OFFH-CDMA Merged M-sequence/Hyperbolic Code for Two Weights One Length WDM System

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Abstract: - In this paper, we propose an extension of two dimensions (2D) code cardinality for slightly the same performance (BER). The code, developed for optical code division multiple access (OCDMA) system is generated by: hyperbolic code for the Time Spreading and merged M-sequences for the Frequency-Hopping, in this work we consider the Optical Fast Frequency Hopping technique (OFFH). For the families codes we apply the constraint of cross-correlation which is equal to one in order to decrease the Multiple Access Interference (MAI). The frequency spacing is selected according to WDM standardization. Optical delay lines and Fiber-Bragg gratings (FBG) are the elements constituting the encoder/decoder utilized to generate/decode the code words. For the same transmission rate (the same code length), compared with the 2D code employing prime code for the time spreading, the extension will increase the cardinality to P(P-1)2 users (proposed code) than P2 users (comparison code) for a given optical LAN code words group and two qualities of service (QoS) are proposed rather than one.

Key-Words: - OFFH, OCDMA, MAI, FBG, QoS

1 Introduction

Until our days, the OCDMA is declared as being the most reliable access technique for the optical networks thanks to the various advantages which it presents. By assigning each user one code word (signature), high level security is providing, no high-speed electronic data processing circuits required in TDMA are employed, burst traffic model appropriation, variety services constraints (quality of services QoS) and different rates accommodation [1], etc.

OCDMA incoherent detection, based in unipolaire codes {0,1}, has the most part in research work because of the multiusers asynchronous mode in sharing bandwidth [5] and the simple realization of the encoder and decoder compared to these in coherent detection. Several physical realizations of this technique with FBG have been reported according to the developed code. The performances in term of BER and cardinality (total number of code words) are modulated by the code dimensionality and the correlation properties. Optical Fast Frequency Hopping CDMA (OFFH-CDMA) [2] is one of the newest techniques that employs two-dimensional coding [1, 2] to increasing the numbers of subscribers.

In this paper, we propose an algebraic construction of 2D code: Merged M-sequence/hyperbolic code (MMS/HC). The performance of the proposed scheme is compared with that of Merged M-sequence/prime code [3].

2 OFFH and Code Construction

In the OFFH technique, frequency changes at every '1' chip of the code word and the data bit, modulated by the code word, pulse of duration Tb and bandwidth Bb (broad band) is subdivided in time into L pulses of chip duration Tc = Tb/L. Each of these L pulses is centered at a different frequency according to the OFFH-CDMA code. The number of FBG employing to encode data is a function of the code length L and they are placed in the same order as the distribution of the frequencies Fi in the chips time slots. At the receiver, the order of the FBG must be the symmetrical one of that of the encoder in order to reconstitute the desired combination Frequency/Time (F/T) of a giving code word.

The 2D MMS/HC code used here, will generate codes words for an OFFHCDMA system but with a weight code *w* lower than the classic OFFH one (w=L). The aim of weak weight code is to reduce the solution cost by the use of a reduced number of FBG and to increase the number of wavelengths combination (code words). The cardinality improvement is done by taking account the minimum cross-correlation (one hit $\lambda_c=1$) and minimum side lobes auto-correlation functions conditions:

1. Each wavelength occurs only once within each sequence code.

2. One-hit condition (minimum cross-correlation) of 2D OCDMA must be at the same frequency-hopping but the different time-spreading, or at the same time-spreading but different frequency-hopping, or different timespreading combined with different time-spreading (delay time.

2.1 Frequency-Hopping construction

The merged M-sequence code, based on M-sequence properties, utilized for wavelength-hopping generation is developed in two steps:

Festival, we must take a basic binary sequence of length $P = 2^{n}$ -1, where n is the number of bits corresponding to the decimal equivalent conversion. The sequence {0 1 1 1 0 0 1} is a basic one because its decimal conversion (P=7, n=3) does not contain a redundancy as shown in Table [1], hence it's suitable for OFFH system where we use a unit of non repeated frequencies in a giving order. The source light used in OFFH is generally a broad band source that contains unique wavelengths. A cyclic rotation of the last bit in the basic sequence (column one and two of Table [1]).

In second step, with each M_q sequence we can have other P Hopping frequency patterns by adding a decimal number in each stage, according to this way: $[q + j] \mod$ P. These code words will present the Hopping Frequencies of the 2D coding system.

Table 1. Set of merged M-sequences (P=7).

Basic sequence : $M_0 = 3$	$\begin{array}{c} 0 1 1 1 0 0 1 0 1 \\ 7 6 4 1 2 5 \end{array}$	Merged-M-sequences				
Basic codes (rotation rule)	M-sequences	$M_{0,i}(j) = M_0(i) + M_0(j)$	$H_{\lambda} = C_0 \boldsymbol{.} M^{modif}{}_{\lambda}$			
$C_0 = 0 \ 1 \ 1 \ 1 \ 0 \ 0 \ 1$ $C_1 = 1 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0$ $C_2 = 0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 0$ $C_3 = 0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 0$	$M_0 = 3 7 6 4 1 2 5$ $M_1 = 5 3 7 6 4 1 2$ $M_2 = 2 5 3 7 6 4 1$ $M_3 = 1 2 5 3 7 6 4 1$	$ \begin{array}{l} M_{0,0}^{*} = 1 \ 5 \ 4 \ 2 \ 6 \ 7 \ 3 \\ M_{0,1}^{*} = 5 \ 2 \ 1 \ 6 \ 3 \ 7 \ 4 \\ M_{0,2}^{*} = 4 \ 1 \ 7 \ 5 \ 2 \ 3 \ 6 \\ M_{0,2}^{*} = 7 \ 4 \ 3 \ 1 \ 5 \ 6 \ 2 \end{array} $	$ H_{00} = 0 5 4 2 0 0 3 H_{01} = 0 2 1 6 0 0 7 H_{02} = 0 1 7 5 0 0 6 H_{02} = 0 4 3 1 0 0 2 $			
$C_{4}^{3} = 1 \ 0 \ 0 \ 1 \ 0 \ 0$	$M_4 = 4 \ 1 \ 2 \ 5 \ 3 \ 7 \ 6$ $M_5 = 6 \ 4 \ 1 \ 2 \ 5 \ 3 \ 7$ $M_6 = 7 \ 6 \ 4 \ 1 \ 2 \ 5 \ 3$					

For example the M_0 elements group (q = 0... P-1) will be given by (column 3 in Table [1]):

$$M'_{0,i}(j) = [M_0(i) + M_0(j)]; i, j = 0..P - 1$$
 (1)

In all, we have P^2 hopping patterns for a given number *P*: *P* sequences \mathbf{M}_q with *P* hopping patterns inside each one. Within a hopping pattern the occurrence of each frequency is one, however for the $\mathbf{M'}_{00}$ and $\mathbf{M'}_{03}$ there's two repeated frequency permutations **1 5**. Hits value is then equal to 2 ($\lambda_c = 2$) between each pair hopping pattern belonging to a giving group. Each hopping pattern $\mathbf{M'}_{q,i}$ will be integrated with the basic C_q generation code to build the final Merged M-sequence Hopping pattern (column 4 in Table [1]):

$$H_{q,i}(j) = C_q(j) \cdot M'_{q,i}(j); \quad i, j = 0, 1, \dots, (P-1) \quad (2)$$

The code weight is now equal to (P+1)/2 and the hit value is reduced from 2 to 1. The repeated wavelength permutation is again equal to one. Using a lower number of FBG, the implementation cost will be minimized.

2.2 Spreading Time Construction

Hyperbolic code, based on congruence theory [2,5], is chosen to construct the spreading time pattern. It will be used as the time placement operator given by the following expression:

$$y_{l,a}(k) = \left[\frac{l}{k} + a\right] \mod P; \quad k, l, a = 1, 2, \dots, P - 1 \quad (3)$$

Where k denotes the time chip position, (l,a) defines the time codeword and leading to (P-1)(P-1) independents codes. In Table 2, we had reported the (P-1)2 spreading time patterns developed with equation (3) for P=7. The first combination was given by: $y_{1,1}(k) = [1 \ 2 \ 5 \ 6 \ 3 \ 4 \ 7]$. Each time pattern have a P chips which representing the P frequencies positions of a hopping frequency pattern. The generated matrix will represent the spreading time base. The delay times between chips positions is defined by:

$$\tau_{i,j} = \begin{cases} y(i,j+1) - y(i,j) + P - 1 \ ; \ j = 0, ...P - 2 \\ y(i,0) - y(i,j) + P - 1 \ ; \ j = P - 1 \end{cases} (4)$$

Table 2: Hyperbolic code Spreading Time and delays time

	_	k=1 2 3 4 5 6		k =	=1 2 3	456	$\tau_{i,j}$	j=1 2 3 4 5 6 7	$\tau_{i,j}$	j=1 2 3 4 5 6 7
a=1	<i>l</i> =1 2 3 4 5 6	125634713245761462735153726416754231743652	a=3	1 2 3 4 5 1 6	1512 1657 1725 1163 1231 1376	2 6 7 3 1 3 2 3 6 1 5 2 7 7 5 6 2 1 5	i=1 2 3 4 5 6	7 9 7 3 7 9 0 8 5 8 7 8 5 1 9 8 2 11 2 8 2 10 4 10 1 10 4 3 11 7 4 5 4 7 4 12 3 5 9 5 3 5	i=1 2 3 4 5 6	10 2 7 10 7 2 4 11 5 8 0 8 5 5 12 1 9 4 9 1 6 6 11 3 8 3 11 0 7 7 4 12 4 7 1 8 10 5 2 5 10 2
a=2	l=1 2 3 4 5 6	$1 3 6 7 4 5 1 \\1 4 3 5 6 1 7 \\1 5 7 3 1 4 6 \\1 6 4 1 3 7 5 \\1 7 1 6 5 3 4 \\1 1 5 4 7 6 3$	a=4	1 1 2 3 1 4 1 5 1 6 1	1 6 2 3 1 7 6 1 1 1 3 6 1 2 7 4 1 3 4 2 1 4 1 7	7 1 4 2 4 3 4 7 2 6 3 1 1 6 7 3 2 6	i=1 2 3 4 5 6	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	i=1 2 3 4 5 6	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
a=3	<i>l</i> =1 2 3 4 5 6	1 4 7 1 5 6 2 1 5 4 6 7 2 1 1 6 1 4 2 5 7 1 7 5 2 4 1 6 1 1 2 7 6 4 5 1 2 6 5 1 7 4	a=5	1 1 2 1 3 1 4 1 5 1 6 1	1734 1172 1247 1315 1453 1521	1 2 5 3 5 4 5 1 3 7 4 2 2 7 1 4 3 7	1 2 3 4 5 6	9 9 0 10 7 2 5 10 5 8 7 1 5 6 11 1 9 4 9 8 0 12 4 3 8 3 11 1 6 7 11 5 4 7 2 7 10 5 2 12 3 3	1 2 3 4 5 6	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

Using the operator below, the time pattern can be spread out on P^2 chips length by forming P blocs from the P code word elements (F/T):

Table [3] present the $y_{1,1,m}$ under group with P =7.

$$Y_{l,a,m} = \begin{cases} 1 & if \quad m = \left[\frac{l}{k} + a\right]_{P} + [k,P]_{P} \\ for \ l, a, k = 0, \dots P - 1 \ ; \ m = 0, \dots P^{2} - 1 \\ 0 & \text{otherwise} \end{cases}$$
(5)

2.2 Construction of Frequency-Hopping/ Spreading Time

The modified Merged/Hyperbolic code will be obtained now by placing the frequencies of a hopping frequency pattern $H_{q,i}(j)$ in the '1' chips positions of a spreading time pattern:

$$Y_{l,a,m}.H_{q,i}(j) = \begin{cases} H_{q,i}(j) \text{ if } m = \left[\frac{l}{k} + a\right]_{P} + [k.P]_{P} \\ 0 \text{ otherwise} \end{cases}$$
(6)

Each $y_{l,a,m}$ defines one group of P code words, Table [3] show the $y_{1,1,m}$ H₀ under group.

Table 3: Merged M-sequence/ Hyperbolic code words of $y_{1,1,m} H_0$

Time spre	ding $y_{1,1}(k) = [1]$	2	5	6	3	4	7]
H _{0,i}	$Y_{1,1,m} = 1000000$	0100000	0000100	0000010	0010000	0001000	0000001
054200	$Y_{1,1,m}.H_{0,0} = 0000000$	0500000	0000400	0000020	0000000	0000000	0000003
021600	$Y_{1,1,m}. H_{0,1} = 0000000$	0200000	0000100	0000060	0000000	0000000	0000007
0 1 7 5 0 0	$Y_{1,1,m}$. $H_{0,2} = 0000000$	0100000	0000700	0000050	0000000	0000000	0000006
043100	$Y_{1,1,m}$. $H_{0,3} = 0000000$	0400000	0000300	0000010	0000000	0000000	0000002
0 6 5 3 0 0	$Y_{1,1,m}$. $H_{0,4} = 0000000$	0600000	0000500	0000030	0000000	0000000	0000004
076400	$Y_{1,1,m}$, $H_{0,5} = 0000000$	0700000	0000600	0000040	0000000	0000000	0000005
032700	$Y_{1,1,m}$. $H_{0,6} = 0000000$	0300000	0000200	0000070	0000000	0000000	0000001

Each user will have a code with characteristics given by Table [3]: a length of P^2 chips, in which only (P+1)/2chips are different from 0. The $y_{1,1,m}$. H_0 under group developed in Table [3] was the first group in the $Y_{l,a,m}$. H_0 group who contains $P(P-1)^2$ code words, more than code words group developed with the Merged Msequence/Prime code : P^2 [3]. The total Modified Msequence/Hyperbolic code cardinality will be divided in P groups $(Y_{l,a,m}.H_0, ..., Y_{l,a,m}.H_6)$, each one comprises $P(P-1)^2$ code words. By adopting this approach we can establish a system of P OFFH CDMA optical LAN. Each OLAN group code words was generated by $Y_{l,a,m}.H_a$ sequences.



Fig.1: First OLAN encode/decoder schema formed by the $Y_{l,a,m}$. H_0 group code words

Fig.1 presents the solution of the proposed Merged MMS/HC for the first OLAN generated by the Y_{l.a.m}. H₀ group. The solution adopts the serial reconfigurable FBGs (filters) and delay lines, with a conventional correlation receiver (CCR) for the decoder. Each FBG was used to reflect one frequency in the unit (F1,... F7). Taking account the WDM specification, the frequency spacing is 0.8 nm, such as the first frequency F1 is 1550 nm. The lengths of the delay lines are designed according to the delay times between chips positions and expressed in period chip time T_C (Tc=T_B/L, where T_B is the bit period time and $L=P^2$). The FBGs order of the decoder is the reverse of the encoder in such a way that only, for example for the user 1, the F3, F2, F4 and F5 separated respectively by 21 (1+7 +7+6=21), 7, 9 and 8T_C constitute the unique combination which can reconstitute the hopping frequency/time spreading pattern, user 1, optical power encoder on the photodiode (PD) detector. Each time-spreading under group has the same delay lines, so every codeword in the same timespreading under group differs only in the frequencies, as shown in Table 3 and figure 2. The estimation decision (bi) is the result of comparison between the bit signal integration and an appropriate threshold (TH) (0< TH≤ w).

Since, the number of 1 in the basic sequence ($C_0 = 01$ 11001) is 4, the weigh of all code words generated with this basic sequence is w1 = 4 = (P+1)/2. If we consider the complement of the C_0 sequence, $\overline{C_0} = 1000110$ is a new basic sequence with a weight equal to w2 = 3 = (P-1)/2, so it can be considered like a basic sequence for a new application with new weight with the same length

P. Table [4], like Table [3], show the first under group $y_{1,1,m} H_{0,i}$ of the $\overline{C_0}$ family. C_0 and $\overline{C_0}$ are two orthogonal sequences, consequently, each user inside an OLAN will be assigned by two code words from each family (C_0 and $\overline{C_0}$) having the same length P^2 , but the first one (from C_0) will have a weight equal to w1 for a giving QoS whereas the second (from $\overline{C_0}$) will have a weight equal to w2 for another QoS.

Table 4: Modified M-sequence/ Hyperbolic code code words for $y_{1,1,m} H_{0,i}$ of $\overline{C_0}$ family

H _{0,i}	$Y_{1,1,m} = 1000000 010$	00000 00	00100 000	00010 001	0000 000	1000 000	0001
	$Y_{1,1,m}.H_{0,0} = 6000000$	0000000	0000000	0000000	0010000	0007000	0000000
6000170	$Y_{1,1,m}$. $H_{0,1} = 2000000$	0000000	0000000	0000000	0040000	0003000	0000000
2000430	$Y_{1,1,m}$. $H_{0,2} = 3000000$	0000000	0000000	0000000	0050000	0004000	0000000
3000540	$Y_{1,1,m}$. $H_{0,3} = 7000000$	0000000	0000000	0000000	0020000	0001000	0000000
7 0 0 0 2 1 0	$Y_{1,1,m}$. $H_{0,4} = 5000000$	0000000	0000000	0000000	0070000	0006000	0000000
5 0 0 0 7 6 0	$Y_{1,1,m}$. $H_{0,5} = 4000000$	0000000	0000000	0000000	0060000	0005000	0000000
4 0 0 0 6 5 0	$Y_{1,1,m}$. $H_{0,6} = 1000000$	0000000	0000000	0000000	0030000	0002000	0000000
1 0 0 0 3 2 0							

3 Analysis Performance

The performance analysis of the proposed 2D code is expressed in term of BER which varying according to user number. It has been shown that in the ideal chip synchronous case [5], the CCR makes errors only when the sent bit is a zero data. The errors are due to MAI issued from the use of unipolar codes. The number of users interfering with the desired user has a binomial distribution with parameters (N – 1) and P_I, where N is the number of active users in a giving group (OLAN) and P_I is the probability that a pulse which belongs to a specific user hits one of the pulses of the desired user. The performance result of the proposed 2D code is compared to that of the Merged M-sequence/ prime code (MMS/PC) performance. The upper bound error probability of the (F/T) system is given by [4]:

$$P_{er} = \frac{1}{2} \sum_{i=TH}^{N-1} {N-1 \choose i} \left(\frac{\hat{H}}{2L}\right)^i \left(1 - \left(\frac{\hat{H}}{2L}\right)\right)^{N-1-i}$$
(7)

Where TH, is the threshold value.

For the Merged MMS/ prime code, the average hit probability is $\frac{\hat{H}}{2L}$ with L is the code length (L=P²) and \hat{H} is the average hit value inside a H_q group (the number of interfering user hits with the desired user), giving by:

$$\widehat{H} = (w (P-1) + (hit)P)/(P^2 - 1)$$
 (8)

w is the code weight, (P-1) is the number of users other than the desired user in each $H_{q,i}$ hopping group, (hit) is the total number of hits between the interfering H_i users group (OLAN), *P* is the number of elements in the

interfering $H_{q,i}$ hopping group, P^2-1 is the total number of users other than the user of interest in H_q group.

In our study and for a given weight w_i , the average hit value is equal to [3]:

$$\widehat{H} = \frac{w_i((P-1)^2 - 1) + (hit)(P-1)^2}{P(P-1)^2 - 1}$$
(9)

For example, to determinate the hit value inside the $Y_{l,a,m}$. H_0 , with w= 4 and P =7, we had P hopping group H_{00}, \ldots, H_{06} with the following frequency sets : $H_{00} = [5, 4, 2, 3]$; $H_{01} = [2,1, 6, 7]$; $H_{02} = [1,7, 5, 6]$; $H_{03} = [4, 3, 1, 2]$; $H_{04} = [6, 5, 3, 4]$; $H_{05} = [7, 6, 4, 5]$; $H_{06} = [3, 2, 7, 1]$. Each $H_{0,i}$ hopping group have (P-1)2 elements ($y_{1,1,m}, \ldots, y_{p,p,m}$). We assume that the $Y_{1,1,m}$. $H_{0,0}$ is the desired user, so H_{00} had 1 hit (2) with H_{01} , 1 hit (5) with H_{02} , 3 hits (2,3,4) with H_{03} , ... and 2 hits (2,3) with H_{06} . By summing, the hit number is then equal to 12.

Reporting these values in equation 9 and 7, the average hit value \hat{H} of our 2D code is 2.27 whereas the Merged MMS/PC is 2.25. Consequently, The P_{er} will increase, but it is necessary to note that this small difference is the price of the increasing cardinality from P2 to P(P-1)². Table [5] show a comparison between the average hit probability of the MMS/PC and MMS/HC versus for w=(P+1)/2 and versus the weight for the MMS/HC.

Table 5. Average hit probability Vs code weight and P

	$\hat{H} / 2L$ MMS/PC for $w_I = 4$	$\hat{H} / 2L$ MMS/HC for $w_1=4$	\hat{H} / 2 L MMS/HC for $w_2=3$
P=7	0.023	0.02325	0.01305
P=15	0.0094	0.009478	0.007256
P=31	0.0043	0.004296	0.003776





(b) Compared MMS/HC BER Vs weights.

From Table [5] and for w=4, the average hit probability of our 2D code is slightly grater than the MMS/PC, so the Per will increase but it is necessary to note that this small difference is the price of the increasing cardinality from P^2 to $P(P-1)^2$ inside any OLAN. Fig.2 (a) shows the Compared BER of MMS/PC and MMS/HC for w1=4, the zooming regions show the slight variation between the two codes. However the Fig.2 (b) show that the second QoS (w2) have an inferior BER than the first one (w1) this is because it contains less frequencies inside a code word, so less probability of hitting.

We can also remarque, that for P=31 and for a BER equal to 10⁻⁹ the permitted simultaneous number of users is bounded by 500, whereas, the theoretical total number of user given by $P(P-1)^2$ is 27900.

In Fig.3 we had taken the assumption that we have 250 users emitting a w1weight application and 250 users emitting a w2 weight application.



Fig.3 BER of composed emission of two QoS

In this case w34, the BER is bounded by the w1 and w2 BER application, the result can be explain that more we have less number of users emitting with w1 and more users emitting with w2, the total BER will decrease than the w1 BER and will approach the w2 BER.

4 Conclusion

This work has presented an extension cardinality of OFFH-CDMA code based in Merged M-sequence/ Hyperbolic code. With the same basic sequence we generate two families of code words having the same length and suitable for supporting two types of QoS for each OLAN user. For a slight difference with the compared code the cardinality of a generated OLAN containing P under groups have passed to $P(P-1)^2$ users inside each one rather than P^2 .

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