AN LMI Approach to Computation State Feedback Control in the Linear Discrete-Time System with Limited Input

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Abstract: Main subject of paper is on the local stability in the linear discrete-time system with limited input and main base of solution problems is linear matrix inequality (LMI). LMI is proposed in order to compute a saturating state feedback that stabilized the system with respect to a given set of admissible initial state.

Key-Words: Control saturation, discrete time system, LMI and state feedback

1 Introduction
It is well known that the global stability can be achieved only when the open loop system is not strictly unstable, i.e., in the discrete time system, it has its pole inside or on the unite circle of complex plane ([1], [2]). However, the physical interest of the global stability is questionable since, in general, the system is restricted to operate in a limit zone of the state space. This work focused on the local stabilization problem.

Given a set of admissible initial conditions $x_0$ to be stabilized, our objective is to compute a saturating state feedback control law that guarantees both the asymptotic convergence to the original of all trajectories emanating from $x_0$ and a certain degree of time-domain performance for the close-loop system in a neighborhood of the origin. In this aim, we use a local representation of the saturated system deduced from the difference inclusions theory. This representation consists in a polytopic model valid in a certain polyhedral set in the state space. Based on this model, some conditions expressed as linear matrix inequalities (LMIs).

2 Problem Statements
Consider a linear discrete-time system

$$x(k+1) = Ax(k) + Bu(k)$$

(1)

Where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$ are respectively the state vector and the control vector. Matrices $A$ and $B$ are real constant matrices of appropriate dimensions.

For system (1) we suppose that the following assumption hold.

a) The control vector is subject to amplitude constraints which define the polyhedral compact region $\Omega \in \mathbb{R}^m$

$$\Omega = \{ u \in \mathbb{R}^m : -\rho \leq u \leq \rho \}$$

(2)

b) The pair $(A,B)$ is controllable.

c) The region of admissible initial states, denoted by $x_0$, is known.

d) All the eigenvalue of are located inside or on the unit disk of all the complex plain.

Consider the saturating feedback control law

$$u(k) = \text{sat}(Fx(k))$$

(3)

Where each component is defined, $\forall i = 1,...,m$, as follows:

$$\text{sat}(F_i x(k)) = \begin{cases} -\rho_{(i)}, & \text{if } F_i x(k) \leq -\rho_{(i)} \\ F_i x(k), & \text{if } -\rho_{(i)} \leq F_i x(k) \leq \rho_{(i)} \\ \rho_{(i)}, & \text{if } F_i x(k) > \rho_{(i)}. \end{cases}$$

(4)

By applying this control law to system (1) the closed-loop system becomes nonlinear:

$$x(k+1) = Ax(k) + B \text{sat}(Fx(k))$$

(5)

and when control inputs do not saturated, the evolution of the close-loop system is described by the following linear model:
x(k+1) = (A + BF)x(k)  \tag{6}

Now compute a matrix F such that:
1) All the trajectories of system (5) emanating from $x_0$ converge asymptotically to the origin.
2) A certain degree of performance is guaranteed when the system operates inside the region of linear behavior.

3 Methods of Calculated the F Matrix

Two methods exist for calculate the F matrix:
1) Solution of reccaty equation
2) Solution linear matrix inequality (LMI).

3.1 Solution of reccaty equation

The state feedback can be calculated by Solution this equation with considering a proper cost functions as below:

$$ J = \frac{1}{2} x \left( X^T Q X + U^T R U \right) $$  \tag{7}

Where R and Q are weighted matrices.

Since in this paper, x convergence to origin is important, then we assume that $R = 1$ and most notice to Q matrix than determines speed and rate of convergency to origin. By attention to equation (8) can seen that if $q(i)$ is bigger, then $x(i)$ is smaller and the rate of convergency to origin is faster and need lees time to movement from $x_0$ to origin.

More specifically, it is proven in [3] and [4] that under assumptions a, b, c and d there always exist $\varepsilon > 0$ which that the parameter-dependent riccati equation has a solution

$$ P(\varepsilon) = A^T P(\varepsilon) A + d_n - A^T P(\varepsilon) B (B^T P(\varepsilon) B + I_m)^{-1} B^T P(\varepsilon) A $$  \tag{9}

$P(\varepsilon)$ and the control law $U(k) = Fx(k) = - (B^T P(\varepsilon) B + I_m)^{-1} B^T P(\varepsilon) A x(k)$ is such that the eigenvalue of $(A + BF)$ are inside the unite disk.

3.2 Solution of Linear Matrix Inequality

In LMI solution, restriction of $U$ and the other necessary conditions which resulting from limitation on initial conditions, limitation on convergency regions and limitation on linear operation regions, was assumed and state feedback matrix obtain by using those conditions.

Consider the following data:
1) A vector $\rho$ of control bound;
2) A set of initial conditions $x_0$ defined as a union of ellipsoidal sets and polyhedral set described by its vertices
3) A region $D$, contained in the unit disk of the complex plane, defined as:

$$ D = \left\{ z \in \mathbb{C} : (H + zQ + z^T Q^T) < 0 \right\} \tag{11} $$

Where $H$ and $Q$ are real matrices and $z$ is complex number with its conjugate $\overline{z}$. We assume that if the poles of $(A + BF)$ are located in the region $D$ the time-domain requirement in the zone of linear behavior of the system (5) is satisfied.

If exist matrices $W, W^T > 0, W \in \mathbb{R}^{n \times n}, Y \in \mathbb{R}^{m \times n}$ satisfying the following matrix inequality:

$$ H_{\bar{\theta},j} W + Q_{\bar{\theta},j} (AW + BY) + Q_{\bar{\theta},j} (AW + BY)^T < 0 $$

$$ \begin{bmatrix} W & W A^T + Y B^T \end{bmatrix} > 0 $$
In (12) inequalities, Y and W are unrecognized and obtained by solution (12). The state feedback matrix can obtain by $F = Y W^{-1}$. The F matrix guarantees that the all eigenvalue of $(A+BF)$ in the unit disk.

### 4 The Numerical Examples
Consider the example treated in [5] for which system (1) is described by the following matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 2\sqrt{2} & -4 & 2\sqrt{2} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \rho = 4$$

The set of initial conditions is given by the hypercube $X_0 = [-10 10 10 10]^T$.

The F matrix calculated via the riccati equation as follow: $F = [-0.0174 0.0369 -0.0350 0.0124]$. Fig. 1 shows the convergency of four states to origin and fig. 2 shows the control signal.

The F matrix calculated via the LMI approach as follow $F=[0.1534 0.3328 0.3207 0.1161]$. Fig. 3 and fig. 4 shows the convergency by state feedback which obtains by LMI approach.

Fig. 1. Time response of four states convergency to origin by state feedback obtains from riccati equation.
By attention to fig 1,2,3,4 was seen that the time of convergency to origin by state feedback via riccati approach is 1500 and we need to a large amount of control signal. But in LMI approach time of convergency very little and we need a little amount of control signal. Consider the second example as below:

$$A = \begin{bmatrix} 0.9995 & 0.0100 \\ -0.1000 & 0.9995 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0000 \\ 0.0100 \end{bmatrix}, \quad \rho = 5, x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T$$

A fig. 5, 6 shows the convergency, control signal and state trajectories via riccati approach. Figs 7, 8 show the convergency, control signal and state trajectories via LMI approach.
By attention to previous discussions in this work, become clear that in the riccati approach, constrain on the inputs was not purpose and then the riccati approach response was suboptimal response. But in the LMI approach all necessary constrain proposed and thus the LMI approach was optimal.

References