Simulation of a Random Perturbation upon the Gyroscopic Stability

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Abstract: - The paper studies by help of Mathcad programs, the behavior of a dynamically tuned gyroscope motion equations. To achieve the simulation we used different random perturbations and studied their effect on the results of the motion equations proving the fact that the stability is assured and also the benefits of using a dynamically tuned gyroscope.

Key-Words: - dynamically tuned gyroscope, random perturbations, Mathcad, simulation, motion equations

1 Introduction
According to the technical documentation concerning the gyroscopic devices, the use of classical gyroscopes does not offer any more possibilities to achieve a significant progress. Their stability against various types of perturbations can be improved only by adding new frames or at the high costs of supplementary control loops. This is the reason why we considered appropriate to turn to the study of dynamically tuned gyroscopes.

![Fig.1](image1)

A dynamically tuned gyroscope consists of a symmetrical or non symmetrical rotor (1), which rotates at a high angular velocity and has such an elastic connection to the base frame, that a gyroscopic torque occurs as a result of its transport motion. This torque will generate the oscillation of the rotor plane with respect to the axis of rotation, representing the rate of the given input angular velocity. In order to decrease the interaction torque between the base frame and the rotor we need to accomplish the dynamical tuning, meaning, the elastic joint becomes a “zero torque” joint for small angular velocities of the base frame. This can be done by choosing such parameters of the torsion bars (3,4), of the inner rotor (2) and of the angular velocity of the rotor, that the torsional rigidity of the torsion bars balances (as an average per rotation) the rotor turn.

As the gyroscope is an important part of all kinds of control devices located on an aircraft, ships or even land vehicles, which may be subjected to various perturbations, the indications provided by the gyroscopic devices should be reliable in spite of these perturbations and of course it is safer to simulate them first on a computer model, simulated in a Mathcad program (fig.2)

![Fig.2](image2)

The Mathcad representation above shows us the two rotors of the dynamically tuned gyroscope, that are connected by elastic joints between them and also with the high speed rotating shaft.

2 Problem Formulation
We need to start with the motion equations of the dynamically tuned gyroscope, that were determined by help of Lagrange equations leading to:
\[
\begin{align*}
I_x^2 \ddot{\chi}_x + c_x \ddot{\chi}_x + (I_y^2 - J_y^2 - J_z^2) \ddot{\psi}_2 + [k_x + \Omega^2 (J_y^2 - J_z^2)] \chi_x = & \\
\Omega (\omega_y \sin \Omega t - \omega_x \cos \Omega t) (J_y^2 - J_z^2 - J_x^2) - \\
(\omega_y \cos \Omega t + \omega_x \sin \Omega t) J_y^2 \\
(I_x^2 + J_z^2) \ddot{\chi}_z + c_z \ddot{\chi}_z + (J_x^2 + J_y^2 + J_z^2) \ddot{\psi}_2 + \\
[k_z + \Omega^2 (J_x^2 - J_y^2)] \chi_z = & \\
\Omega (\omega_y \cos \Omega t + \omega_x \sin \Omega t) (J_x^2 + J_y^2 + J_x^2 - J_y^2 + J_z^2 - J_y^2) \\
(\omega_y \sin \Omega t - \omega_x \cos \Omega t)
\end{align*}
\]

where \( J_x, J_y, J_z \) with the corresponding superscripts represent the moments of inertia of the rotors 1 and 2, \( \chi_2 \) and \( \nu_2 \) the displacement of the rotors with respect to the shaft, \( c \) the damping coefficients, \( k \) the elastic constants and \( \Omega \) the angular velocity.

It is obvious that the motion equations consist of a system of non-homogeneous differential equations, very difficult to approach by classical methods. The non-homogeneity is due to the various perturbations that may affect the gyroscope. Our goal is to prove that the solutions are stable for a random type perturbation.

The characteristic equations solutions are complex conjugated, as we can see in the graphic representation in fig.3.

Then we need to deal with the homogeneous solutions of the motion equations that yield a generally form:

\[ x_0(t) = Z(t) \cdot C \]

where \( Z(t) \) is a matrix of the eigenvalues for the characteristic equations and \( C \) is a matrix of constant values given by the initial conditions.

### 3 Problem Solution

In order to find the general solution, it is necessary to add to the homogeneous solution a particular one corresponding to the perturbation type expressed using a function \( f(t) \).

Because not every perturbation can be expressed exactly in terms of a mathematical function we decided to consider a random function, generated by the computer in order to be able to cover all perturbations possibilities. This time the program requests a different form for the general solution, which was expressed as follows:

\[ xg^{<i>} = xo(i \cdot \tau) + xp^{<i>} \]

where \( xo \) is the homogeneous solution and \( xp \) the particular solution, having the following expression:

\[ \begin{align*}
\sum_{s=0}^{\infty} \phi(i-s) \cdot \tau_1 \cdot f((i-s) \cdot \tau)
\end{align*} \]

The random function \( f \) generated by the computer was represented in fig.4.

The representations of the homogeneous solutions (continuous line) by comparison to the general solutions (dotted line) can be seen in fig.5, first for one of the rotors.

The homogeneous solution is represented by a continuous line, while the perturbed solution was represented by help of a dotted line.

Analyzing the representations we may conclude that the instability caused by a random perturbation is a
little stronger but for a very short time and anyway it
does not exceed some limits.

If we use a matrix projector we are able to find a 3D
representation, including the time t as the third
coordinate, obtaining thus a much more suggestive
image of the gyroscope rotors behaviour during the
random perturbation.

The 3 D representation was presented in fig.7.

It shows that as time passes, the solution obtained
after applying the perturbation given by the function
f, comes always closer to the vicinity of the non
perturbed solution (the homogeneous one).

In the previous figures, for clarity reasons we
represented only the solutions corresponding to one
of the rotors, as the evolution is similar for both of
them.

To prove this, we also can examine fig.8, where the
continuous line represents the homogeneous solution
of the first equation, while the dotted line is the
general solution. The homogeneous solution of the
second equation is a dot-point line, while the
interrupted line is the general solution of the second
equation. The phases plane representation for both
rotors is shown in fig.9.

Both solutions show the tendency of fast damping of
the perturbation and returning close to the vicinity of
the homogeneous solution, proving that the stability
is assured.

Though the chosen function expressing the
perturbation is completely random, the system
proves to be stable, the values of the general
solution (perturbed) will reorganize in a very short
time, around 5-7 seconds, around a slightly different
position from the initial one, but stable. This means
that after a perturbation, the systems finds
immediately a new equilibrium position, regardless
of the perturbation type, which is essential when
using gyroscopic devices in acquiring vital
information.

4 Conclusion

As the classical gyroscopes do not provide the
required stability by themselves but only equipped
with high costs devices, we considered another type of gyroscopes, namely the dynamically tuned gyroscopes. Their motion equations can be determined based on simple mathematical models and proved to be stable. If different types of perturbations occur, it is much easier and less expensive to check first the stability using computer programs, which due to their flexibility allow changing the parameters as the situation requires. After considering several types of perturbations and their influence on the solutions stability, we found that they are stable regardless of the perturbation. Besides, the elastic joint gyroscopes are easier to be manufactured by using electro-erosion, which provides easily coaxial torsion bars and the necessary dimensions. An interesting problem for the future research can be the study of the material influence upon the gyroscope behaviour, the change of material leading to the change of elasticity degree of the torsion bars. We also would have to consider the internal friction phenomena in the torsion bars material, which may influence their deformation, if the material did not have suitable elastic characteristics. The time limits may also be changed, also the geometric and mechanic characteristics of the gyroscope and each time, after minor adjustments the program we set in Mathcad will provide all the necessary information, guiding this way the experimental research.

References: