Optimal determination of partial transmission ratios for four-step helical gearboxes with first and third step double gear-sets for minimal mass of gears

VU NGOC PI
Faculty of Mechanical, Maritime and Materials Engineering
Delft University of Technology
Mekelweg 2, 2628 CD Delft
THE NETHERLANDS

Abstract: - This paper introduces a new study on the applications of optimization and regression analysis techniques for optimal determination of partial transmission ratios of four-step helical gearboxes with first and third step double gear-sets in order to get the minimal mass of gears. In the study, based on the moment equilibrium condition of a mechanic system including gear units and their regular resistance condition, an optimization program for determining the partial ratios of the gearboxes is carried out. From the results of the optimization program, explicit models for calculation of the partial ratios are introduced by using regression analysis. Using these models, the calculation of the partial ratios becomes accurate and simple.

Key-Words: - Gearbox design; Optimal design; Helical gearbox; Transmission ratio.

1 Introduction

It is known that, in optimal gearbox design, the prediction of partial transmission ratios of the gearbox has a very important role. This is because the size, the dimension, the mass, and the cost of a gearbox depend strongly on the partial ratios. For that reason, optimal determination of the partial ratios of gearboxes has been subjected to many researches.

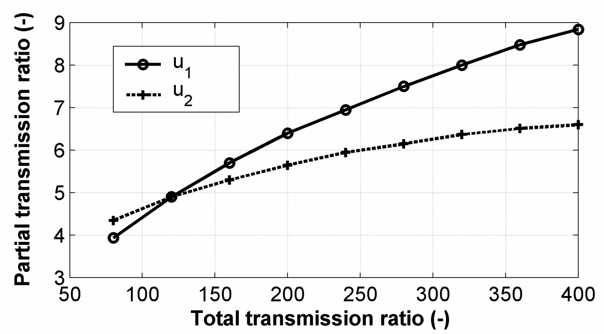

Fig. 1 Determination of partial ratios of three-step helical gearbox [1]

To date, there have been many studies on the prediction of the partial ratio of helical gearboxes. For the gearboxes, there have been several following methods for the determination of the partial ratios:

-By graph method: in this method, the partial ratios were determined graphically. For example, for a three helical gearbox, the partial ratios of steps 1 and 2 (u_1 and u_2, respectively) are predicted from the total transmission ratio u_h (see Figure 1) for getting the minimal mass of the gearbox. The transmission ratio of the third step u_3 is then determined from the total ratio and the ratios of step 1 and 2: $u_3 = u_h / (u_1 \cdot u_2)$. The graph methods have been used to find the partial ratios of two, three and four-step helical gearboxes ([1], [2] and [3]).

-By "practical method": in this method, the partial transmission ratios are predicted based on practical data. Using this method, G. Milou et al. [4] found that the weight of two-step helical gearbox will be minimal if the ratio a_{w2} / a_{w1} is from 1.4 to 1.6 (a_{w1}, a_{w2} are the center distances of the first and the second-step, respectively). From this, the authors proposed the tabulated optimal values of the partial ratios.

-By models: in this method, from the results of optimization problems, models for determination of the partial ratios have been found in order to get different targets, such as for getting minimal gearbox mass of two and three-step gearboxes [5], for minimal mass of gears of three-step gearboxes [6], or for minimal gearbox length of three-step helical gearboxes [7].

From previous studies, it is understandable that there have been many researches on the prediction of the partial ratios for two, three and four-step helical gearboxes. However, there have not been studies on the optimal calculation of the partial ratios for four-step helical gearboxes with first and third-step double gear-sets. This paper presents a new result for optimal determination of partial ratios for four-step helical gearboxes with first and third-step double gear-sets in order to get the minimal mass of gears.

2 Determination of mass of gears

The mass of gears of a four-step helical gearbox with first and third-step double gear-sets can be determined as follows:

$$G = 2 \cdot G_{1a} + G_2 + 2 \cdot G_{3a} + G_4 \qquad (1)$$

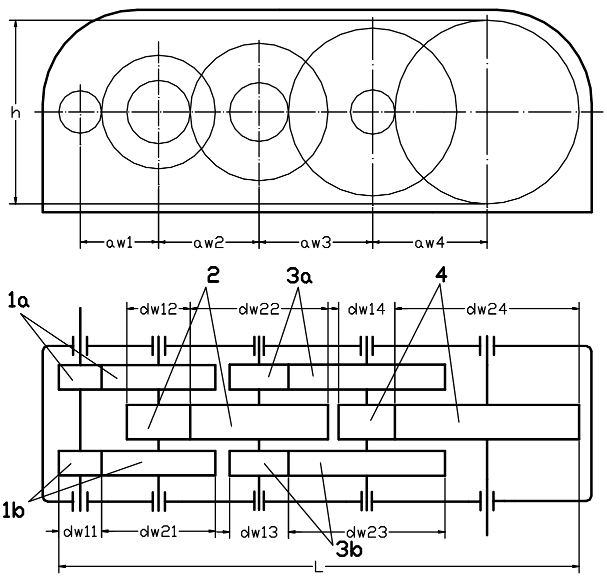

Fig. 2 Calculating schema for four-step helical gearbox with first and third-step double gear-sets

In which, G_{1a}, G_2, G_{3a} and G_4 are the mass of gears of gear unit 1a, unit 2, unit 3a and unit 4, respectively (see Figure 2). G_{1a} can be calculated as follows:

$$G_{1a} = \frac{\pi \cdot \rho \cdot d_{w11}^2 \cdot b_{w1} \cdot (e_1 + e_2 \cdot u_1^2)}{4} \qquad (2)$$

Where, d_{w11} is pitch diameter (m), b_{w1} is the face width (m) of gear unit 1; ρ is the density of the gear material (kg/m^3); e_1, e_2 are volume coefficients of gear 1 and 2, respectively. The volume coefficient of a gear is ratio of the actual volume to the theoretical volume of the gear. For a helical gear unit, we can have e_1=1; e_2=0.6 [2].

With $\psi_{bd1} = b_{w1} / d_{w11}$ is diameter coefficient of the first helical gear step, Equation 2 can be rewritten as follows:

$$G_{1a} = \frac{\pi \cdot \rho \cdot \psi_{bd1} \cdot d_{w11}^3 \cdot (1 + 0.6 \cdot u_1^2)}{4} \qquad (3)$$

For the first helical gear unit, the allowable torque on the driving shaft 1 can be determined by the following equation [7]:

$$[T_{11}] = \frac{\psi_{bd1} \cdot d_{w11}^3 \cdot u_1 \cdot [K_{01}]}{2 \cdot (u_1 + 1)} \qquad (4)$$

Where

$$[K_{01}] = \frac{[\sigma_{H1}]^2}{K_{H1} \cdot \left(Z_{M1} \cdot Z_{H1} \cdot Z_{\varepsilon 1} \right)^2}$$

In the above equations, Z_{M1}, Z_{H1}, $Z_{\varepsilon 1}$ are coefficients which consider the effects of the gear material, contact surface shape, and contact ratio of the first gear unit when calculate the pitting resistance; $[\sigma_{H1}]$ is allowable contact stresses of the first helical gear unit.

From Equation 4 we have

$$\psi_{bd1} \cdot d_{w11}^3 = \frac{2 \cdot (u_1 + 1) \cdot [T_{11}]}{u_1 \cdot [K_{01}]} \qquad (5)$$

Substituting (5) into (3) we get

$$G_{1a} = \frac{2 \cdot \pi \cdot \rho \cdot [T_{11}] \cdot (u_1 + 1) \cdot (1 + 0.6 \cdot u_1^2)}{4 \cdot [K_{01}] \cdot u_1} \qquad (6)$$

Calculating in same way, the following equations were found for the second, the third and the fourth steps:

$$G_2 = \frac{2 \cdot \pi \cdot \rho \cdot [T_{12}] \cdot (u_2 + 1) \cdot (1 + 0.6 \cdot u_2^2)}{4 \cdot [K_{02}] \cdot u_2} \qquad (7)$$

$$G_{3a} = \frac{2 \cdot \pi \cdot \rho \cdot [T_{13}] \cdot (u_3 + 1) \cdot (1 + 0.6 \cdot u_3^2)}{4 \cdot [K_{03}] \cdot u_3} \qquad (8)$$

ISBN: 978-960-6766-67-1

ISSN: 1790-2769

$$G_4 = \frac{2 \cdot \pi \cdot \rho \cdot [T_{14}] \cdot (u_4 + 1) \cdot (1 + 0.6 \cdot u_4^2)}{4 \cdot [K_{04}] \cdot u_4} \quad (9)$$

Based on the moment equilibrium condition of the mechanic system including the gear units and the regular resistance condition of the system we have:

$$\frac{T_r}{2 \cdot T_{11}} = \frac{[T_r]}{2 \cdot [T_{11}]} = u_1 \cdot u_2 \cdot u_3 \cdot u_4 \cdot \eta_{brt}^4 \cdot \eta_o^4 \quad (10)$$

In the above equation, T_r and $[T_r]$ are torque and allowable torque on the output shaft (N/m); T_{11} and $[T_{11}]$ are torques and allowable torque of the and driving shaft of gear unit 1 (N/m); η_{brt} is helical gear transmission efficiency (η_{brt} is from 0.96 to 0.98 [7]); η_o is transmission efficiency of a pair of rolling bearing (η_o is from 0.99 to 0.995 [7]).

Choosing $\eta_{brt} = 0.97$, $\eta_o = 0.992$ and substituting them into (10) we have

$$[T_{11}] = \frac{[T_r]}{1.7146 \cdot u_1 \cdot u_2 \cdot u_3 \cdot u_4} \quad (11)$$

Substituting (11) into (6) we have

$$G_{1a} = \frac{2 \cdot \pi \cdot \rho \cdot [T_r]}{4 \cdot [K_{01}]} \cdot \frac{(u_1 + 1) \cdot (1 + 0.6 \cdot u_1^2)}{1.7146 \cdot u_1^2 \cdot u_2 \cdot u_3 \cdot u_4} \quad (12)$$

For the second helical gear unit we also have:

$$\frac{T_r}{T_{12}} = \frac{[T_r]}{[T_{12}]} = u_2 \cdot u_3 \cdot u_4 \cdot \eta_{brt}^3 \cdot \eta_o^3 \quad (13)$$

With $\eta_{brt} = 0.97$ and $\eta_o = 0.992$ we can get $[T_{12}]$ from Equation 13:

$$[T_{12}] = \frac{[T_r]}{0.8909 \cdot u_2 \cdot u_3 \cdot u_4} \quad (14)$$

From Equations 7 and 14 we have the following equation:

$$G_2 = \frac{2 \cdot \pi \cdot \rho \cdot [T_r]}{4 \cdot [K_{02}]} \cdot \frac{(u_2 + 1) \cdot (1 + 0.6 \cdot u_2^2)}{0.8909 \cdot u_2^2 \cdot u_3 \cdot u_4} \quad (15)$$

In exactly similar manner, we can get the mass of gears of unit 3a and unit 4 as follows:

$$G_{3a} = \frac{2 \cdot \pi \cdot \rho \cdot [T_r]}{4 \cdot [K_{03}]} \cdot \frac{(u_3 + 1) \cdot (1 + 0.6 \cdot u_3^2)}{1.8518 \cdot u_3^2 \cdot u_4} \quad (16)$$

$$G_4 = \frac{2 \cdot \pi \cdot \rho \cdot [T_r]}{4 \cdot [K_{04}]} \cdot \frac{(u_4 + 1) \cdot (1 + 0.6 \cdot u_4^2)}{0.9622 \cdot u_4^2} \quad (17)$$

Substituting (12), (15), (16) and (17) into (1) with the note that $u_1 = u_h / (u_2 \cdot u_3 \cdot u_4)$, we have the following equation for the calculation of the mass of gears:

$$G = \frac{2 \cdot \pi \cdot \rho \cdot [T_r]}{4 \cdot [K_{01}]} \cdot \left\{ \frac{(u_h + u_2 \cdot u_3 \cdot u_4) \cdot \left[1 + 0.6 \cdot u_h^2 / \left(u_2^2 \cdot u_3^2 \cdot u_4^2\right)\right]}{1.7146 \cdot u_h^2} + \frac{(u_2 + 1) \cdot (1 + 0.6 \cdot u_2^2)}{0.8909 \cdot k_{C2} \cdot u_2^2 \cdot u_3 \cdot u_4} + \frac{(u_3 + 1) \cdot (1 + 0.6 \cdot u_3^2)}{1.8518 \cdot k_{C3} \cdot u_3^2 \cdot u_4} + \frac{(u_4 + 1) \cdot (1 + 0.6 \cdot u_4^2)}{0.9622 \cdot k_{C4} \cdot u_4^2} \right\} \quad (18)$$

Where, $k_{C2} = [K_{02}]/[K_{01}]$, $k_{C3} = [K_{03}]/[K_{01}]$ and $k_{C4} = [K_{04}]/[K_{01}]$.

3 Optimization problem and results

3.1 Optimization problem

From Equation 18, the optimization problem for determining the partial transmission ratio for getting the minimal mass of gears can be written as follows:

The objective function is:

$$\min G = f(u_h; u_2; u_3; u_4) \quad (19)$$

With the following constraints:

$$u_{h\min} \le u_h \le u_{h\max}$$

$$u_{2\min} \le u_2 \le u_{2\max}$$

$$u_{3\min} \le u_3 \le u_{3\max}$$

ISBN: 978-960-6766-67-1

ISSN: 1790-2769

$$u_{4\min} \le u_4 \le u_{4\max} \tag{20}$$

$$k_{C2\min} \le k_{C2} \le k_{C2\max}$$

$$k_{C3\min} \le k_{C3} \le k_{C3\max}$$

$$k_{C4\min} \le k_{C4} \le k_{C4\max}$$

To solve the above optimization problem, a computer program was built. The following data were used in the program: u_2, u_3 and u_4 were from 1 to 9 [1], k_{C2}, k_{C3} and k_{C4} are from 1 to 1.3 [7] and u_h was from 50 to 400.

3.2 Results and discussions

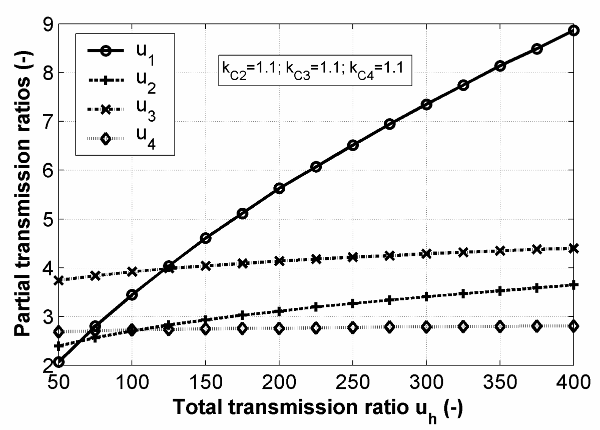

Fig. 3: Partial transmission ratios versus the total transmission ratio

The relation between the partial transmission ratios and the total transmission ratio when the coefficients k_{C2}, k_{C3} and k_{C4} equal 1.1 is shown in Figure 3. It can be seen that the larger of the total ratio u_h the larger of the partial ratios. Also, the increase of u_1 when the total ratio increases is much larger than that of u_4. This is because with the increase of the total ratio u_h, the torque on the output shaft T_r is much larger than that on the driving shaft of the first gear unit T_{11}. Therefore, the increase of the partial ratio u_4 should be much smaller than that of u_1 in order to reduce the mass of the gears.

From the results of the optimization program, regression analysis was carried out and the following models were found for determining the optimal values of the partial ratios of the second, the third and the fourth steps:

$$u_2 \approx 0.0969 \cdot \frac{k_{C2}^{0.9547}}{k_{C3}^{0.8039} \cdot k_{C4}^{0.1128}} \cdot u_3^{2.0605} \cdot u_4^{0.3553} \cdot u_h^{0.0334} \tag{21}$$

$$u_3 \approx 2.5041 \cdot \frac{k_{C3}^{0.4274}}{k_{C2}^{0.3962} \cdot k_{C4}^{0.0228}} \cdot u_2^{0.3423} \cdot u_4^{0.0703} \cdot u_h^{0.008} \tag{22}$$

$$u_4 \approx 2.2918 \cdot \frac{k_{C4}^{0.2883}}{k_{C2}^{0.0761} \cdot k_{C3}^{0.2101}} \cdot u_2^{0.0623} \cdot u_3^{0.0743} \cdot u_h^{0.0024} \tag{23}$$

The above regression models fit very well with the data. The coefficients of determination for the models 21, 22 and 23 were $R^2 = 0.9991$, $R^2 = 0.9994$ and $R^2 = 0.9979$, respectively.

Equations 21, 22 and 23 are used to determine the transmission partial ratios u_2, u_3 and u_4 of the second, the third and the fourth helical gear units. After finding u_2, u_3 and u_4, the transmission ratio of the first gear unit u_1 is calculated as follows:

$$u_1 = \frac{u_h}{u_2 \cdot u_3 \cdot u_4} \tag{24}$$

4 Conclusion

It can be concluded that the minimal mass of gears of four-step helical gearboxes with first and third-step double gear-sets can be obtained by optimal splitting the total transmission ratio.

Based on the results of the optimization problem, models for determination of the optimal partial ratios of three-step helical gearboxes with first and third-step double gear-sets for getting the minimal mass of gears have been proposed.

By using explicit models, the partial ratios of the gearboxes can be predicted accurately and simply.

References:

[1] V.N. Kudreavtev; I.A. Gierzaves; E.G. Glukharev, *Design and calculus of gearboxes* (in Russian), Mashinostroenie Publishing, Sankt Petersburg, 1971.

[2] Trinh Chat, Optimal calculation the total transmission ratio of helical gear units (in Vietnamese), *Scientific Conference of Hanoi University of Technology* (1996) 74-79.

[3] A.N. Petrovski, B.A. Sapiro, N.K. Saphonova, About optimal problem for multi-step gearboxes

(in Russian),*Vestnik Mashinostroenie*, No. 10 (1987) 13-14.

[4] G. Milou; G. Dobre; F. Visa; H. Vitila, Optimal Design of Two Step Gear Units, regarding the Main Parameters, *VDI Berichte* No 1230, 1996, 227-244.

[5] Romhild I. , Linke H., Gezielte Auslegung Von Zahnradgetrieben mit minimaler Masse auf der Basis neuer Berechnungsverfahren, *Konstruktion* 44 (1992) 229- 236.

[6] Vu Ngoc Pi, Nguyen Dang Binh, Vu Quy Dac, Phan Quang The, Optimal calculation of total transmission ratio of three-step helical gearboxes for minimum mass of gears *(In Vietnamese)*, *Journal of Science and Technology of 6 Engineering Universities*, Vol. 55 (2006) 91-93.

[7] Vu Ngoc Pi, A new study on optimal calculation of partial transmission ratio of three-step helical reducers, *Accepted paper for the 3rd IASME / WSEAS International Conference on Continuum Mechanics*, Cambridge, UK, February 23-25, 2008.

[8] Trinh Chat, Le Van Uyen, *Design and calculus of Mechanical Transmissions* (in Vietnamese), Educational Republishing House, Hanoi, 1998.