Optimal Design of the Linear Delta Robot for Prescribed Cuboid Dexterous Workspace based on performance chart

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Abstract: - Determine an optimal set of design parameter of PR whose DW fits a prescribed workspace as closely as possible is an important and foremost design task before manufacturing. In this paper, an optimal design method of a linear Delta robot (LDR) to obtain the prescribed cuboid dexterous workspace (PCDW) is proposed. The optical algorithms are based on the concept of performance chart. The performance chart shows the relationship between a criterion and design parameters graphically and globally. The kinematic problem is analyzed in brief to determine the design parameters and their relation. Two algorithms are designed to determine the maximal inscribed rectangle of dexterous workspace in the O-xy plane and plot the performance chart. As an applying example, a design result of the LDR with a prescribed cuboid dexterous workspace is presented. The optical results shown that every corresponding maximal inscribed rectangle can be obtained for every given RATE by the algorithm and the error of RATE is less than 0.05. The method and the results of this paper are very useful for the design and comparison of the parallel robot.

Key-Words: - Parallel Robot, Cuboid Dexterous Workspace, Optimal Design, performance chart

1 Introduction

The parallel robots (PRs) have a number of advantages over traditional serial robots due to their particular architecture [1]. Such mechanisms have attracted more and more researchers’ attention in machining applications. However, there are some classic disadvantages associated with the PRs, which have prevented their application. The uppermost disadvantage is its small and irregular-shaped workspace, and poor dexterity. To overcome this disadvantage, Lou et al. [2-4] proposed methods to find a set of parameters of a PR with a maximal workspace which should satisfy two requirements simultaneously: (i) the prescribed dexterity, and (ii) a regular shape.

But how to design a PR to fit a prescribed workspace as closely as possible [10~18] became more and more important for industrial application, and that is the foundation of produce a more compact and economical PR. This problem was initially proposed by Gosselin and Guillot [10]. Merlet [11] has paid attention to the optimal problem of design parameters of the Gough-type PR to fit prescribed workspace. Boudreau and Gosselin [12] determined some parameters of the planar PR whose workspace approximates the prescribed one by using a genetic algorithm. Recently, a combined method with the genetic algorithm and fuzzy logic was proposed by Laribi et al. [13] to obtain parameters of the Delta robot having the approximate prescribed cuboid workspace. Kosinska et al. [14] presented a numerical method for the determination of the parameters of the Delta robot having the prescribed cuboid and well-conditioned workspace. Zhao et al. [16] proposed a method to minimize length of leg of PRs for a desired cylindrical dexterous workspace. However, the aforementioned methods have three common disadvantages: (i) the objective function involve a nonlinear constrain, and (ii) the reasonability of the PR to the prescribed operation task workspace, and (iii) these methods can provide an optimal result, but the consumer cannot know how optimal the result is; at least, they did not give us any proof about it [17]. The design method in this paper is based on the concept of performance chart, which can show the relationship between a criterion and design parameters graphically and globally [18]. Liu et al. [17, 18] proposed an optimal kinematic design method to determine the geometric parameters of a three translational DoFs parallel manipulator. He

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established a geometric design space based on the involved geometric parameters, which can embody all basic similarity manipulators. But his task workspace is a cylinder that cannot meet the industrial requirements as most industrial 3-axis machine tools have a cuboid workspace. When prescribed workspace is a cuboid, the problem became more difficult to deal with.

2 Kinematic Problem of a Linear Delta Robot

2.1 The Architecture and Inverse Kinematics

The LDR presented in this paper and its CAD model is shown in Fig.1 and its geometry is shown in Fig.2, which is modified from the parallel manipulators in [7]. All passive joints are revolute pair, and with three prismatic actuated joints in this mechanism. It is different with the general LDR. Because three points A₁, A₂, and A₃, which are points of intersection between three prismatic actuated joints and the base, make up an isosceles right triangle. But it is a regular triangle one in other LDR, i.e. in [18]. This LDR reveals still pure translational motion.

Attach a Cartesian coordinate system O-xyz to the base. Its origin O is located in the middle point of the +z-axis is normal to the base and the +x-axis is point to OA₁. Similarly, another moving Cartesian coordinate system P-uvw is located in the middle point of line B₁B₂ of the moving platform. Suppose that the radii of the base and the moving platform are L₁ and L₃ respectively, i.e. \( OA_1 = OA_2 = OA_3 = L_1 \), \( PB_1 = PB_2 = PB_3 = L_3 \), and the length of the parallelogram joint is equal to \( L_2 \). The \( \phi_i = (i - 1)\pi / 2 (i = 1, 2, 3) \) denotes the angle from x-axis to line \( OA_i \).

In order to determine the workspace of the LDR, it’s necessary to analyse the inverse kinematics problem. According to the geometry, a loop-closure of each limb is holding with respect to the O-xyz

\[
OP = OA_i + A_iD_i + D_iB_i + B_iP_i, \quad i = 1, 2, 3
\]

(1)

The coordinate of the point P in the moving platform can be written as \( u = [x, y, z]^T \), and \( v = [d_1, d_2, d_3]^T \) denote the actuated joints variables.

Transform vector equations (1) into three scalar equations:

\[
(x - x_i)^2 + (y - y_i)^2 + (z - d_i)^2 = L_i^2, \quad i = 1, 2, 3
\]

(2)

where \( x_i = R\cos\phi_i, \ y_i = R\sin\phi_i \), and \( R = L_1 - L_3 \). The inverse kinematic solution of the LDR with respect to the assembly mode in Fig. 1 can be obtained as

\[
d_i = z + \sqrt{L_i^2 - (x - x_i)^2 - (y - y_i)^2}, \quad i = 1, 2, 3
\]

(3)

Thus, if the coordinate \( u \) of the point P in the moving platform is given, all passive revolute joints variables are determined uniquely, respectively.

2.2 Jacobian Matrix and Singularity

The Jacobian matrix is defined as the matrix that maps the relation between the velocity of the moving platform and the actuated joint rates. Equations (2) can be differentiated with respect to time to obtain the velocity equations

\[
(z - d_i)\dot{d}_i = (x - x_i)\dot{x} + (y - y_i)\dot{y} + (z - d_i)\dot{z}
\]

(4)

It can be rearranged into matrix form

\[
Av = Bu
\]

(5)

where \( \dot{v} = [\dot{d}_1, \dot{d}_2, \dot{d}_3]^T \) denotes the actuated joints rates and \( u = [\dot{x}, \dot{y}, \dot{z}]^T \) denotes the vector of output velocities. The matrixes A and B are respectively written as
\[ A = \text{diag}(z - d_1, z - d_2, z - d_3), \]
\[ B = \begin{bmatrix} x - x_1 & y - y_1 & z - d_1 \\ x - x_2 & y - y_2 & z - d_2 \\ x - x_3 & y - y_3 & z - d_3 \end{bmatrix} \]  
\[ J = \begin{bmatrix} x - R \cos \phi_1 & y - R \sin \phi_1 & 1 \\ q_1 & q_1 & 1 \\ x - R \cos \phi_2 & y - R \sin \phi_2 & 1 \\ q_2 & q_2 & 1 \\ x - R \cos \phi_3 & y - R \sin \phi_3 & 1 \\ q_3 & q_3 & 1 \end{bmatrix} \]  
\[ k(J) = \|J\|^{-1}, \text{ and } \eta(J) = \frac{1}{k(J)} \]

The reciprocal of the condition number of the Jacobian matrix \( \eta \) is defined as the local condition index (LCI) to evaluate the control accuracy, dexterity and isotropy of a manipulator, which can be written as

\[ R = |L_1 - L_3| = L_2, \]  
which means that three links are all in a plane parallel to the \( O-xy \) plane. To avoid this singularity, let \( R = |L_1 - L_3| \neq L_2 \).

Therefore, in order to avoid these singularities with respect to the assembly mode in Fig.2, the relation of the design parameters \( R \) and \( L_2 \) should be

\[ R = L_1 - L_3 > 0, \text{ and } R < L_2 \]  

3 Determining Workspace
3.1 The Reachable and Dexterous Workspace in O-xy Plane

The reachable workspace (RW) is defined as the set of points where the point P in the moving platform can be accessible no matter what the orientation is [22]. When the various physical constraints are considered, its RW can be defined as

\[ W_R = \{ u \in \mathbb{R}^3 | f(u, \alpha) \in [u_{\min}, u_{\max}], g(u, \alpha) \in [w_{\min}, w_{\max}] \} \]

where \( f(u, \alpha) \) and \( g(u, \alpha) \) denote the inverse kinematic maps of actuated and passive joints respectively. The intervals \([u_{\min}, u_{\max}]\) and \([w_{\min}, w_{\max}]\) denote respectively the constraints of the actuated joints and the passive joints. [21]

The dexterous workspace (DW) is defined as a subset of WR, i.e., the set of points satisfies the prescribed dexterity. The DW to the prescribed \( \eta \) is defined as

\[ W_D = \left\{ u \in W_R \mid \eta^{\min} \leq \eta(J) \right\} \]

The algorithms of searching the boundaries of the DW are proposed and discussed in detail in paper [21]. If a set of design parameters and \( \eta^{\min} \) are given, a unique RW and DW are obtained by means of these algorithms. The DWs with respect to several different \( \eta^{\min} \) are shown in Fig.3. From the Fig.3 known that the DW of LDR is symmetric about the y-axis.
Algorithm 1: Determining the maximal inscribed rectangle DW (MIRDW)
S1: Determine the boundary of the RW and DW in the O-xy plane using the algorithm proposed in [25].
S2: Select a suitable positive real number as neighborhood radius $\varepsilon$. Then, the point $u0 = [0, y_{min}+\varepsilon]$ as the search starting point, and the search times is calculated with $maxi = \frac{[y_{max} - y_{min}]}{\varepsilon} - 2$.
Here, $y_{min}$ and $y_{max}$ is the minimal and maximal coordinates of the boundary points of DW in O-xy plane on the y-axis respectively.
S3: Let $\theta = \frac{\pi}{360}$, the loop times is defined as $maxloop = \frac{2\pi}{\theta}$. The coordinates of the boundary points of DW in O-xy plane are expressed as $(x(k), y(j))$.

\[
\text{for(}i=1:1:\text{maxi)} \\
\quad xci=0; yci= y_{min}+\varepsilon; \\
\text{for(}j=1:1:\text{maxloop)} \\
\quad \text{if } |yci - y(j)| < \varepsilon/2 \\
\quad \quad x=x(j); \\
\quad \quad \text{break; } \\
\quad \text{end; } \\
\text{end } \\
\text{for(}k=1:1:\text{maxloop)} \\
\quad \text{if } |x - x(k)| < \varepsilon/2 \\
\quad \quad y=y(k); \\
\quad \quad \text{break; } \\
\quad \text{end; } \\
\text{end; }
\]

Considered that the DW is symmetry to y axis, its width $W_i = 2|x|$ and its length $L_i = |yci - y|$ can be calculated respectively, so an inscribed rectangle of the DW is found and its area is calculated from $S_i = 2|x||yci - y|$. At the same time, its width length ratio RATE can be calculated from $RATE = W_i / L_i$, where $W_i = 2|x|, L_i = |yci - y|$.

4 The Optimal Procedure
4.1 Normalization on the geometric parameters
For the size information of PCDW with specified shape and dexterity is given as length, width, and height, the PCDW can be depicted with a triple positive real $(L_p, W_p, H_p)$. As we only consider the problem in the O-xy plane, $S_p = W_p \cdot L_p$ and $RATE = W_p / L_p$ can express the MIRDW of the PR. According to the analysis in section II, the Jacobian matrix J, the RW and MIRDW are related to parameters $R$ and $L_2$. As each of the parameters $R$ and $L_2$ have any value between 0 and infinite theoretically, it is necessary to normalize $R$ and $L_2$ firstly. We use the normalized algorithm proposed by Liu et al. in [20]. The parameters $r$ and $l$ are normalized from the parameters $R$ and $L_2$, respectively.

\[
r + l = 2, \quad 2 > l > 1 \quad (12)
\]

A set of $r$ and $l$ can be written as $a^* = [l, r]$. Formula (12) is the analysis results from the normalized procure based on formula (9).

A scale factor of zoom on shape preserving can be calculated from formula (13).

\[
D = \sqrt{\frac{S_p}{S_{MIR}}} \quad (13)
\]

Where $S_{MIR}$ is the maximal rectangle area which is calculated from a set of $r$ and $l$ given arbitrarily but meets the formula (12).

4.2 Performance charts
Algorithm 2: Plot the performance charts.
For a given normalized parameters \( a^* \), \( \eta_{\text{min}} \) and \( \text{RATE} \), the \( S_{\text{MIR}} \) can be achieved numerically from Algorithm 1 by searching the MIRDW.

S1. Let the initial value of \( l_{\text{min}} \) is 1 and its maximal value \( l_{\text{max}} \) is 2.

S2. \( \varepsilon_1 \) is step length, and the loop times \( T \) is calculated from \( |l_{\text{max}} - l_{\text{min}}| / \varepsilon_1 \).

\[
\text{for } (i=1:1:T) \quad l = l_{\text{min}} + i \cdot \varepsilon_1; \\
r = 2.0 - l;
\]

Search the maximal area of inscribed rectangles and save \( S_{\text{MIR}} \) as result (i).

S3. Plot the curve of result (i) changed with the \( l \). Search the maximal result and return this \( l_{\text{max}} \) value. The performance charts are shown in Fig. 5 when the maximal error of \( \text{RATE} \) is 0.1. The optical results of various input variable are shown in Table 1.

Obviously, more large value of \( \eta_{\text{min}} \), obtain more smaller \( l_{\text{max}} \) and \( S_{\text{MIR}} \). When \( \eta_{\text{min}} \) is more than 0.5, no DW can be obtained, and no \( l_{\text{max}} \) or \( S_{\text{MIR}} \) can be obtained from the performance chart.

From Table 1, it is observed that \( l_{\text{max}} \) and \( S_{\text{MIR}} \) changed not too large when the \( \eta_{\text{min}} \) is the same. Every corresponding maximal inscribed rectangle can be obtained for every given \( \text{RATE} \) by algorithm 1 and 2 and the error of \( \text{RATE} \) is less than 0.05.

<table>
<thead>
<tr>
<th>Input variables</th>
<th>Optical results</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_{\text{min}} )</td>
<td>Rate</td>
</tr>
<tr>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>0.2</td>
<td>0.7</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>
5 Case Study

5.1 The Design Objects and Design Results
In this section, four cases of PCDWs are considered to design the LDR. The characteristic lengths of four cases of PCDWs and the four groups of optimal results are listed in Table 2.

1) Design Option: In order to ensure accuracy of the results, design option of ε, ε₁ and maximal error of RATE must be given appropriately. But, a very small value of ε and ε₁ will waste the calculated time and a very small value of the maximal error of will make it become difficult to search the real MINDW. Let ε = 0.002, ε₁ = 0.01 and maximal error of RATE is 0.05.

2) Implement Optimal Design: In order to obtain the factor D of zoom on shape preserving, problem (13) must be solved firstly. Set \(a_0 = [1.24, 0.76]\). The \(S_{\text{MIR}}\) is calculated by algorithm 1, when \(n_{\text{min}} = 0.3\). The factors D of four cases of PCDWs are calculated respectively.

5.2 Discussion of the Optimal Results
If the normalized solutions are the same, the different PCDW can be obtained by magnifying an identical normalized MIRDW with different zoom factor D. So we choose some classic PCDW with the same \(H_p\) as optical case.

From Table 2, each case of MIRDW is close to corresponding PCDW and named these MIRDWs as the operation tasks state of the LDR. Thus, in order to measure the LDR whether it fit to the specified operation task, the error ΔE and ΔR are defined as formula (14), (15).

\[
\Delta E = \sqrt{\Delta S / (S_p)} \quad (14)
\]

\[
\Delta R = |W_p / L_p - W_{\text{MIR}} / L_{\text{MIR}}| \quad (15)
\]

The ΔE and ΔR of four cases are calculated and listed in Table 2. It is observed that the ΔE of case 2 is maximal, but case 4 is minimal. The ΔE and ΔR is used to explain the reasonability of LDR for a prescribed operation task.

Table 2 The results of optimal design to the four cases of PCDWs

<table>
<thead>
<tr>
<th>Cases</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W_p) (mm)</td>
<td>40</td>
<td>50</td>
<td>70</td>
<td>100</td>
</tr>
<tr>
<td>(L_p) (mm)</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>(H_p) (mm)</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>(W_{\text{MIR}} / L_{\text{MIR}})</td>
<td>0.4</td>
<td>0.5</td>
<td>0.7</td>
<td>1.0</td>
</tr>
<tr>
<td>(W_{\text{MIR}})</td>
<td>0.37</td>
<td>0.38</td>
<td>0.44</td>
<td>0.49</td>
</tr>
<tr>
<td>(L_{\text{MIR}})</td>
<td>0.86</td>
<td>0.84</td>
<td>0.67</td>
<td>0.51</td>
</tr>
<tr>
<td>(D)</td>
<td>112.03</td>
<td>124.69</td>
<td>153.08</td>
<td>201.04</td>
</tr>
<tr>
<td>(r)</td>
<td>0.76</td>
<td>0.76</td>
<td>0.76</td>
<td>0.78</td>
</tr>
<tr>
<td>(S_{\text{MIR}})</td>
<td>0.319</td>
<td>0.322</td>
<td>0.299</td>
<td>0.247</td>
</tr>
<tr>
<td>(W_R) (mm)</td>
<td>41.28</td>
<td>47.67</td>
<td>68.061</td>
<td>98.494</td>
</tr>
<tr>
<td>(L_R) (mm)</td>
<td>96.90</td>
<td>104.89</td>
<td>102.86</td>
<td>101.53</td>
</tr>
<tr>
<td>(H_R) (mm)</td>
<td>200.0</td>
<td>200.0</td>
<td>200.0</td>
<td>200.0</td>
</tr>
<tr>
<td>(R) (mm)</td>
<td>85.14</td>
<td>94.76</td>
<td>116.34</td>
<td>156.53</td>
</tr>
<tr>
<td>(L_2) (mm)</td>
<td>138.92</td>
<td>154.62</td>
<td>189.82</td>
<td>244.83</td>
</tr>
<tr>
<td>(\Delta S) (%) (\sqrt{\Delta S_S})</td>
<td>0.198</td>
<td>0.480</td>
<td>0.626</td>
<td>0.338</td>
</tr>
<tr>
<td>(\Delta E) (%) (\sqrt{\Delta E_S})</td>
<td>0.70</td>
<td>0.98</td>
<td>0.95</td>
<td>0.58</td>
</tr>
<tr>
<td>(\Delta R) (%) (\sqrt{\Delta R_S})</td>
<td>0.03</td>
<td>0.05</td>
<td>0.0383</td>
<td>0.04</td>
</tr>
</tbody>
</table>

6 Conclusions
This paper presents an optimal design method of a LDR for the PCDW based on performance chart. In order to determine the boundary of the MIRDW, an algorithm is designed. Then an explicit design procedure is proposed to solve the optimal design problems. The results of the optimal design not only provide a set of optimal design parameters of LDR, but also give the performance chart of LDR for a PCDW.

The whole procedure of optimal design is demonstrated by four cases of PCDWs, and their results are compared. Some advantages of the proposed method are summarized as follows: (i) one performance criterion corresponds to one chart, which can graphically and globally show the relationship between the criterion and design.
parameters; (ii) the optimum design process can consider multi-objective functions or multi-criteria, and also guarantees the optimal result; (iii) a set of tools is proposed which is consisted of two algorithms; (iv) there are no nonlinear constraints in the proposed design procedure.

The results of this paper will be useful in the optimal design of other PR, in particular who has parallel arrangement actuated prismatic joints. It can also be applied to compare and select PRs for a specified operation task.

References: