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Radar: signal analysis during operation time using wavelets

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Abstract: - Over-the-horizon-Radars (OTHRs) operate in the High Frequency band (3 to 30 MHz); they are able to detect targets beyond the horizon and are employed in many applications such as scientific meteorological studies, surveillance and oceanic platform control. The radar operates for long periods of time without interruption, this requires analyzing the echo signal during the time of operation. In this article an adaptation of Mallat's algorithm is proposed; the method compute the wavelet's coefficients of consecutive intervals of the signal in a multiresolution analysis framework. The coefficients are calculated and used efficiently to estimate the radial velocity of the target over the time.

Key-Words:- Wavelets, multiresolution, radar, signal segmentation.

1 Introduction

High Frequency (HF) radio frequencies (RF) are between 3 and 30 MHz and are extensively used for medium and long-range communications, taking advantage of the reflection of the waves off the ionosphere.

Over-the-horizon-Radars (OTHRs) operates in the High Frequency band ([13]). They are able to detect targets beyond the horizon and are employed in many applications such as scientific meteorological studies, surveillance and oceanic platform control. Some OTHRs utilize the electromagnetic waves' reflection off the ionosphere while others operate in a surface wave propagation mode (ground wave).

A moving target reflects radar's transmitted signal s_t of frequency ν_t , producing a radar return signal s_r of frequency ν_r. The frequency shift

$$\nu_0 = |\nu_r - \nu_t| \tag{1}$$

is proportional to the radial velocity of the target with respect to the source of radiation (Doppler effect). Depending on the direction of the target's motion, this frequency shift may be positive or negative. V can in this way be estimated by measuring ν_0.

Signals s_t and s_r are mixed by the radar system, producing a signal s_m that is low pass filtered obtaining a new signal $g(t)$ of frequency ν_0 ([11]). The radial velocity V can then be calculated using (([5],[6])

$$V \cong \frac{c\ \nu_0}{2\ \nu_e}. \tag{2}$$

where c is the speed of light. Knowing ν_0 and assuming $c = 3\ 10^8$ m/sec, with (2) we can estimate V ([11],[12]) with relative error less than $1.9\ 10^{-4}$ for velocities less than 100 Km/h.

The precision of the obtained V value will depend on the frequency approach; in this sense, the convenience of detecting ν_0 using an appropriate bandpass filter, with adjustable range, it suggests the employment of wavelets. The cubic spline orthogonal wavelets ([14]), in a multiresolution analysis context, allows us to approach ν_0 in a range or band, associated to certain scale or resolution level ([7],[15]); finally, the approach is obtained from the wavelet coefficients of this level using appropriate Fourier matrices ([10]).

This article presents an adaptation of Mallat's algorithm, the proposed method operate over time intervals of the signal. The coefficients are calculated and used efficiently to estimate the radial velocity of the target over time.

We use the following notation:

t : time (sec). Δt : sampling rate interval (sec).
ν : frequency (Hz). $\tau = t/\Delta t$: normalized time.
$\nu_s = 1/\Delta t$: sampling rate frequency (Hz).
$f = \nu/\nu_s$: normalized frequency.

2 Multiresolution analysis

A multiresolution analysis of $L^2(\mathbb{R})$ ([3],[7]) consist in a collection of nested subspaces $V_j,\ j \in \mathbb{Z}$ such that:

- $V_j \subset V_{j+1}$
- $\bigcap_{j\in\mathbb{Z}} V_j = \{0\}$ and $\bigcup_{j\in\mathbb{Z}} V_j$ is dense in $L^2(\mathbb{R})$
- $s(\tau) \in V_j \Leftrightarrow s(2\tau) \in V_{j+1}$
- $s(\tau) \in V_0 \Leftrightarrow s(\tau - n) \in V_0\ ,\ n \in \mathbb{Z}$
- There exists a scaling function $\phi \in V_0$ such that the family $\{\phi(\tau - k), k \in \mathbb{Z}\}$ is an orthonormal basis of V_0.

Let W_j be the orthogonal complement of V_j in V_{j+1}:

$$V_{j+1} = V_j \oplus W_j \ \text{ and } \ V_j \perp W_j \tag{3}$$

and let $P_j s$ and $Q_j s$ the orthogonal projections of the signal s on Vj and W_j respectively. Then

$$P_{j+1}s = P_j s + Q_j s\ , \tag{4}$$

particularly:

$$P_0 s = \sum_{j=-\infty}^{-1} Q_j s \tag{5}$$

On the other hand there is a second function

$$\psi \in W_0 \subset V_1\ , \tag{6}$$

called *mother wavelet*, such that the family

$$\{\psi(\tau - k), \ \ k \in \mathbb{Z}\} \tag{7}$$

is an orthogonal basis of W_0.

In correspondence, the full family

$$\{\psi_{j,k}(\tau) = 2^{j/2}\psi(2^j\tau - k), \ \ j,k \in \mathbb{Z}\} \tag{8}$$

is an orthonomal basis of $L^2(\mathbb{R})$ and

$$Q_j s(\tau) = \sum_{k=-\infty}^{\infty} < s, \psi_{j,k} > \psi_{j,k}(\tau) \tag{9}$$

Denote

$$s_j[k] = P_j\ s(2^{-j}k) \tag{10}$$

and

$$d_j[k] =< s, \psi_{j,k} > \tag{11}$$

the wavelet coefficients. For special choices of V_0 (i.e. spline space functions of odd degree) there is a cardinal function $L \in V_0$ such that

$$P_0 s(\tau) = \sum_{k=-\infty}^{\infty} s_0(k) L(\tau - k) \tag{12}$$

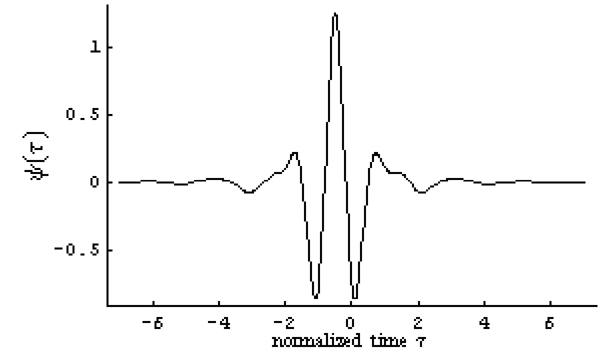

Figure 1: Orthogonal cubic spline (OCS) wavelet.

Then there are a discrete filter g such that

$$d_j[k] = 2^{-j/2}\ (g * s_{j+1})[2k]\ ; \tag{13}$$

on the other hand,

$$s_j[m] = s_{j+1}[2m] - 2^{j/2}\ (d_j * r)[m] \tag{14}$$

where

$$r[m] = \psi(m) \text{ with } \ m \in \mathbb{Z}\ ; \tag{15}$$

we refer to [8] for filter details.

3 Spline wavelet packets

Let $\psi(\tau)$ the orthogonal cubic spline (OCS) wavelet depicted in Fig.1 and

$$\hat{\psi}(f) = |\hat{\psi}(f)| e^{-i\pi f} \tag{16}$$

its Fourier transform ([2]).

It is well known that $\hat{\psi}(f)$ is a bandpass filter on $1/2 \leq |f| \leq 1$, but it is not localized at any frequency; that is, the module of the frequency response $|\mathcal{F}(\psi)(f)| \equiv |\hat{\psi}(f)|$ is almost constant on the bandpass, this is illustrated in Fig.2 for $f > 0$.

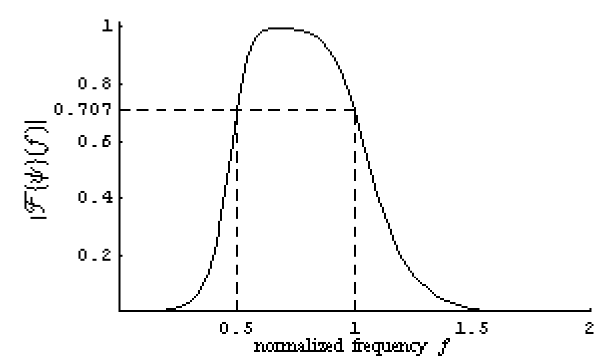

Figure 2: OCS-bandpass filter.

Then the wavelet transform gives us a time-scale representation rather than a time-frequency one. To over come this disadvantage we have proposed in [12] a new family of spline wavelet packets.

For this purposes, given $N = 2^p$ a fixed parameter called *size block*, we define the frequencies

$$f_n = \begin{cases} 1 + n/N & \text{if} \quad 1 - N/2 \le n \le 0 \\ n/N - 1 & \text{if} \quad 1 \le n \le N/2 \end{cases} ; \quad (17)$$

localized on $1/2 \le |f_n| \le 1$.

Defining the Fourier matrices

$$e_{N,n} = \left(\frac{1}{\sqrt{N}} e^{-i\pi f_n (2k+1)} \right)_{0 \le k < N} \quad (18)$$

with $1 - N/2 \le n \le N/2$, the family $e_{N,n}$ is an orthogonal basis of $\mathcal{C}^N$.

The wavelet packets are defined in W_0 as

$$\Theta_{N,n}(\tau) = \sum_{k=0}^{N-1} e_{N,n}[k] \; \psi(\tau - k) \quad (19)$$

for $1 - N/2 \le n \le N/2$; then the family

$$\{\Theta_{N,n}(\tau - b\,N),\; 1 - \frac{N}{2} \le n \le \frac{N}{2}, b \in \mathbb{Z}\} \quad (20)$$

is an orthogonal basis of the subspace W_0. The extension to other subspaces W_j is analogous.

If $d_j[k]$ mentioned in (13) are the wavelet coefficients for a given signal, the orthogonal wavelet packet transform in W_j is defined as

$$w_{j,n}[r] = \sum_{k=0}^{N-1} e_{N,n}[k] \; d_j[k] \; ; \quad (21)$$

for real signals, the values $|w_{j,n}[r]|^2 + |w_{j,-n}[r]|^2$ represent the local contribution of the frequency $2^j |f_n|$. We refer to [12] for details.

We only need N entries (the size block) $d_j[k]$ to compute the values $w_{j,n}[r]$ for each r. Then we can identify $N/2$ frequencies $2^j|f_n|$.

This suggests an efficient procedure to optimize the memory used in the computational implementation.

4 Method description

With a constant size block $N = 2^p$ and assuming that $s \in V_0$, we denotes for $j = 0$, $k = 0, 1, 2, \cdots$:

$$S_0(k) = (\; s_0[k\;2^p], \cdots, s_0[(k+1)\;2^p - 1]\;) \quad (22)$$

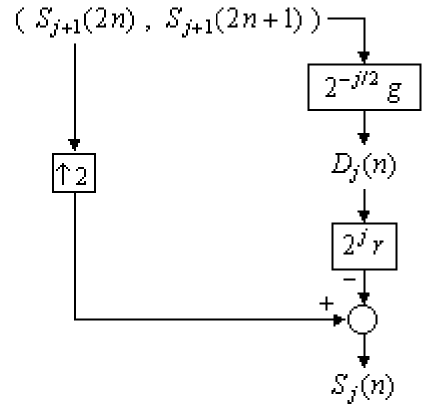

Figure 3: Computing method of $D_j(n)$ and $S_j(n)$.

where $s_0[\,\cdot\,]$ are the values of the sampled signal; and for $j \le -1$:

$$S_j(k) = (\; s_j[k\;2^p], \cdots, s_j[(k+1)\;2^p - 1]\;) \quad (23)$$

$$D_j(k) = (\; d_j[k\;2^p], \cdots, d_j[(k+1)\;2^p - 1]\;) \quad (24)$$

all these vectors have $N = 2^p$ entries.

For each pair $(S_{j+1}(2n), S_{j+1}(2n+1))$ it is possible to compute the vectors $D_j(n)$ and $S_j(n)$ as showed in Fig. 3, where:

$D_j(n)$: is analized with wavelet packets and erased from memory.

$$\xi_{\uparrow 2}(r) = \begin{cases} \xi(k) & \text{for } r = 2k, k \in \mathbb{Z} \\ 0 & \text{otherwise} \end{cases}$$

$S_j(n)$: is stored in memory until the next iteration, erasing $S_{j+1}(2n)$ and $S_{j+1}(2n+1)$.

The block generation is ordered as follows:

(O_1) : $S_j(n) < S_j(n+1)$
(O_2) : $S_j(2n+1) < S_{j-1}(n) < S_j(2n+2)$

the iterative process is depicted in Fig.4, it begins computing $S_0(0)$ and $S_0(1)$ to obtained $S_{-1}(0)$; the last one is stored in memory until it can be used with $S_{-1}(1)$ to obtained $S_{-2}(0)$.

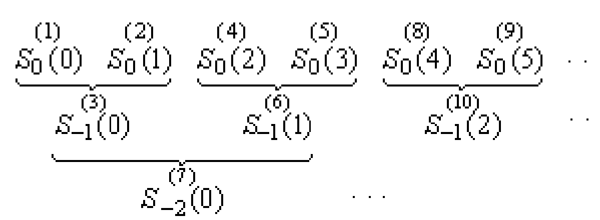

Figure 4: Order generation blocks $S_j(n)$.

Then, it is necessary to store in memory the N entries of the vector $S_j(2n)$ until to can computed $S_{j-1}(n)$ using the pair $(S_j(2n), S_j(2n+1))$.

Since r and g are IIR filters with exponential decay, in practice we implement appropriated finite version of them, see [12] and [14] for details.

5 Application

We apply the proposed method to obtain the radial velocity of a target detected by a radar; this velocity V is around 20 km/hour or 5.56 m/sec.

The signal was provided by a Doppler radar with an emission frequency $\nu_e = 10^{10}$ Hz.

We analyze the beat signal obtained in the audio frequency band, digitalized with a sampling frequency $\nu_s = 44100$ Hz through a 16-bit A/D converter.

Since we can assume that the target velocity is bounded by $V \leq V_{max} = 10$ m/sec, the associated frequency ν_0 must be bounded by $\nu_{max} = 667$ Hz.

We analyze the signal in the multiresolution scheme, for levels $j = -1, -2, \ldots$; we remark that each level j is associated with the frequency band $[\ 2^{j-1}\nu_s\ ,\ 2^j\nu_s\ ]$.

For these reasons we will search the frequency ν_0 in the rank $j \leq -6$. Computing the energy

$$E_j = \sum_k |d_j[k]|^2 \quad (25)$$

and the percent energy distribution

$$PE_j = 100\frac{E_j}{E_T} \quad \text{with} \quad E_T = \sum_{j=-1}^{-10} E_j\ .$$

Computing PE_j during a signal interval of three blocks of 2048 values ($\cong$ 46.44 mseg), we obtained the results depicted in Fig.5.

We can see that the top energy correspond to levels $j = -6$ and $j = -7$, that suggest a variation of the target velocity during the process. Applying wavelet packets to each block we detect the greatest amplitude for the following frequencies:

355.33 Hz, 315.56 Hz and 333.46 Hz,

corresponding to the velocities

5.33 m/sec, 4.73 m/sec and 5.00 m/sec,

respectively.

These describe the velocity variations, just as before it was supposed.

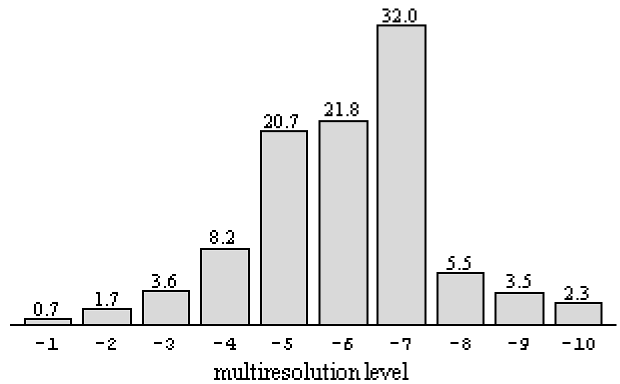

Figure 5: Percent energy distribution.

6 Conclusion

Wavelet analysis is a powerful tool for signal processing, particularly for Doppler radar signals. The algorithm used is based on a multiresolution analysis with appropriated orthogonal wavelets to obtain the resolution level corresponding to the band of frequency where is included the Doppler shift; finally, this shift is estimated from the wavelet packet coefficients using Fourier matrices.

The iterative proposed process of calculation for successive intervals of the signal with fixed size blocks, is an effective procedure to optimize the amount of memory required. It can be used to perform the analysis of signal during the time of radar's operation.

Further these results, a broad way for wavelet applications in radar signal processing is open. By example, we refer to another recent correlated develops using wavelets in [1], [4], [9].

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