Self-Tuning Fuzzy Sliding-Mode Control for Time-Delay Chaotic Systems

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Abstract: This paper presents a new self-tuning control scheme for a class of time-delay chaotic systems. It is implemented by using fuzzy sliding-mode control with self-tuning method. The adjustment of the membership functions is performed within a stable range derived from a Lyapunov stability analysis. The adaptation laws are derived using a Lyapunov stability analysis, so that both system tracking stability and error convergence can be guaranteed in the closed-loop system.

Key-Words: Fuzzy, Sliding-Mode, Chaotic, Time-Delay

1 Introduction

Time delays can be found in various engineering systems such as chemical processes, physical systems, biological systems, social systems, and chaotic systems [1]. Over the past few years many chaotic systems have been developed and analyzed [2-8]. Chaotic systems are essentially nonlinear systems having the following properties of being broadband, noise-like, and difficult to predict [22]. A fundamental characteristic of a chaotic system is its extreme sensitivity to the initial conditions; this means that small differences in the initial state can lead to extraordinary differences in the system state. Recently, an adaptive method [3] has been applied to deal with the problem of controlling uncertain chaotic systems. However, the presently papers are mainly to attention controlling chaotic systems without time delay.

On the other hand, intelligent modeling and control methodologies based on fuzzy logic and neural networks [4-6,23] have also been used. Some researchers [9-11] have investigated the similar between a simple FLC and an SMC with a boundary layer. Hwang and Lin [9], and Palm [11] combined the attractive features of fuzzy logic control (FLC) and sliding-mode control (SMC). The developed and proposed a fuzzy sliding-mode controller (FSMC) for a second order nonlinear system. They proved that the chattering phenomenon inherent to the SMC can be improved by using FSMC and that the ultimate error bound of states can be obtained asymptotically even when system dynamic uncertainties exist. Choi, et al. [14] used the signed-distance fuzzy logic control (SDFLC) technique (equivalent the pseudo SMC) to improve FSMC performance. In this way, the number of fuzzy rules can be greatly reduced, although the tuning of the rule base is still done in a trial-and-error manner.

In order overcome this problem, a self-tuning fuzzy sliding-mode control (STFSMC) design is used greatly reduced the number of fuzzy rules making tuning of the rules easier. With this approach, the fuzzy rules can be automatically adjusted by an adaptive law until satisfactory system response is achieved.

This paper discusses a self-tuning fuzzy controller design method for time-delay chaotic systems in the presence of completely unknown nonlinearities and disturbances. The proposed scheme combines the advantages of the adaptive control, fuzzy control and sliding-mode control strategies without exact system model information. It has on-line learning ability to deal with the parametric uncertainty and disturbance by adjusting the control parameters.

In the proposed STFSMC system the main tracking controllers is a fuzzy sliding-mode controller which is used to approximate an ideal controller. We utilize an adaptive process to adjust the parameters of the STFSMC. Under the proposed control law, it is guaranteed that the system orbits of time-delay chaotic systems can be asymptotically driven to arbitrarily desired trajectories even with uncertainties. A Duffing-Holmes time-delay system is used as an illustration of the effectiveness of the proposed control design method.

2 Problem Formulation

Consider a class of chaotic systems described as

\[ \dot{X}(t) = FX(t) + G(X(t)) + K_i(\dot{X}(t + \tau) - X(t)) \] (1)

where \( F \in \mathbb{R}^{n \times n} \) and \( G(X(t)) \) is a continuous nonlinear function, and \( X(t) \in \mathbb{R}^n \) is the state vector. Then a time-delay chaotic system can be
obtained as follows[25]:
\[ \dot{X}(t) = FX(t) + G(X(t)) \]  
and
\[ \dot{X}(t + \tau) = F X(t + \tau) + G(X(t + \tau)) + K_1(X(t) - \dot{X}(t + \tau)) \]

where \( \dot{X}(t) \) is the \( n \)-dimensional state vector of the response system, \( \tau \) is a finite time-delay constant, \( G: \mathbb{R}^n \rightarrow \mathbb{R}^n \) is a continuous function, \( K_1 \) and \( K_2 \) are diagonal matrices. Assume that

\[ G(X(t + \tau)) - G(X(t)) = H_{X,X} X(t + \tau) - X(t) \]

where \( H_{X,X} \) is a matrix with elements depending on \( X(t) \) and \( X(t + \tau) \). Let \( e(t) = X(t + \tau) - X(t) \), then by subtracting (2) to (3), we can take the error dynamical system:

\[ e(t) = (F + H_{X,X} - (K_1 + K_2))e \]

In order to realize the chaos synchronization with time-delay, we should select the suitable parameter matrices \( K_1, K_2 \) which can mak

\[ \lim_{t \to \infty} e(t) = 0 \]

Since it is difficult even impossible to measure all the state variables in practical achieves, here the state variables are selected to define a sliding surface on the phase plane

\[ s(t) = e^{(n-1)} k_{n-2} e^{(n-2)} + \cdots + k_1 e + k_0 = 0 \]

This sliding variable \( s \) will be used as the input signal for establishing a fuzzy logic control system to approximate the ideal control law \( u_{eq} \). With this perfect control law, the closed loop control system demonstrates an asymptotical stable dynamic behavior [17]

\[ s(t) + ks(t) = 0 \]

Since \( k \) is a real value, the sliding surface variable \( s \) will gradually converge to zero. Based on the definition of sliding surface variable \( s \) in (8), the selected system-states output errors will gradually converge to zero, too. The control law may have certain difference with the ideal control law \( u \), then the following equation can be obtained:

\[ s(t) = -ks(t) + [u_{FLC}(t) - u(t)] \]

Multiplying both sides of the aforementioned equation with gives

\[ s(t) s(t) = s(t)[-ks(t) + [u_{FLC}(t) - u(t)]] \]

Based on the Lyapunov theorem, the sliding surface reaching condition is \( s < 0 \). If a control input \( u_{FLC}(t) \) can be chosen to satisfy this reaching condition, the control system will be converge to the origin of the phase plane.

### 3 Self-Tuning Fuzzy Sliding-Mode Control

In Fig. 1, let \( V(e,e) \) be the point of intersection of the switching line and the line perpendicular to the switching line from an operating point \( W(e_1,e_1) \). Next evaluate \( d \).

The distance between \( V(e,e) \) and \( W(e_1,e_1) \) can be expressed as

\[ d = \frac{|e+ke|}{\sqrt{1+k^2}} \]

so without loss of generality, Eq. (11) can be rewritten as

\[ s = \begin{cases} 1 & \text{for } s > 0, \\ 0 & \text{for } s = 0, \\ -1 & \text{for } s < 0. \end{cases} \]

Now, we choose a Lyapunov function

\[ V(e,e) = \frac{e+ke}{\sqrt{1+k^2}} \]

\[ s = \begin{cases} 1 & \text{for } s > 0, \\ 0 & \text{for } s = 0, \\ -1 & \text{for } s < 0. \end{cases} \]
\[ V = \frac{1}{2} d_i^2, \quad (15) \]

Then
\[ \dot{V} = d_i \dot{d}_i = \frac{s}{\sqrt{1+k^2}} \cdot \frac{\dot{s}}{\sqrt{1+k^2}} = \frac{ss}{1+k^2} \quad (16) \]

Hence, it is seen that if \( s > 0 \), then \( d_i > 0 \); decreasing \( u \) will make \( ss \) decrease so that \( \dot{V} < 0 \) and that if \( s < 0 \), then \( d_i < 0 \); increasing \( u \) will make \( ss \) decrease so that \( \dot{V} < 0 \). Thus we can ensure that the system is asymptotically stable. The fuzzy rule table can be established in one-dimensional space \( d_i \) instead of a two-dimensional space of \( e \) and \( e \).

From the above relation, we can conclude that \( u \propto -d_i \). \( (17) \)

Hence, the fuzzy rule table can be established on a one-dimensional space of \( d_i \) instead of a two-dimensional space of \( e \) and \( e \). The control action can be determined by \( d_i \) only.

The triangular-typed functions and singletons are used to define the membership functions of IF-part and THEN-part, which are depicted in Figs. 2, respectively. The defuzzification of the control output is accomplished by the method of center-of-gravity \( (16) \)

\[ u = \sum_{j=1}^{n} \mu_j \theta_j \quad \sum_{j=1}^{n} \mu_j \]

where \( \mu_j \) is the firing weight of the \( j \)th rule.

\[ (18) \]

\[ \begin{array}{cccccc}
NB & NS & ZE & PS & PB \\
1 & & & & \\
\end{array} \quad (a) \]

\[ \begin{array}{cccccc}
NB & NS & ZE & PS & PB \\
-1 & -0.5 & 0 & 0.5 & 1 \\
\end{array} \quad (b) \]

The on-line parameters tuning algorithm adjusts the consequent parameters for system control. The modification rule is derived from the chain rule to decrease the value of \( d_i \dot{d}_i \) with respect to \( \theta_j \). The equation for the modification of the consequent parameter is

\[ \dot{\theta}_j = -\gamma \frac{\partial d_i(t) \dot{d}_i(t)}{\partial u} \frac{\partial u}{\partial \theta_j} = \gamma d_i(t) \phi_j(t) \quad (19) \]

where \( \gamma \) is the proper rate value. The defuzzification membership functions can now be regulated directly through the modification of consequence parameter \( \dot{\theta}_j \). The on-line modification rule also has the effect of improving the stability and increasing the speed at which the sliding surface can be reached.

\[ \begin{array}{ll}
\text{IF} & s(t) > p \quad \text{then} \quad \dot{\theta}_j = \gamma d_i(t) \phi_j - \nu d_i \theta_j \\
\text{else} & \dot{\theta}_j = 0. \quad (20) \\
\end{array} \]

where \( p \) is real number.

The parameters update \( \theta_j \) and the robust term \( u_g \)

\[ \dot{\theta}_j = \gamma d_i(t) \phi_j - \nu d_i \theta_j \quad (21) \]

\[ u_g = -g_i \tanh(d_i) - g_2 d_i \quad (22) \]

where \( \lambda, \nu, g_i, \) and \( g_2 \) are positive value.

A optimal consequent parameters vector \( \theta^* \) exist for STFSMC, which makes the control force \( u(d_i, \theta^*) \) approximate the ideal equivalent control law with an error smaller than \( \xi \)

\[ \sup \{ u(d_i, \theta^*) - u_{eq} \} \leq \xi, \quad (23) \]

and

\[ u_{eq} = \theta^* \phi + \xi. \quad (24) \]

Define \( \zeta = \theta^* - \theta \) as the approximate error vector between the optimal value and the current estimated value of the fuzzy consequent parameters.

Choose the Lyapunov function

\[ V = \frac{\sqrt{1+k^2}}{2} d_i^2 + \frac{1}{2\gamma} \zeta^T \zeta, \quad (25) \]
where \( \gamma \) is a positive constant. Then, the variation of this function (23) with respect to time is now
\[
\dot{V} = \frac{1}{\gamma} \left[ \gamma k - \left( \xi T + \gamma \right) \right]
\]
\[
= d_{ss} \left[ -ks + (\xi T T T s T s - \phi_i + \phi_j) \right] - \xi r d_i(t) \phi_j(t) - \frac{\partial d_i(t) \theta_j(t)}{\gamma} d_{ss} \theta_j(t)
\]
\[
\geq |d_{ss}| \left[ -k \mid s \mid + \xi \right]
\]
(26)

If \( |s| \geq \beta / k \), then \( \dot{V} \leq 0 \) which implies the presence of sliding surface. This means that Lyapunov function will be decrease gradually and the sliding surface variable \( s \) will be converge into the \( s = 0 \) in the phase plane [21].

4 Computer Simulation Results

The system of interest here is a Duffing-Holmes with time-delay system, which can be described by
\[
x_1(t) = x_2(t)
\]
\[
x_2(t) = -p_1 x_2(t - \tau) - p_2 x_2(t - \tau) - x_1(t)
\]
\[
+ q \cos(\omega_0 t) + u(t).
\]
(27)

Here the objective is to use STFSMC to let system output trace the desired \( \sin(\omega_0 t) \) input. We simply choose \( k = 1.8 \). The parameter value of \( p_1 = -1 \), \( p_2 = 0.25 \), \( q = 7 \), \( \omega_0 = 1.0 \) and \( \tau = 0.5 \). The Duffing-Holmes oscillator displays chaotic behavior [6,8]. The control aim is to drive the time-delay chaotic system to the following trajectory
\[
x_d(t) = M \sin(\omega_0 t).
\]
(28)

Obviously, the desired trajectory \( x_d(t) \) with \( M = 1 \), \( \omega_0 = 1.1 \) does not belong to the embedded orbits of the strange attractor. The simulation results for the Duffing-Holmes chaotic system, given the initial condition \( x_1(0) = 1 \) and \( x_2(0) = 1 \). These simulations exhibit the time-delay chaotic responses that compare the SMC and STFSMC system are shown in Fig. 3-Fig. 10. From the simulation results, the proposed method is applied to a time-delay nonlinear chaotic system and exhibit that the proposed method cannot only stabilize the chaos systems, but has strong robustness. The stability of the designed closed-loop system is thus proved.
5 Conclusion

A self-tuning fuzzy sliding-mode controller for Duffing-Holmes with time-delay system is proposed. The important characteristics for employing the STFSMC are:
(1) mathematical model of the system is without accurate;
(2) its adaptive law can reduce dependency on expertise;
(3) the stability of the closed-loop system is guaranteed by means of the Lyapunov stability theory; the control parameters and actions of a robust controller can be adjusted in response to parameter uncertainties.

References:


