Bifurcation-Analysis Technique for Nonlinear, Parametric Effects in 3D Magnetic Nanodevices at Microwaves and Photonics

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Abstract: - The developed technique involves numerical determination of bifurcation points of the nonlinear Maxwell equations complemented by the Landau-Lifshitz equation with the exchange term taking into account the electrodynamical boundary conditions for 3D magnetic nanodevices. A special computational algorithm has been developed for determination of bifurcation points of the nonlinear Maxwell operator including the Landau-Lifshitz equation. The bifurcation points are found from the eigenvalues of the characteristic equation of the linearized Maxwell operator via the use of necessary and sufficient conditions for the existence of a bifurcation point. The original computational algorithm is improved by combining it with a qualitative method of analysis, based on Lyapunov stability theory. The instabilities of parametric excitation process of magnetostatic dipole-dipole waves (MSW) and dipole-exchange spin waves (SW) in periodic bandgap structures of nano-sized magnetic particles is simulated using bifurcation-analysis technique.

Key-Words: - bifurcation points, Maxwell operator, mathematical simulation, nonlinear, diffraction, boundary problem

1 Introduction

Up to now, the bifurcation analysis has been performed to study nonlinear systems with lumped parameters that are described by ordinary differential equations (DEs). Investigation of nonlinear electrodynamic systems that are characterized by distributed parameters and described by partial DEs has certain peculiarities and encounters substantial mathematical difficulties. The first the bifurcation-analysis technique for 3D nonlinear magnetic nanoparticle devices is developed. This technique, based on the bifurcation theory [1,2] and is a pioneering approach in nanoelectrodynamics taking into account the constrained geometries.

2 Solution of 3D Nonlinear Diffraction Problem

The mathematical simulation of magnetic nanoparticle devices is based on the solution of boundary problems for nonlinear Maxwell's equations

$$\mathrm{rot}\overline{\mathrm{H}} = \varepsilon_0 \varepsilon \frac{\partial \overline{E}}{\partial \mathrm{t}}, \mathrm{rot}\overline{\mathrm{E}} = - \frac{\partial \overline{B}(\overline{\mathrm{H}})}{\partial \mathrm{t}}, \quad (1)$$

complemented by the Landau-Lifshitz equation of motion of magnetization vector in ferromagnet [1]

$$\partial \vec{M} / \partial t = -\gamma[\vec{M}, \vec{H}_{eff}] - (\alpha / M)[\vec{M}, \partial \vec{M} / \partial t], \quad (2)$$

where $\overline{\mathrm{E}}, \overline{\mathrm{H}}$ are the electric- and magnetic-field intensity vectors, respectively; $\overline{B}(\overline{H})$ is the magnetic induction vector; ε is the relative permittivity; ε_0, μ_0 are the where $\vec{M}$ is the magnetization vector, γ is the gyromagnetic ratio, α is the damping constant, $\vec{H}_{eff} = \vec{H} + \vec{H}_{ex}$ is the effective field, $\vec{H}$ is the external magnetic field, $\vec{H}_{ex} = (2A/\mu_0 M_s)\Delta\vec{M}$ is the exchange field, A is the exchange constant of the ferrite, M_s is the saturation magnetization.

Let us reduce nonstationary nonlinear Maxwell's equations (1) combined with the Landau-Lifshitz equation (2) to systems of stationary equations. Assuming that the electromagnetic fields of transmitters having frequencies $\omega_1, \omega_2, .., \omega_n$ are harmonic, the fields $\overline{\mathrm{E}}(\mathrm{t})$, $\overline{\mathrm{H}}(\mathrm{t})$ and the magnetization $\overline{\mathrm{M}}(\mathrm{t})$ were represented in the form of series in terms of the combination frequencies ω_m:

$$\begin{aligned} \overline{\mathrm{E}}(\mathrm{t}) &= \sum_{m=-\infty}^{\infty} \overline{\mathrm{E}}(\omega_m) \cdot \exp(\mathrm{i}\omega_m \mathrm{t}), \\ \overline{\mathrm{H}}(\mathrm{t}) &= \sum_{m=-\infty}^{\infty} \overline{\mathrm{H}}(\omega_m) \cdot \exp(\mathrm{i}\omega_m \mathrm{t}), \quad (3) \\ \overline{\mathrm{M}}(\mathrm{t}) &= \sum_{m=-\infty}^{\infty} \overline{\mathrm{M}}(\omega_m) \cdot \exp(\mathrm{i}\omega_m \mathrm{t}). \end{aligned}$$

Substituting these series (3) into (1) and (2), we obtain the following systems of stationary nonlinear equations for each combination frequency:

$$\begin{aligned} & rot\overline{H}(\omega_m) = i\omega_m \varepsilon_0 \varepsilon(\omega_m) \overline{E}(\omega_m), \\ & rot\overline{E}(\omega_m) = -i\omega_m \mu_0 \overline{H}(\omega_m) - i\omega_m \mu_0 \overline{M}(\omega_m), \quad (4) \\ & -i\omega_m \mu_0 \overline{M}(\omega_m) = \omega_r(\chi_0 \overline{H}(\omega_m) - \overline{M}(\omega_m)) - \\ & -\gamma \sum_{i=-q}^{q} \sum_{j=-q}^{q} \gamma_{ij} (\overline{M}(\omega_i) \times \overline{H}(\omega_j)) \end{aligned}$$

where m=0,1,2,…,q, q – number of frequencies taken into account.

Let us consider the nonlinear diffraction boundary problem for the electromagnetic waves propagating in waveguiding structures (WGS) containing a nonlinear gyromagnetic ferromagnet discontinuity, placed between cross-sections S_1, S_2.

A few monochromatic waves, having frequencies $\omega_1, \omega_2, .., \omega_n$, are incident on the input cross-sections S_1. The waves are the fundamental and higher-order modes of the WGS, having known magnitudes $C^{+}_{k(\beta)}(\omega_n)$ (β is the index of cross-sections, k are the indices of eigenwaves of WGS). The magnitudes $C^{-}_{k(\beta)}(\omega_m)$ of reflected modes vs. combination frequencies ω_m are determined by the numerical method of autonomous blocks with Floquet channels (FABs) [4]. (The local co-ordinate systems are used on cross-sections S_1, S_2).

The approximate solution of the diffraction boundary problem can be found in the form of reduced series:

$$\overline{E}^N(\omega_m) = \sum_{n=1}^{N} a_n^t(\omega_m)\overline{e}_n^t(\omega_m) + a_n^z(\omega_m)\overline{e}_n^z(\omega_m)$$

$$\overline{H}^N(\omega_m) = \sum_{n=1}^{N} b_n^t(\omega_m)\overline{h}_n^t(\omega_m) + b_n^z(\omega_m)\overline{h}_n^z(\omega_m), \quad (5)$$

$$\overline{M}^N(\omega_m) = \sum_{n=1}^{N} \mu_0 \cdot \mu_1(\omega_m)\cdot(d_n^t(\omega_m)\cdot \overline{h}_n(\omega_m) + $$

$$+ d_n^z(\omega_m)\cdot \overline{h}_n^z(\omega_m)),$$

where m=0,1,2,…,q, N is the number of accounted eigenwaves of WGS; $a_n^t(\omega_m)$, $b_n^t(\omega_m)$, $a_n^z(\omega_m)$, $b_n^z(\omega_m)$, $d_n^t(\omega_m)$, $d_n^z(\omega_m)$ are the unknown functions of the longitudinal coordinate variable z; $\overline{e}_n^t(\omega_m)$, $\overline{h}_n^t(\omega_m)$, $\overline{e}_n^z(\omega_m)$, $\overline{h}_n^z(\omega_m)$ are the transverse and longitudinal components of eigenwaves in WGS filled by a medium having a dielectric constant $\varepsilon_1(\omega_m)$ and magnetic permeability $\mu_1(\omega_m)$.

Substituting (5) into the Landau-Lifshitz equation (2) and projecting, using the basis functions $\overline{e}_n^t(\omega_m)$, $\overline{h}_n^t(\omega_m)$, $\vec{e}_k^z(\omega_m)$, $\vec{h}_k^z(\omega_m)$, a system of ordinary nonlinear DEs is obtained:

$$\frac{db_k^t(\omega_m)}{dz} = -i\Gamma_\kappa(\omega_m)\cdot b_k^z(\omega_m) +$$

$$+ i\omega_m \sum_{n=1}^{N}(A_{kn}^t(\omega_m)b_n^z(\omega_m) - B_{kn}^t(\omega_m)a_n^t(\omega_m)),$$

(6)

$$\frac{da_k^t(\omega_m)}{dz} = -i\Gamma_\kappa(\omega_m)a_k^z(\omega_m) + i\omega_m \sum_{n=1}^{N}(C_{kn}^t(\omega_m)(a_n^z(\omega_m) -$$

$$- d_n^t(\omega_m)) - D_{kn}^t(\omega_m)b_n^t(\omega_m))$$

Equations (6) are solved jointly with the system of nonlinear algebraic equations:

$$\sum_{n=1}^{N}(B_{kn}^z(\omega_m)a_n^z(\omega_m) - A_{kn}^z(\omega_m)b_n^t(\omega_m)) = 0, \quad (7)$$

$$\sum_{n=1}^{N}(C_{kn}^z(\omega_m)(a_n^t(\omega_m) - d_n^z(\omega_m)) - D_{kn}^z(\omega_m)b_n^z(\omega_m)) = 0,$$

$$\sum_{n=1}^{N}(\alpha_r D_{kn}^z(\omega_m)b_n^z(\omega_m) - \beta_r C_{kn}^z(\omega_m)d_n^z(\omega_m)) = 0,$$

$$\sum_{n=1}^{N}(\alpha_r D_{kn}^t(\omega_m)b_n^t(\omega_m) - \beta_r C_{kn}^t(\omega_m)d_n^t(\omega_m) -$$

$$-\gamma \sum_{i=-q}^{q}\sum_{j=-q}^{q}\gamma_{ij}(X_{kn}(\omega_i,\omega_j)\cdot d_n^t(\omega_i)\cdot b_n^z(\omega_j) +$$

$$+ Y_{kn}(\omega_i,\omega_j)\cdot d_n^z(\omega_i)\cdot b_n^t(\omega_j)) = 0$$

where m = 0, 1, 2,…, q; k, n = 1, 2,…, N; $\alpha_r = \dfrac{\omega_r \chi_0}{\mu_0}$, $\beta_r = \omega_r + i\omega_m$, $\Gamma_\kappa(\omega_m)$ are the propagation constants of eigenwaves of WGS; ω_r is the relaxation frequency; and $\chi_0 = M_0/H_0$ is the static magnetic susceptibility.

Let us formulate the boundary conditions for stationary nonlinear Maxwell's equations (4). Defining conditions of non-asymptotic radiation [5], and taking into account the electrodynamical boundary conditions for the tangential components $\overline{E}_\tau(\omega_m), \overline{H}_\tau(\omega_m)$ on the cross-sections S_1, S_2, we obtain the following boundary conditions:

$$C^{+}_{k(\beta)}(\omega_m) + C^{-}_{k(\beta)}(\omega_m) = a^t_{k(\beta)}(\omega_m),$$

(8)

$$C^{+}_{k(\beta)}(\omega_m) - C^{-}_{k(\beta)}(\omega_m) = b^t_{k(\beta)}(\omega_m),$$

where β =1, 2; k, n = 1, 2,…, N; m = 0, 1, 2,…, q.

The algorithm for the solution of nonlinear diffraction boundary problem was developed using the cross-sections method [5]. It reduces the nonlinear boundary problem to the system of nonlinear DEs (6) with boundary conditions (8), solved together with the system (7). A short-cut numerical method allows to transform this boundary problem for the system of nonlinear algebraic equations (7) to the Cauchy problems with variable initial conditions relative to $C^{-}_{k(\beta)}(\omega_m)$ (β =1, 2; k = 1, 2,…, N; m = 0, 1, 2,…,q).

3 Numerical Approach for Determining Bifurcation Points of Nonlinear Maxwell Operator Including Landau-Lifshitz Equation

Branching points of the nonlinear Maxwell equations are analyzed under the assumption that one solution in the neighborhood of a singularity point is known [1]. A decomposition computational algorithm can be used to find this solution [4]. However, a situation is possible when second solution y exists in the neighborhood of a branching point and is very close to first solution. When nonlinear boundary problems of electromagnetics are solved with the use of traditional computational algorithms, such branching points may be missed. Therefore, the computational algorithm should be improved via its combination with a qualitative method of analysis of branching points [2]. Let us construct an algorithm that implements numerical analysis of bifurcation points of the nonlinear Maxwell operator.

Let's write the system of nonlinear DEs (6) with the system of the nonlinear algebraic equations (7) in a symbolic form, as:

$$\frac{dy_i}{dz} = \Phi_i(y_1, y_2, ..., y_n), \quad \psi_j(y_1, y_2, ..., y_n) = 0, \tag{9}$$

where $i=1,2...,m;\ j=m+1,\ m+2,...n$; $y_i = y_i(z)$ are the unknown functions of coordinate z, compiled on functions, obtained by the cross-section method [5].

To obtain the solutions $\tilde{\tilde{y}}_i$ of system (9), the nonlinear diffraction boundary problem was solved using the decomposition computational algorithm by the numerical FABs method [5]. But using this algorithm alone, there is a probability to miss a second unknown solution y_i, appearing at the bifurcation point. That's why it is necessary to use a special numerical method to determine and analyze the bifurcation points of nonlinear Maxwell's operator for the 3D boundary problems. This numerical approach consists in the following.

System (9) is transformed by substitution $x_i = y_i - \tilde{y}_i$ into:

$$\frac{dx_i}{dz} = \Phi_i(x_1, x_2, ..., x_n), \quad \Psi_j(x_1, x_2, ..., x_n) = 0, \tag{10}$$

where

$$\Phi_i(x_1, x_2, ..., x_n) = \varphi_i(x_1 + \tilde{y}_1, ..., x_n + \tilde{y}_n) - \varphi_i(\tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_n)$$

$$\Psi_j(x_1, x_2, ..., x_n) = \varphi_j(x_1 + \tilde{y}_1, ..., x_n + \tilde{y}_n).$$

Then functions Φ_i and Ψ_j identically vanish for $x_i = 0$, consequently, the solution $x_i = 0$ of system (10) is a fixed point, stationary relative to variable z (coordinate z is the longitudinal WGS axis).

As the first approximation, let's reduce the system of nonlinear DEs (10) to a system of linear DEs. For this purpose it is necessary to represent functions Φ_i and Ψ_j by their generalized Taylor's series in the neighborhood of fixed points $x_i = 0$, and to take into account the first order partial derivatives.

For this purpose, we develop functions Φ_i and Ψ_j as a generalized Taylor series in the neighborhood of the stationary point $x_i = 0$ and take into account zero- and first-order partial derivatives. Then, we obtain the system of linear DEs

$$\frac{dx_i}{dz} = \sum_{k=1}^{n} \frac{\partial \Phi_i(0,0,...,0)}{\partial x_k} x_k, \tag{11}$$

$$\sum_{k=1}^{n} \frac{\partial \Psi_i(0,0,...,0)}{\partial x_k} x_k = 0,$$

$$i = 1,2,...,r,\ j = r+1, r+2, ..., n.$$

Equations (11) can be represented in an expanded form as

$$\begin{cases} \dfrac{dx_1}{dz} = a_{11}(z)x_1 + a_{12}(z)x_2 + ... + a_{1n}(z)x_n = x_1', \\ \dfrac{dx_2}{dz} = a_{21}(z)x_1 + a_{22}(z)x_2 + ... + a_{2n}(z)x_n = x_2', \\ \cdots \\ \dfrac{dx_m}{dz} = a_{m1}(z)x_1 + a_{m2}(z)x_2 + ... + a_{mn}(z)x_n = x_m', \\ 0 = a_{m+1,1}(z)x_1 + a_{m+1,2}(z)x_2 + ... + a_{m+1,n}(z)x_n, \\ 0 = a_{m+2,1}(z)x_1 + a_{m+2,2}(z)x_2 + ... + a_{m+2,n}(z)x_n, \\ \cdots \\ 0 = a_{n1}(z)x_1 + a_{n2}(z)x_2 + ... + a_{nn}(z)x_n, \end{cases} \tag{12}$$

where coefficients $a_{ij}(z)$ $(i,j = 1,2,...,n)$ are constructed from the partial derivatives entering (11).

Equations (12) can be represented in the matrix form. This procedure results a system of linear ordinary DEs, written in a matrix form as:

$$A(z)\vec{x} = \frac{d\vec{x}}{dz} \tag{13}$$

where $\vec{x}$, $\frac{d\vec{x}}{dz}$ are the vector-functions having components $x_1, x_2, ..., x_m$ and $x_1', x_2', ..., x_m'$;

$$A(z) = A_{11}(z) - A_{12}(z)A_{22}^{-1}(z)A_{21}(z).$$

Here,

$$A_{11}(z) = \begin{pmatrix} a_{11}(z) & a_{12}(z) & ... & a_{1m}(z) \\ a_{21}(z) & a_{22}(z) & ... & a_{2m}(z) \\ . & . & . & . \\ a_{m1}(z) & a_{m2}(z) & ... & a_{mm}(z) \end{pmatrix},$$

$$A_{12}(z) = \begin{pmatrix} a_{1\,m+1}(z) & a_{1\,m+2}(z) & ... & a_{1n}(z) \\ a_{2\,m+1}(z) & a_{2\,m+2}(z) & ... & a_{2n}(z) \\ . & . & . & . \\ a_{m\,m+1}(z) & a_{m\,m+2}(z) & ... & a_{mn}(z) \end{pmatrix},$$

$$A_{21}(z)=\begin{pmatrix} a_{m+1\ 1}(z) & a_{m+1\ 2}(z) & \dots & a_{m+1\ m}(z) \\ a_{m+2\ 1}(z) & a_{m+2\ 2}(z) & \dots & a_{m+2\ m}(z) \\ . & . & . & . \\ a_{n1}(z) & a_{n2}(z) & \dots & a_{nm}(z) \end{pmatrix},$$

$$A_{22}(z)=\begin{pmatrix} a_{m+1\ m+1}(z) & a_{m+1\ m+2}(z) & \dots & a_{m+1\ n}(z) \\ a_{m+2\ m+1}(z) & a_{m+2\ m+2}(z) & \dots & a_{m+2\ n}(z) \\ . & . & . & . \\ a_{n1}(z) & a_{n2}(z) & \dots & a_{nn}(z) \end{pmatrix}.$$

Note that the function $x_i = y_i - \tilde{y}_i$ is the difference between unknown solution y_i and known solution $\tilde{\tilde{y}}_i$.(System of nonlinear DEs (10) is linearized in the neighborhood of $\tilde{\tilde{y}}_i$.) Function $\tilde{\tilde{y}}_i$ is a solution obtained with the use of a standard computational method, for example, the method of cross sections [6] that is based on the decomposition approach using the FABs [4].

System of linear ordinary DEs (13) has the trivial solution $x_i = 0$ $(y_i - \tilde{y}_i = 0)$.It is necessary to determine nontrivial solutions to system (13). If such solutions exist, the values of parameters determine the branching points. We find the solution of matrix equation (13) in the form of its Fourier series:

$$\vec{x} = \sum_{p=-\infty}^{\infty} \vec{\alpha}_p \cdot \exp(i\frac{2\pi p}{l} z), \tag{14}$$

where $\vec{\alpha}_p$ is a vector having components $\alpha_{1p}, \alpha_{2p}, \dots, \alpha_{mp}$, and l is the length of the nonlinear ferromagnet discontinuity placed in the WGS between cross-sections z = 0 and z = l.

Vector function $\vec{x}(z)$ from (14) satisfies the boundary conditions $\vec{x}(0) = \vec{x}(l)$, which are fulfilled only in theneighborhood of a branching point where the quantity $\vec{x} = \vec{y} - \tilde{\tilde{y}}$ is determined approximately by the zeroth term of the Taylor series.

Substituting (14) into (13) we obtain the matrix equation:

$$A(z)\vec{\alpha}_p = (i\frac{2\pi p}{l})\vec{\alpha}_p,$$

where p = 0, ±1, ±2, … , ±m.

If system of linear DEs (13) has a nonzero solution, eigenvalues λ (at least one eigenvalue) of matrix **A**(z) coincide with the values $i2\pi p/l$ (p = 0, ±1, ±2, …, ±m) for any z from [0, l] [2]. The converse is not always valid. Matrix **A**(z) for any z from [0, l] may have eigenvalues λ coinciding with $i2\pi p/l$, but branching points do not necessarily exist [2].

The behavior of a solution near a branching point is analyzed numerically under the assumption that one solution $\tilde{y}_i$ in the neighborhood of a bifurcation point is known. In contrast to the definition of a branching point, the definition of a bifurcation point assumes that one solution of the family of solutions $\tilde{y}_i (i = 1,2,...,n)$ defined at all of the values of the calculation parameter is known a priori. Note that we are dealing with the discrepancy of solutions y_i from specified family $\tilde{y}_i (i = 1,2,...,n)$.

The bifurcation points exist when the necessary and sufficient conditions for their existence are satisfied [2]. The necessary condition consists in that the eigenvalues of matrix **A**(z) for any z on [0, l] are equal $i2\pi p/l$; the sufficient condition is that each of odd-multiple, simple eigenvalues of matrix A(z) for any z = $i2\pi p/l$ are the bifurcation points [2]. Using the auxiliary computing algorithm, the necessary and sufficient conditions in the neighborhood of the given value of calculation parameter are examined. If both conditions are satisfied, then at this bifurcation point the new solution y_i, describing the onset of self-oscillations, exists at the given value of the bifurcation parameters.

4 Numerical Technique of Bifurcation Analysis of Nonlinear Phenomena (Parametric Instabilities) in Magnetic Nanoparticle Devices

To solve the 3D nonlinear diffraction boundary problem for magnetic nanoparticle devices at the electrodynamic accuracy level a computational algorithm was developed based on the decomposition approach by FABs [5]. The FAB method is computing efficient in solving the problems of diffraction of TEM-wave in periodic bandgap structures of nano-sized ferromagnetic particles .

However, in general, it is not easy to see the physical meaning of numerical solutions. That's why for the analysis of nonlinear phenomena, related to the parametric instability, a special computational algorithm to determine the bifurcation points, developed by us, is used.

The numerical technique to investigate the nonlinear effects involves finding the bifurcation points of the nonlinear Maxwell operator including the Landau-Lifshitz with the exchange term. Our original computational algorithm was improved by combining it with a qualitative method of analysis, based on Lyapunov stability theory [7].

According to the linearization principle [2] the detection of the bifurcation points of the nonlinear operator is reduced to determining the eigenvalues of the linearized operator.

Earlier the nonlinear Maxwell's operator (the full Maxwell's equations (1) complemented by the Landau-Lifshitz equation (2)) for WGS, containing the nonlinear ferromagnet discontunity, was reduced to the linearized Maxwell's operator (13).

The partial solutions of the system of linear ODEs (13)

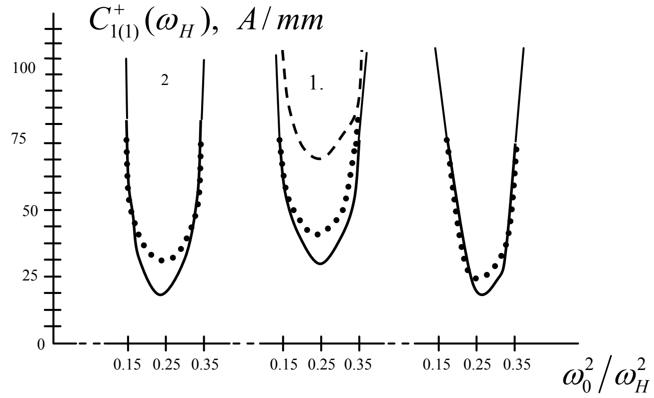

Fig. 1. The threshold of parametric instability of MSW and SW in the periodic bandgap structures of ferromagnetic spheres: the radius R=0.1 mm, the separation of spheres h ----- h= 0.55 mm, …….- h=0.35 mm; —— h=0.215 mm. Curve 1 – f_0 = 9.330 GHz; 2 – 8.125 GHz; 3 – 9.946 GHz, H_0=3330 Oe, $C^+_{1(1)}(\omega_H)$ - magnitude of the incident pumping wave; $\omega_0 = 2\pi f_0$ - frequency of the signal wave, ω_H - frequency of the pumping wave.

are the exponential functions:

$$x_m = \alpha_m \cdot e^{\lambda_m \cdot z}, \tag{15}$$

Substituting (2) into (1), we obtain the following matrix equation:

$$\mathbf{A} \cdot \vec{\alpha} = \lambda \cdot \vec{\alpha} \tag{16}$$

where $\vec{\alpha}$ is a vector having components $\alpha_1, \alpha_2, ..., \alpha_m$; λ and $\vec{\alpha}$ are the eigenvalues and eigenvectors of matrix $\mathbf{A}$.

The eigenvalues λ_m are the propagation constants of "weakly" nonlinear waves in WGS (or the eigenfrequencies of "weakly" nonlinear oscillations in resonator structures); the components of eigenvectors $\vec{\alpha}$ are the components of the electromagnetic fields of these waves.

Using numerical methods (e.g. QR-algorithm) to solve the matrix equation (13) the eigenvalues λ_m and eigenvectors $\vec{\alpha}$ of matrix $\mathbf{A}$ are determined.

According to the Lyapunov method [7] if the real parts of complex eigenvalues λ_m of matrix $\mathbf{A}$ are negative, then the solution of a system of linear ODEs (13) is asymptotically stable. If at least one of the real parts of λ_m is positive, then the solution of system (13) is unstable. The change of the sign of the real part of λ_m occurs in the bifurcation points; therefore it is essential to determine the bifurcation values of parameters where parametric instabilities occur.

Hence, it is necessary to find the bifurcation points of the nonlinear Maxwell's operator for the 3D nonlinear magnetic nanodevices, and to monitor the bifurcation points depending on the control parameters by using our algorithm.

Accurate modeling of nonlinear propagating of

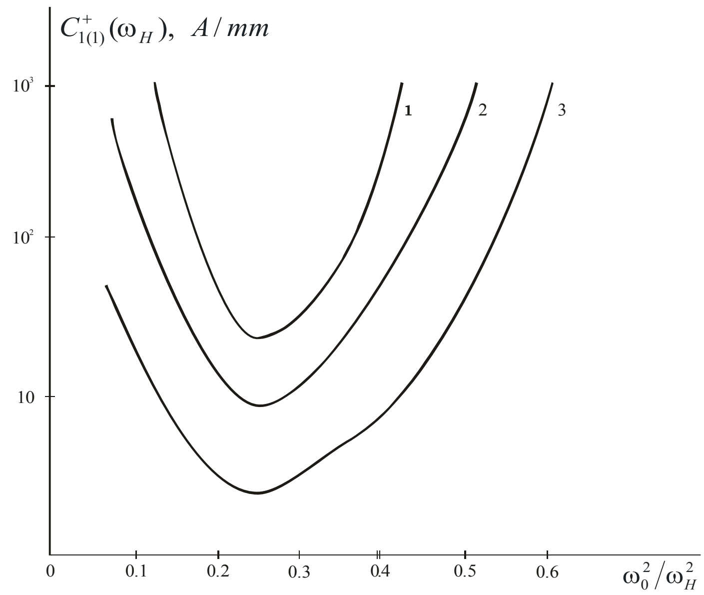

Fig. 2. The threshold of parametric instability of MSW and SW in the periodic bandgap structures of ferrite nanospheres: the radius R=250 nm; the separation of spheres h 1- h=3000 mm; 2 - h=750 nm; 3 - h=600 nm;, H_0=3330 Oe, $\omega_0 = 2\pi f_0$ - frequency of the signal wave, f_0 = 9.330 GHz, ω_H - frequency of the pumping wave; $C^+_{1(1)}(\omega_H)$ - magnitude of the incident pumping wave.

ectromagnetic waves in periodic bandgap structures of nano-sized ferromagnetic particles, embedded in a nonmagnetic matrix, and their interactions with dipole-dipole MSW and "short" dipole-exchange SW were made by using FABs numerical method [4]. A bias magnetic field is applied normal to the direction of the propagating ectromagnetic wave (TEM-wave, $C^+_{1(1)}(\omega_H)$ magnitude, frequency ω).

The instability regions of parametric generation of MSW and SW in the periodic bandgap structures of ferromagnetic nanospheres depending on the magnitude $C^+_{1(1)}(\omega_H)$ of the incident pumping wave and normalized frequency are simulated, when the sizes of magnetic particles are reduced to the order of exchange length. The threshold magnitudes of the pumping EMW $C^+_{1(1)}(\omega_H)$, where the nonlinear processes and the parametric instability excitation of MSW and SW occur, are determined by computing the bifurcation points of the nonlinear Maxwell's operator.

The results for the first order processes of the parametric excitation [3] in case of transverse pumping in periodic bandgap structures of ferromagnetic

nanospheres, are shown in Figs. 1 and 2. According to the Lyapunov stability theory [7] the curves in Figs. 1 and 2 divide the instability regions
for the parametric generation of MSW and SW from the stable regions.
The thresholds were calculated at microwave and photonics frequencies for different shapes (nanospheres and nanowires) and various separations of ferromagnetic nanoparticles in the periodic bandgap structures, taking into account constrained geometries. The results of computing by using the bifurcation points of the nonlinear Maxwell's operator permit to analyze and optimize geometries, sizes of magnetic nanoparticles in bandgap structures and parameters of magnetic nanomaterials at microwaves and photonics.

4 Conclusions

The bifurcation-analysis technique, developed in this paper, permits to determine the new solutions of nonlinear Maxwell's equations in the neighborhood of bifurcation points; to follow the changes of branching points depending on the values of control parameters and to determine the bifurcation values of parameters and their sensitivity to the transition regimes of nonlinear nanodevices.
With this algorithm, substantial computational difficulties caused by the ambiguity of computation parameters in the neighborhoods of singularity points can be overcome.
Using these mathematical models it will be possible to estimate the efficiency of nonlinear effects: nonlinear diffraction, parametric interactions, multistability, generation of higher order time harmonics, solitons, dynamic chaos, taking into account the constrained geometries in 2D or 3D microwave and photonics devices, based on magnetic nanoparticles or magnetophotonic crystals.
Using this technique the reliable engineering methods applicable in CAD for the numerical computation of electromagnetic properties of magnetic nanocomposite materials, applied in the modern high nanotechnology, and 3D nanodevices may be developed.

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