with increasing the angles $\omega$, thermal boundary layer grows. As $y$ increases from the upper stagnation point ($y = 0$), the cooled fluid falls down due to the gravity hence the thickness of the thermal boundary layer is expected to grow. The minimum and maximum temperatures in Fig.3 are 283 and 303 on the cylinder surface and thermal boundary layer, respectively.

Velocities contours for the above cylinder are presented in Fig. 5. Because of gravity effect, air flow descends on the cylinder surface. This figure shows that by moving from surface, flow gets higher velocity. As expected, the Local Nusselt numbers versus angles (from $\omega = 0$ to falling plume. These contours clearly show that in free flow plume there is not high amount of velocities. velocity value is zero on the cylinder surface due to non-slip condition and it is maximum in about the middle of $\omega = 180$) for various Rayleigh numbers are illustrated in Fig. 6. As expected, by moving from top stagnation point ($\omega = 0$) to the position with $\omega = 180$, local Nusselt numbers show a decreasing trend.

Calculated local Nusselt numbers for $R_{ad} = 10^6$ are compared with a numerical result [1] for a cooled cylinder and a experimental results of [9] for a hot cylinder in Fig. 7. Agreement between presented results and [1] results are observable in this figure. Difference between local Nusselt numbers on cooled and hot cylinder is because of the nature of the problem. The gravitational acceleration has an opposite direction with the plume for hot cylinder but, in this problem gravitational acceleration has a similar direction with the plume and flow is the aiding type. For such difference, heat transfer coefficient (or Nusselt number) of cooled cylinder must be greater than hot cylinders.
Determining average Nusselt number for a cooled circular cylinder is the main purpose in this work. This parameter is presented in Fig. 8 for various Rayleigh numbers. In addition, this figure shows two comparison of calculated results with [1,10] results. High accuracy of presented results is observable by this comparison. Average Nusselt number for a hot circular cylinder is illustrated in this figure. These Nusselt numbers are calculated from a Churchil correlation [10] for hot cylinder. Fig. 8 clearly shows that average Nusselt number around a cooled cylinder is higher than around a hot cylinder. Gravity effect is the reason of this difference. For a cooled cylinder, gravity accelerate the fluid flow have the same direction but, for a hot cylinder this direction is opposite. In the mentioned condition, convective heat transfer in aiding flow has a higher value compared with convective heat transfer in opposing flow.

For aforementioned conditions one correlation, $\text{NU}_D = 0.557R^{0.251}_{\alpha D}$ for $R_{\alpha D} \ll 10^7$, is extracted from numerical result to easily compute average Nusselt number for cooled pipe surfaces.

5 Conclusion
1-Natural convection around a cooled circular cylinder is investigated, and a correlation for calculating average Nusselt number in $R_{\alpha D} \ll 10^7$ is presented.
2-Natural convection from cold cylinder is aiding flow but natural convection from hot cylinder is opposing flow.
3-Natural convection heat transfer rate in aiding flow is higher than convective heat transfer in opposing flow. Convection heat transfer in 
4-Results from finite volume method are in a good agreement with results with other schemes.

References: