Passivation by Feedback on Control Backstepping
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Abstract: - This paper aims at summing up some concepts related to the design of a certain type of control laws. These concepts concern exclusively the backstepping, strict-feedback and passivity. Firstly, the concepts of passivity, dissipativity and optimality have been put forward. Secondly, the main issues on the backstepping theory on general plan have been detailed. The backstepping theory has been set out specifically in accordance with the concepts of passivity. Notion of passivity is linked to other notions mentioned above and are necessary for the synthesis of control laws. All these concepts approached previously are necessary for the synthesis of the control laws. It's the reason why apart from the analysis on part of the backstepping technique, as mentioned above, the technique of passivation by feedback effect has briefly been analyzed (i.e. how to make the system passive by feedback). The characteristics of a passive system and the limitations of the technique of passivation by feedback effect has been examined. One of these limitations has got its solution by recursive the control called ‘‘backstepping’’.

Key-Words: backstepping, strict-feedback, passivity, dissipativity and optimality.

1. Introduction

Most of the control laws aim at to ensure the asymptotic stability of control systems to guarantee and justify the studied element performances. Other methods can improve that stability by more performant practices. Su and Stepanenko [1] used the ‘‘integrator backstepping’’ technique for setting up a control law adaptive hybrid for the joints of the robots manipulators. The parameters adaptation was made by the direct method. The developed control schema was made with a view to getting simple calculations, avoiding derived terms and the inverse in the matrices of linear zed dynamic known as the regressor, and the inverse of matrices of inertias. The semi global asymptotic stability of the system is established within the meaning of Lyapunov. In [2], the concepts of standard geometrical control have correctly been analyzed and developed with a lot of examples justifying the system stability. R. Kelly and R Carelli [3] presented a unified approach for the design and the analysis of controllers for the robot-like manipulators. The concerned control manages five quite precise aspects namely: the position, the impedance, the force, the hybrid force/position and the hybrid /force impedance. In the law design of unified control, all the above aspects can be found. The functioning principle of the control algorithm is that of making an adequate choice for carrying out principle type of aspect and cancelling the four others. In other words, if the unified control is devoted for example to follow up force impedance, then the expressions of the concerned control by position, impedance, force, hybrid force/position and hybrid impedance/force is automatically put to 0. We hereby give details on the notion of passivity, its link with stability and the optimality and so called technique of passivation by feedback (or how to make a system passive by feedback). We will focus on the characteristics of a passive system and the limitations of the technique of passivation by feedback. One of these limitations is given by recursive the control called ‘‘backstepping’’.

So this paper is organized as follows: In section 2, we will focus on the concept of passivity. Section 3 will reveal the concepts of stability of the passive systems. Section 4 puts forward the concepts of passivity, optimality and stability. Sections 5 and 6 lay an emphasis on the theories related to backstepping and passivation by the design method.
recursive `backstepping' followed by one conclusion to section 7.

2. Concept of passivity
Given \( H_1 \) the dynamic system describes by the following equation.

\[
\begin{align*}
\dot{x} &= f(x) + g(x)u \\
y &= h(x) + j(x)u
\end{align*}
\]

with \( x \in \mathbb{R}^n \), \( y, u \in \mathbb{R}^m \)

Let us imagine that the upgrade in the energy of the expression (1) is due to an external source. (Example: a circuit RLC connected to a battery). On this basis, we can respectively set out definition of dissipativity and passivity.

2.1 Dissipativité Definition
Let us suppose that with the system (1) is associated a function \( W : \mathbb{R}^m \times \mathbb{R}^m \rightarrow \mathbb{R} \) called rate of provisioning locally integrable for all \( u \in U \subset \mathbb{R}^m \), i.e.

\[
\int_{t_0}^{t} W(u(t), y(t)) dt < \infty
\]

\( \forall t_0 \leq t_1 \).

Let us consider \( x \) a subset linked \( \subset \mathbb{R}^m \) including the origin. We say that (1) is dissipative on \( x \) with a supplying rate \( W(u, y) \) if function exists \( S(x) \), \( S(0) = 0 \) such as \( \forall x \in X, S(x) \geq 0 \).

\[
S(x(T)) - S(x(0)) \leq \int_{0}^{T} W(u, y) dt
\]

(3)

\( \forall u \in U \) and \( \forall T \geq 0 \) such as \( x(t) \in X \)

\( \forall t \in [0,T] \).

\( S(x) \) function of memory is called.

2.2 Passivity Definition
The system (1) is called passive when it is dissipative with a provisioning rate

\[
W(u, y) = u^T y
\]

In the case of electric circuit \( S \) is energy and \( W \) is the power provided by the internal source. The integral of (4) given by \( \int_{0}^{T} W(u, y) dt \) is therefore the energy provided by the external source. A system is then dissipative if its energy increase is inferior or equal to provided energy. If \( S(x) \) is derivable, the derived expression is equivalent to:

\[
\dot{S}(x) \leq W(u, y)
\]

(5)

2.3 Example 1
Let us consider an electrical circuit governs by the equation of following state:

\[
\begin{align*}
\frac{di}{dt} &= \frac{1}{L}v \\
\frac{dv}{dt} &= \frac{1}{RC}(v - v_c) \\
i &= i_L + \frac{1}{R}(v - v_c)
\end{align*}
\]

(6)

\( v \) and \( i \) are respectively input and output of the system. The energy stored in inductance is \( \frac{1}{2} L i_L^2 \) and that stored in the capacitance is \( \frac{1}{2} C v_c^2 \).

The total circuit energy is therefore

\[
E = \frac{1}{2} L i_L^2 + \frac{1}{2} C v_c^2
\]

(7)

\[
\frac{dE}{dt} = v i - \frac{1}{2}(v - v_c)^2 \leq v i
\]

(8)

The system is dissipative. Also,

\[
W(u, y) = v i
\]

the system is thus passive.

A fundamental property of passive systems concerns the interconnection. This property is given by the following theorem:

2.4 Theorem 1
Let us suppose that \( H_m \) and \( H_n \) are passive. Then the systems go by parallel interconnection or by feedback of \( H_m \) and \( H_n \) are also passive.

3. Stability of the passive systems
We can easily remark that the concept of memory function is related to function of Lyapunov. In fact, this similarity is not fortuitous because a close relationship exists between the properties of dissipativity and stability.

Let us note that it is not necessary that the memory function \( S(x) \) of a dissipative system (or passive) be (strictly) negative. It can be definite semi-negative. Whereas the Lyapunov function can be defined (strictly negative). Consequently, if a dissipative
system has an unstable no observable part, its point of balance can be unstable.

3.1 Example 2
Let us suppose the system below put in the form of variables of states

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= u \\
y &= x_2
\end{align*}
\]

(10)

This system is passive thanks to the candidate function below

\[
S(x) = \frac{1}{2}x_2^2
\]

(11)

To rule out that situation, conditions are imposed on the stability of the non observable part.

4. Passivity, optimality and stability
There is a close link (under certain conditions) between concepts of passivity and stability. In fact, a passive system has a certain phase margin. Moreover, it is well-known that an optimal system is stable and has a stability margin. We can ask ourselves if there exists a link between the passivity and optimality. The answer is that the issue is affirmative, as regard all the developments that follow.

4.1 Commande stabilisante optimale
Given to determine the control \( u(x) \) for the system

\[
\dot{x} = f(x) + g(x)u
\]

(12)
such as \( u(x) \) asymptotically stabilize the system for on the one hand \( x = 0 \) and on the other function minimizes the coast function. That coast function is given by the expression

\[
J = \int_0^\infty (L(x) + u^TR(x)u)dt, \quad L(x) \geq 0, \quad R(x) > 0 \quad (13)
\]

\( \forall x \)

4.2 Theorem Optimality and Stability
Let us suppose that \( V(x) \in C^1 \) satisfied the equation with Hamilton-Jacobi-Bellman below

\[
L(x) + \frac{1}{4}L_sV(x) - \frac{1}{4}L_sV(x)R^{-1}(L_sV(x))^T = 0
\]

(14)
such as \( V(0) = 0 \) and the following control

\[
u^*(x) = -\frac{1}{2}R^{-1}(x)(L_sV(x))^T
\]

(15)

(14) stabilisez asymptotically for \( x = 0 \).

Then \( u^*(x) \) is the optimal control which minimizes the function coast \( J \) and \( \lim_{t \to \infty} x(t) = 0 \).

\( V(x) \) is the optimal value of \( J \).

4.3 Optimality and passivity
In the control expression (13), Let us set out \( R(x) \equiv I \)

(16)

In other words

\[
J = \int_0^\infty (L(x) + u^TR(x)u)dt
\]

(17)

The control law

\[
u = -k(x)
\]

(18)
is optimal for \( J \) if and only if

\[
\begin{align*}
\dot{x} &= f(x) + g(x)u \\
y &= k(x)
\end{align*}
\]

(19)
is ZED (zero - stable state) and dissipative with like rate of provisioning.

\[
w(u, y) = u^Ty - \frac{1}{2}y^Ty
\]

(20)

and a memory function \( S(x) \in C^1 \) such as

\[
S(x) = \frac{1}{2}V(x)
\]

(21)

Passivation by feedback is a design technique which consists of making a passive system in closed loop by a particular form of the control law. The system which has become passive is obviously stable. It has go a certain stability margin (i.e., can tolerate an uncertainty of modeling).The principle comes from the fact for the passive system given by

\[
H_2: \begin{align*}
\dot{x} &= f(x) + g(x)u \\
y &= h(x)
\end{align*}
\]

(22)

\( \forall x \)

A law of control

\[
u = -y = -h(x)
\]

(23)
permits to stabilize in an asymptotic way the balance point if \( H_2 \) is ZED.

Then for

\[
\dot{x} = f(x) + g(x)u
\]

(24)
The technique consists in determining an output

\[
y = h(x)
\]

(25)
and a control law

\[
u = a(x) + b(x)v
\]

(26)
such as

\[ H_3: \begin{cases} \dot{x} = f(x) + g(x)(a(x) + b(x)v) \\ y = h(x) \end{cases} \] (27)

is passive. If \( H_3 \) is ZED,

\[ v = -y \] (28)

then permits to stabilize it in an asymptotic way. However two restrictions limit the use of passivation by feedback. Restrictions are imposed by the following proposals:

4.4 Proposal 1

if

\[ H_4: \begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \] (29)

is passive with \( S(x) \in \mathbb{C}^2 \), then \( H_4 \) is of relative degree 1.

Then, to make the \( H_4 \) system passive by feedback, it is worth determining an output \( y = h(x) \) so that \( H_4 \) is of relative degree 1. This research is all the more complex as the control of \( H_4 \) is high. The following limitation relates to the stability of the dynamics of the zero.

4.5 Proposition 2

Let us take back the previous system as follows:

\[ H_5: \begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \] (30)

is passive with \( S(x) \in \mathbb{C}^2 \).

(30) is weakly at minimum out phase as follows.

We say that \( H_5 \) is minimal out of phase if the zero dynamic is asymptotically stable. It is known as weakly at no minimal out of phase if that dynamic is stable in the was of Lyapunov. The theorem below completes this proposal.

4.4 Theorem 1

Let custom supposes the following relation:

\[ \frac{\partial h}{\partial x}(0) = m \] (31)

then \( H_5 \) is passive by feedback with \( S(x) \in \mathbb{C}^2 \) if and only if \( H_5 \) is slightly with minimal dephasing and if it has a relative degree 1.

The two restrictions on the stability of the dynamics of the zero and relative degree can be solved by the design methods recursive known as 'backstepping' and 'forwarding'.

The 'backstepping' makes it possible to solve the problem of the relative degree while the 'forwarding' that of no minimal dephasing slightly.

The following sections respectively present the theory of the backstepping and passivation based on the recursive method 'backstepping'.

5 Theory on the backstepping

The backstepping is a method which is to date of topicality in the recursive design of the control laws. It can be defined as a way of organizing a system in several under systems in cascade which arise in several forms. The exploitation of the methodology of design on a general level by feedback which leads to the installation of a control law, systematically associates a function of Lyapunov or the equivalent.

The important properties of the local or total stability relative to the future control law are obtained by arising a stabilizing function with each stage of the system in cascade. These stabilizing functions are dependant between it and the last acting as control law of all the system. One of the advantages which the method of the backstepping gets is that to keep the properties of the initial system in the control law obtained. This constitutes to some extent the characteristic of the backstepping compared to other methods such as the linearizing feedback.

5.1 Assumption 1

Let us consider the following system:

\[ \dot{x} = f(x) + g(x)u \] (32a)

\[ f(0) = 0 \] (32b)

where \( x \in \mathbb{R}^n \) is the vector of state and \( u \in \mathbb{R} \) is the system input.

There exists for (32) a control law of per continuously derivable information feedback such as:

\[ u = \alpha(x) \] (33a)

\[ \sigma(0) = 0 \] (33b)

and a positive definite function not limited:

\[ V: \mathbb{R}^n \to \mathbb{R} \] such as:

\[ \frac{\partial V}{\partial x}(x)[f(x)+g(x)\alpha(x)] \leq -W(x) \leq 0 \] (34)

\( \forall x \in \mathbb{R}^n \)

where \( W: \mathbb{R}^n \to \mathbb{R} \) is semi-definite positive.

Under this assumption, the control

\[ u = \alpha(x) \] (35)
applied to the system considered, guaranteed that x(t) is limited overall, via the theorem of LaSalle-Yoshizawa as explained on page 24 of the reference [4].

5.2 Lemma 1 (Integrator Backstepping)
That is to say the system expressed in (32) that we increase by an integrator such as:

\[
\begin{align*}
\dot{x} &= f(x) + g(x)\xi \\
\dot{\xi} &= u
\end{align*}
\]  

(36)

By supposing that the first under system of (36) satisfied assumption 1 with

\[ \xi \in \mathbb{R} \] and which is regarded as its control.

if W(x) is definite positive, then

\[ V_s(x, \xi) = V(x) + \frac{1}{2}[\xi - \alpha(x)]^2 \]  

(37)

is a function of control of Lyapunov (clf) for (36) as defined on page 25 of [4].

The control of the system is then defined as follows:

\[ u = -C(\xi - \alpha(x)) + \frac{\partial \alpha}{\partial x}(x)[f(x) + g(x)\xi] + \frac{\partial V}{\partial x}(x)g(x) \]  

(38)

where C>0 is considered as a winning

and \( \frac{\partial \alpha}{\partial x}(x) \) and \( \frac{\partial V}{\partial x}(x) \) are respectively the derivatives of stabilizing clf function.

If W(x) is only semi-definite positive, then there exists an control by feedback which makes

\[ \dot{V}_s \leq -W_s(x, \xi) \leq 0 \]  

(39)

However, W_s(x, \xi) > 0 when W(x) > 0

where \( \xi \neq \alpha(x) \). This guaranteed that

\[
\begin{bmatrix}
    x(t) \\
    \xi(t)
\end{bmatrix}
\]

converge and is limited narrow as the proof of this lemma given on page 34 of [4].

Only from that assumption and this lemma, we can extend these concepts to different forms of the backstepping as we will show it below.

5.3 Methodology of the systems design

strict-feedback
The use of the systems strict-feedback is justified for our backstepping because its configuration is appropriate to the robotic systems. It is given by the following form:

\[
\begin{align*}
\dot{x} &= f(x) + g(x)\xi_1 \\
\dot{\xi}_1 &= f_1(x, \xi_1) + g(x, \xi_1) \\
\dot{\xi}_2 &= f_2(x, \xi_1, \xi_2) + g(x, \xi_1, \xi_2)\xi_3 \\
&\vdots
\end{align*}
\]  

(40)

where \( x \in \mathbb{R}^n \) and \( \xi_1, \xi_2, \) and \( \xi_3 \) are scalars.

Let us suppose that subsystem x of (40) satisfied assumption 1 it be-with saying that it depends on F(x) and G(x). For this subsystem, \( \xi_1 \) is regarded as the control of entry. Our recursive design starts with under system below:

\[
\begin{align*}
\dot{x} &= f(x) + g(x)\xi_1 \\
\dot{\xi}_1 &= f_1(x, \xi_1) + g(x, \xi_1) \\
\dot{\xi}_2 &= f_2(x, \xi_1, \xi_2) + g(x, \xi_1, \xi_2)\xi_3 \\
&\vdots
\end{align*}
\]  

(41)

if \( f_1 \equiv 0 \) and \( g_1 \equiv 1 \), lemma 1 can be directly applicable while considering \( \xi_2 \) like an control known as virtual, then:

\[ V(x, \xi_1) = V(x) + \frac{1}{2}[\xi_1 - \alpha(x)]^2 \]  

(42)

where \( \alpha(x) \) is a stabilizing information feedback which satisfies (36) for under system X. By introducing a stabilizing function \( \alpha_t(x, \xi_1) \) for the virtual control of (41), we need to calculate the derivative of no positive when

\[ \dot{\xi}_2 = \alpha_t(x, \xi_1) \]  

(43)

\[ \dot{\xi}_1 = -W(x, \xi_1) + \frac{\partial V}{\partial \xi_1}(x, \xi_1)g(x) \]  

(44)

where \( W(x, \xi_1) > 0 \) when \( W(x) > 0 \) or \( \xi_1 \neq \alpha(x) \). if \( g_1(x, \xi_1) \neq 0 \) for all x and \( \xi_1 \) we choose for stabilizing function \( \alpha_t \) such as:

\[ \alpha_t(x, \xi_1) = -\frac{1}{g_1(x, \xi_1)} \left\{ -c_1[\xi_1 - \alpha(x)] - \frac{\partial V}{\partial x}(x)g(x) \\
- f_1(x, \xi_1) + \frac{\partial \alpha}{\partial x}(x)[f(x) + g(x)\xi_1] \right\} \]  

(45)

with \( c_1 > 0 \), which gives

\[ W_t(x, \xi_1) = W(x) - c_1[\xi_1 - \alpha(x)]^2 \]  

(46)
The objective of this step being achieved by the out
of stabilizing function $\alpha_i(x,\xi_i)$, our recursive
method goes on with the following compact notation:

$$
\begin{align}
\dot{X}_i &= F_i(X_i) + G_i(X_i)\xi_2 \\
\dot{\xi}_2 &= f_i(X_i,\xi_2) + g_i(X_i,\xi_2)\xi_3
\end{align}
$$

where $f_i(X_i,\xi_2), g_i(X_i,\xi_2)$ represent respectively
$f_i(x,\xi_1,\xi_2), g_i(x,\xi_1,\xi_2)$ and

$$
X_i = \begin{bmatrix}
x \\
\xi_i 
\end{bmatrix}; F_i(X_i)= \begin{bmatrix}
\partial f(x,\xi) \\
\partial \xi
\end{bmatrix} \text{ and } G_i(X_i) = \begin{bmatrix}
0 \\
g_i(x,\xi)
\end{bmatrix}
$$

The structure of (47) is similar to that of (41), we
rewrite the same iteration white by the introduction of the
following expression:

$$
V_k(x,\xi_1,\xi_2) = V_i(x_i) + \frac{1}{2} \sum_{i=1}^{2} \left[ (\xi_i - \alpha_i(x_i))^2 \right]
$$

where, for an adequate notation, we use
$X_0 = x$ and $\alpha_i(x_0) = \alpha(x)$.

The stabilizing function

$$
\alpha_i(x_i,\xi_i) = [X_i^T \xi_i]^T
$$

for virtual control $\xi_3$ is then determined by:

$$
\dot{V}_k = -W_k(x,\xi_1,\xi_2) + \sum_{i=1}^{2} \left[ (\xi_i - \alpha_i(x_i))^2 \right]
$$

with $W_k(x,\xi_1,\xi_2)>0$ when $W(x,\xi_1)>0$ where $\xi_2 \neq \alpha_i(x_i)$.

It is clear that this procedure determines the next
iteration of $K$, for which the system (40) is stabilized
by a u control, can be rewrite as follows:

$$
\begin{align}
\dot{X}_k &= F_k(X_k) + G_k(X_k)\xi_k \\
\dot{\xi}_k &= f_k(X_k,\xi_k) + g_k(X_k,\xi_k)u
\end{align}
$$

where $X_k = \begin{bmatrix}
X_k \xi_k
\end{bmatrix}; F_k(X_k) = \begin{bmatrix}
f_i(x,\xi) \\
\partial f(x,\xi)
\end{bmatrix} \text{ and }

$$
G_k(X_k) = \begin{bmatrix}
0 \\
g_k(X_k,\xi_k)
\end{bmatrix}
$$

In the same way, the function of Lyapunov for (50) is
the following one:

$$
V_k(x,\xi_1,\xi_2) = V_k(X_k) + \frac{1}{2} \left[ (\xi_k - \alpha_i(x_i))^2 \right]
$$

By calculating the derivative of (52) with the respect of
all the procedure, one finds:

$$
\dot{V}_k = -W_k(x,\xi_1,\xi_2) + \xi_1^T \left( \frac{\partial V_k}{\partial x} \right) g_k
$$

If the condition is nonsingular, i.e.:

$$
\left( \frac{\partial V_k}{\partial x} \right) g_k \neq 0 \
\forall x \in \mathbb{R}^n, \forall \xi \in \mathbb{R}, i = 1, \ldots, k
$$

then the choice for U is the following:

$$
\begin{align}
U &= \frac{1}{g_k} \left( \xi_k - \alpha_i(x_i) \right) - \frac{\partial V_k}{\partial X_k} g_k \\
&\quad \left( F_k(x) + G_k(x)u \right)
\end{align}
$$

The methodology then presented ensures us a
system stabilizing function at each step of the
system. The last stabilizing function acts as control
law of all the system. We easily see that this way of
getting the control law through the backstepping
method, allows us not only to stabilize the system,
but also to keep all the nonlinear elements. This is
not evident for other design methods.

This level, the design of the law of control
backstepping such as it is presented cannot be
applicable in certain field like that of robotics. If the
variables of states are pure scalars whereas work is
carried out with matrices it y a proven difficulty. To
circumvent this difficulty, we will need the passivity
of which the form strict-feedback of the
backstepping which uses the elements in the form of
matrices.

### 6. Passivity by the design method

**recursive “backstepping”**

This method consists in applying passivation to a
subsystem of the system to control it and then to
apply step by step by increasing the control of the
subsystem progressively. The synthesis ends when
we find again all the system. Therefore, to each step of
the design, we determines an output which makes
the subsystem considered passive and a function of
memory which will be used Lyapunov function. Let
us mention that the “backstepping” technique is
easily applied to system having a certain inferior
triangular structure.
6.1 Proposal

Let us suppose that for the system
\[ H_8 : \dot{x} = f(x) + g(x)u \quad (55) \]
there exists a derivable continually function
\[ u = \alpha_0(x) + v_0 \quad (56) \]
and a memory function \( W(x) \) twice continuously derivable (i.e. \( \in C^2 \)), defined positive, not limited and known such as \( H_8 \) be passive with like entry \( v_0 \) and an output
\[ y_0 = (L_0W)^T(x) \quad (57) \]
then the increased system
\[ H_7 : \dot{x} = f(x) + g(x)e \quad (58) \]
\[ \dot{e} = a(x,e) + b(x,e)u \]
\[ (b^{-1}(x,e) \text{existing}), \quad \text{is (globally)} \quad \text{passive by feedback with as output} \]
\[ y = e - \alpha_0(x) \quad (59) \]
and as memory function
\[ V(x,y) = W(x) + \frac{1}{2} y^T y \quad (60) \]
A particular form of the control which makes the system passive is
\[ u = b^{-1}(x,e)\theta(-\alpha(x,e)) - y_0 + \frac{\partial \alpha_0}{\partial t}(f(x)+g(x)e+v) \quad (61) \]
being an auxiliary entry.

The \( H_7 \) system with \( u \) control above is ZED (zero-detectable state) if \( H_7 \) is ZED with like \( v_0 \) input.

Furthermore, if \( W(x) \) is strictly decreasing for \( H_7 \) having as input
\[ u = \alpha_0(x) \quad (62) \]
Then the function candidate
\[ V(x,y) = W(x) + \frac{1}{2} y^T y \quad (63) \]
is strictly decreasing for \( H_7 \) with an auxiliary input
\[ v = -ky \quad (64) \]
\[ k > 0. \]
Generally, the method applied to the form systems
\[ \dot{x} = f(x) + g(x)e_i \]
\[ H_8 : \begin{align*}
\dot{e}_1 &= a_1(x,e_1) + b_1(x,e_1)e_2 \\
& \quad \vdots \\
\dot{e}_n &= a_n(z,e_1,\ldots,e_n) + b_n(z,e_1,\ldots,e_n)u
\end{align*} \quad (64) \]
The procedure of the preceding previous proposal is to apply in a recursive way to \( H_8 \) until \( u \) obtained.
So, to each step we have:
\[ y_i = e_i - \alpha_{i-1}(x,e_1,\ldots,e_{i-1}) \quad (65) \]
\[ \alpha_i = b_i^{-1}(-\alpha_i - y_{i-1} + \frac{\partial \alpha_{i-1}}{\partial t} - y_i) \quad (66) \]
\[ u = \alpha_n + v \quad (67) \]

Conclusion

In this paper, we focused on the backstepping theory and passivity such as it can be exploited for control. We defined and set out definitions as well as passive systems theorems. We also put an emphasis on the link between passivity and other concepts such as the dissipativity, the detectability, observability and the optimability.

By the backstepping and strict-feedback method, we succeed in setting up control laws of which stabilize different steps of the last stabilizing function looks like a system control. By linking up passivity to backstepping, we insure very performant control laws. We ensure of this fact of the very powerful control laws which concervent their properties.

All the definite concepts are of an major importance to accordance white the searched stability. Most of these concepts will be very useful to us in the design of the different control laws considered.

Reference