An Efficient Low-Complexity Joint Multi-User Power Control and Partial Crosstalk Cancellation in xDSL Systems

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ABSTRACT: - Perfect crosstalk cancellation techniques have been proposed to mitigate the effect of crosstalk. However, the online complexity of these crosstalk cancellation techniques grows with the square of the number of lines in the binder. Fortunately, most of the crosstalk originates from a limited number of lines on a limited number of tones. As a result, a fraction of the complexity of perfect crosstalk cancellation suffices to cancel most of the crosstalk. This is known as partial crosstalk cancellation. Because the crosstalk profile changes over time, there is additional requirement that partial crosstalk cancellation provide a very low pre-processing complexity. Also, a much lower online complexity can be obtained if the multi-user power control and partial crosstalk cancellation problems are solved jointly. Currently, this joint problem is formulated as a constrained optimization problem and solved by employing Lagrange dual decomposition method. However, it suffers from per-tone exhaustive search because of non-convexity of its per-tone problem. This paper presents a solution for the joint multi-user power control and partial crosstalk cancellation problem with significantly lower pre-processing complexity than the currently proposed algorithms. The problem is considered as a mixed binary-non-convex problem. Then it is reformulated as a mixed binary-convex problem via a successive convex relaxation. Finally it is solved by an efficient branch and bound method. The complexity analysis of our algorithm shows that it provide much lower pre-processing complexity than currently proposed algorithms, allowing it to work efficiently in time-varying crosstalk environment. Moreover, the analytical and simulation results demonstrate that our algorithm is close to the optimal solution from the crosstalk cancellation point of view.

Keywords: - Digital subscriber line (DSL), Partial crosstalk cancellation, Power control, Convex relaxation, Branch and bound.

1 Introduction

The ever increasing demand for higher data rates forces DSL systems to use higher frequencies, e.g. up to 30 MHz for VDSL2. The major obstacle for performance improvement in modern xDSL systems remains to be crosstalk, which is the interference generated among different lines in the same cable binder. The crosstalk is typically 10-20 dB larger than the background noise [1].

There are two strategies for dealing with this crosstalk. Multi-user power control which is known as spectrum management and crosstalk cancellation. A multi-user power control algorithm chooses the transmit spectra such that crosstalk is avoided. As an example, optimal spectrum balancing (OSB) [2] addresses the spectrum management problem through the maximization of a weighted sum rate across all users, which explicitly takes into account the damage done to the other lines when optimizing each line's spectra. Crosstalk cancellation techniques have been proposed to remove crosstalk [3] [4] [5]. Because most of the crosstalk originates from a limited number of lines on a limited number of tones, a fraction of this complexity suffices to cancel most of the crosstalk. This is called partial crosstalk cancellation [6] [7]. The challenge is then to determine for every user which crosstalk to cancel on which tones. In [6], an algorithm based on resource allocation is presented to solve this problem. It considers fixed complexity budget per user and try to find set of dominant cross-talkers based on line and tone selection while consider flat transmit spectra for every users.

However, it is sub-optimal and has considerable pre-processing complexity. Thus, it is not desirable for time-varying xDSL crosstalk environments.

Recently, there are some available algorithms that solve independently the multi-user power control and partial crosstalk cancellation problem. First a multi-use power control technique chooses transmit spectra to avoid crosstalk, then a partial crosstalk cancellation scheme is used to cancel the remaining crosstalk. This approach can be sub-optimal. It can be seen that for strong crosstalk scenarios the transmit spectra result in long and short lines occupying different frequency bands. When the partial crosstalk cancellation problem is solved, there is not much crosstalk left to cancel. Therefore, only a limited crosstalk cancellation tap budget can be used effectively. A better solution (i.e. lower online complexity) can be obtained if the multi-user power control and partial crosstalk cancellation problems are solved jointly. In [8], multi-user power control and partial crosstalk cancellation is formulated as a constrained optimization problem. The Lagrange multipliers are used to decouple the constrained optimization problem into a series of per-tone unconstrained optimization problems. However, this problem suffers from per-tone exhaustive search because of non-convexity of its per-tone optimization problem. Thus, each per-tone problem still has a computational complexity that is exponential in the number of users. In this paper, we reformulate the non-convex per-tone optimization problem into a mixed binary-convex problem based on a successive convex relaxation technique. Then we solve it by using an efficient branch and bound approach. Branch and bound [9] is a general method for finding optimal solutions of various optimization problems. Branch and bound operations will be proposed which require a limited amount of computation, keeping the pre-processing complexity significantly low. The complexity of the proposed branch and bound procedure will be compared with the complexity of the per-tone exhaustive search.

2 DSL Channel

We consider a DSL network with N users (i.e., lines, transmitting modems) and K tones (i.e., frequency carriers). Assuming the standard synchronous discrete multi-tone (DMT) modulation, each tone is capable of transmitting data independently from other tones, and so the transmit power and the number of bits can be assigned individually for each tone. Transmissions can be modeled independently on each tone k as follows:

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k. \qquad 1 \le k \le K$$

The vector $\mathbf{x}_k \triangleq \left[x_k^1, x_k^2, \ldots, x_k^N \right]^T$ contains the transmitted signals on tone k for all N users, where x_k^n is the transmitted signal by user n on tone k. Vectors $\mathbf{y}_k$ and $\mathbf{z}_k$ have similar structures. The vector $\mathbf{y}_k$ contains the received symbols. The vector $\mathbf{z}_k$ is the vector of additive noise on tone k, containing thermal noise, alien crosstalk, RFI, …. $\left[H_k \right]_{n,m} = h_k^{n,m}$ is an $N \times N$ matrix containing the channel transfer functions from transmitter m to receiver n. The diagonal elements are the direct channels; the off-diagonal elements are the crosstalk channels.

To take crosstalk cancellation into account, in [8] an equivalent channel $\tilde{\mathbf{H}}$ is introduced. This is the same channel as the original channel $\mathbf{H}$, but with off-diagonal elements set to 0 where the crosstalk is cancelled. If user n is canceling crosstalk originating from user m on tone k, then

$$\tilde{h}_k^{n,m} = \begin{cases} h_k^{n,m} & if\ c_k^{n,m} = 0 \\ 0 & if\ c_k^{n,m} = 1 \end{cases} \Rightarrow \left| \tilde{h}_k^{n,m} \right|^2 = (1 - c_k^{n,m}) \left| h_k^{n,m} \right|^2,$$

where $c_k^{n,m}$ is the cancellation tap assigned to the user n to cancel the crosstalk originating from user m on tone k [8].

We denote the transmit power as s_k^n and the noise power as σ_k^n. The DMT symbol rate is denoted as f_s, the tone spacing as Δ_f. The achievable bit loading of user n on tone k, given the transmit spectra of all modems in the system, is

$$b_k^n \triangleq \log_2 \left(1 + \frac{1}{\Gamma} \frac{\left| \tilde{h}_k^{n,n} \right|^2 s_k^n}{\sigma_k^n + \sum_{m \ne n} \left| \tilde{h}_k^{n,m} \right|^2 s_k^m} \right). \qquad (1)$$

Where Γ denotes the SNR-gap to capacity, which is function of the desired BER, the coding gain and noise margin [1]. The data rate and total power for user n is

$$R^n = f_s \sum_k b_k^n \quad \text{and} \quad P^n = \sum_k s_k^n.$$

3 Joint Multi-User Power Control and Partial Crosstalk Cancellation

3.1 Problem formulation

Let $\mathbf{s}=\left[\mathbf{s}^1,\mathbf{s}^2,\ldots,\mathbf{s}^N\right]$ and $\mathbf{c}=\left[\mathbf{c}_1,\mathbf{c}_2,\ldots,\mathbf{c}_K\right]$ denote the matrix of power spectra and crosstalk cancellation configuration where $\mathbf{s}^n$ is the vector of transmit spectra of user *n* and $\mathbf{c}_k$ is the matrix of crosstalk cancellation taps on tone *k*. The joint multi-user power control and partial crosstalk cancellation problem can be formulated as the following weighted sum rate maximization (WSRmax) problem [8]:

$$\max_{\mathbf{s},\mathbf{c}} \sum_{n=1}^{N} \alpha_n R^n \quad \text{subject to:}$$
$$\begin{aligned} & P^n \le P^{n,tot}, R^n \ge R^{n,target}, \\ & 0 \le s_k^n \le s_k^{n,mask} \end{aligned} \qquad \forall n, \forall k \tag{2}$$
$$\begin{aligned} & \sum_k \sum_n \sum_m c_k^{n,m} \le C^{tot}, \\ & c_k^{n,m} \in \{0,1\} \end{aligned} \qquad \forall n, \forall m, \forall k$$

where α_n is the weight assigned to user *n*. Moreover, there are a number of constraints: total power constraint $P^{n,tot}$ and data-rate constraint $R^{n.target}$ for each user, spectral mask constraint $s_k^{n,mask}$ for each user in each tone and total number of cancellation taps constraint C^{tot} [8]. This problem finds the optimal allocation of transmit spectra and set of the cross-talkers to cancel (i.e. allocation of crosstalk cancellation taps) that maximizes the weighted sum rate. The WSRmax problem (2) is a mixed binary-non-convex problem. To find the global optimum, one has to exhaustively search through all possible transmit spectra $\mathbf{s}$ (continuous values) and cancellation tap configurations $\mathbf{c}$ (binary values). The cancellation taps constraint, the total power constraints and the target bit rate constraints are coupled over the tones and also the objective function is coupled over the users. This leads to an exponential complexity in both the number of users and tones, namely $O((B2^{N-1})^{KN})$ where B is the number of possibilities for power loading and 2^{N-1} possibilities for cancellation taps for each user on each tone. Lagrange dual decomposition can be used to derive efficient algorithm for the WSRmax problem (2) which makes the complexity linear in the number of tones [8].

3.2 Efficient resource allocation algorithm

The Lagrangian of WSRmax problem (2), dualized with respect to the total power constraint and the number of cancellation taps constraint, is defined as

$$J = \sum_{n=1}^{N} \alpha_n R^n + \sum_{n=1}^{N} \lambda_n (P^{n,tot} - \sum_{k=1}^{K} s_k^n) + \beta (C^{tot} - \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{m=1}^{N} c_k^{n,m}), \tag{3}$$

where α_n, λ_n and β are Lagrange multipliers, $\boldsymbol{\alpha}=\left[\alpha_1,\alpha_2,\ldots,\alpha_N\right]^T$ and $\boldsymbol{\lambda}=\left[\lambda_1,\lambda_2,\ldots,\lambda_N\right]^T$. Then with this Lagrange dual function, the WSRmax problem (2) can be formulated as follows:

$$\min_{\boldsymbol{\alpha},\boldsymbol{\lambda},\beta} \max_{\mathbf{s},\mathbf{c}} J \quad \text{subject to}$$
$$\begin{aligned} & \alpha_n \ge 0, \lambda_n \ge 0, \beta \ge 0, && \forall n \\ & 0 \le s_k^n \le s_k^{n,mask} && \forall n, \forall k \\ & R^n \ge R^{n,target} && \forall n \\ & \sum_k \sum_n \sum_m c_k^{n,m} \le C^{tot}, \; c_k^{n,m} \in \{0,1\} && \forall n, \forall m \ne n, \forall k \end{aligned} \tag{4}$$

This Lagrangian can be decomposed into the following *K* independent per-tone objective function:

$$J = \sum_{k=1}^{K} J_k + \sum_{n=1}^{N} \lambda_n P^{n,tot} + \beta C^{tot} \quad \text{where } J_k = \sum_{n=1}^{N} \alpha_n f_s b_k^n - \sum_{n=1}^{N} \lambda_n s_k^n - \beta \sum_{n=1}^{N} \sum_{m=1}^{N} c_k^{n,m} \tag{5}$$

The constant has no influence on the maximization and can be discarded. Then (for a particular choice of $\alpha_n, \lambda_n, \beta, \forall n$) the optimization problem (4) can be solved in a per-tone fashion:

$$\mathbf{s}_k^{opt}, \mathbf{c}_k^{opt} = \arg\min_{\mathbf{s}_k,\mathbf{c}_k} (-J_k) \quad \text{subject to:}$$
$$\begin{aligned} & \alpha_n \ge 0, \lambda_n \ge 0, \beta \ge 0 && \forall n \\ & 0 \le s_k^n \le s_k^{n,mask}, c_k^{n,m} \in \{0,1\} && \forall n, \forall m \end{aligned} \tag{6}$$

Note that the sign of the per-tone objective function is changed and the maximization is changed into a minimization for convenience. The original complexity of $O((B2^{N-1})^{KN})$, exponential in *K*, is now reduced to a linear complexity in *K*: $O(K(B2^{N-1})^N)$. Minimization of (6) for given Lagrange multipliers can be performed by an exhaustive search since it is non-convex problem. For each tone, the objective function should be evaluated for all possible combinations of the transmit power levels and cancellation tap configurations of the users [8].

To solve (2) by (6), $\boldsymbol{\alpha}$, $\boldsymbol{\lambda}$ and β should be tuned to enforce the constraints. The following sub-gradient descent form for the Lagrange multipliers update can be used [8]:

$$\begin{aligned} \alpha_n^{t+1} &= \left[\alpha_n^t - \mu\left(\sum_k b_k^n - R^{n,target}\right)\right]^+ \quad ,\forall n \\ \lambda_n^{t+1} &= \left[\lambda_n^t - \mu\left(P^{n,tot} - \sum_k s_k^n\right)\right]^+ \quad ,\forall n \\ \beta^{t+1} &= \left(\beta^t - \mu(C^{tot} - C)\right)^+, \end{aligned} \tag{7}$$

where $(x)^+$ means $\max(0,x)$, t is the iteration number, μ is a step size parameter and C is the total number of cancellation taps corresponding to the Lagrange multipliers at hand. Note that all the Lagrange multipliers are updated in parallel. The joint multi-user power control and partial crosstalk cancellation algorithm is listed in Algorithm 1, adopted from [8].

Algorithm 1 Joint multi-user power control and partial crosstalk cancellation algorithm

1. Set the tolerance and C^{tot}, $P^{n,tot}$, $s_k^{n,mask}$ and $R^{n.target}$;
2. Initialize transmit spectra $\mathbf{s}$ and cancellation taps configuration $\mathbf{c}$;
3. Initialize Lagrange multipliers $[\boldsymbol{\alpha},\boldsymbol{\lambda},\beta]$;
4. distance = $\left\|\left[\mathbf{R}-\mathbf{R}^{target},\mathbf{P}^{tot}-\mathbf{P},C^{tot}-C\right]\right\|$;
5. **repeat** distance > tolerance **do**
6. $\quad \mu = 1$;
7. $\quad$ **repeat**
8. $\quad\quad$ previous_distance = distance;
9. $\quad\quad$ **for** n=$1\ldots N$ (i.e. each user)
10. $\quad\quad\quad$ **for** k=$1\ldots K$ (i.e. each tone)
11. $\quad\quad\quad\quad$ $[\mathbf{s}_k,\mathbf{c}_k]$ = solve mixed binary-non-convex optimization problem (6);
12. $\quad\quad\quad$ **end for**
13. $\quad$ **end for**
14. $\quad C = \sum_k\sum_n\sum_m c_k^{n,m}$;
15. $\quad \mu = \mu \times 2$;
16. $\quad$ **for** n=$1\ldots N$
17. $\quad\quad$ update Lagrange multipliers $\boldsymbol{\alpha}$, $\boldsymbol{\lambda}$ and β based on (7)
18. $\quad$ **end for**
19. $\quad$ distance = $\left\|\left[\mathbf{R}-\mathbf{R}^{target},\mathbf{P}^{tot}-\mathbf{P},C^{tot}-C\right]\right\|$;
20. **until** distance ≤ previous_distance
21. **until** distance > tolerance

where $\mathbf{R}=\left[R^1,R^2,\ldots,R^N\right]$ and $\mathbf{P}=\left[P^1,P^2,\ldots,P^N\right]$.

4 Per-Tone Complexity Reduction

4.1 Successive convex relaxation

The per-tone optimization problem (6) is a combinatorial or mixed binary-non-convex problem. If we rewrite the objective function of (6) in the following form using (1)

$$\begin{aligned} -J_k &= (I) + (II) \\ (I) &= \sum_{n=1}^N \left(\alpha_n f_s \log_2\left(\sum_{m=1}^N \left(\left|\hat{h}_k^{n,m}\right|^2 s_k^m + \Gamma\sigma_k^n\right)\right)\right. \\ &\left. + \lambda_n s_k^n + \beta\sum_{m=1}^N c_k^{n,m}\right) \\ (II) &= \sum_{n=1}^N \alpha_n f_s \log_2\left(\sum_{m\neq n}^N \left(\left|\hat{h}_k^{n,m}\right|^2 s_k^m + \Gamma\sigma_k^n\right)\right) \end{aligned} \tag{8}$$

$$\text{with } \left|\hat{h}_k^{n,m}\right|^2 = \begin{cases} \Gamma\left|\tilde{h}_k^{n,m}\right|^2 = \Gamma(1-c_k^{n,m})\left|h_k^{n,m}\right|^2 & n\neq m \\ \left|\tilde{h}_k^{n,m}\right|^2 = (1-c_k^{n,m})\left|h_k^{n,m}\right|^2 & n = m \end{cases},$$

We observe that the objective function consists of a convex part (I) and a concave part (II). This objective function is a difference of two convex (d.c.) functions which is known to correspond to a hard optimization problem [10]. In this section an efficient low-complexity joint multi-user power control and partial crosstalk cancellation algorithm is presented. The problem (6) is reformulated as a mixed binary-convex problem based on a successive convex relaxation, leading to a much low complexity procedure [9] [10]. Our approach is to consider a relaxation of the non-convex problem (6) by an over-estimator to avoid the d.c. structure. We make use of the following approximation [9]:

$$\log_2(1+x) \geq u\log_2(x) + v. \tag{9}$$

That is tight with equality at an approximation point $\hat{x}$ when the constants u and v are chosen as below:

$$u = \frac{\hat{x}}{1+\hat{x}}, \quad v = \log_2(1+\hat{x}) - \frac{\hat{x}}{1+\hat{x}}\log_2(\hat{x}).$$

Applying (9) to objective function (6) results in the following relaxed objective function

$$\begin{aligned} (J_k)_{relaxed} &= \sum_{n=1}^N \left(-\alpha_n f_s \left(u_k^n \log_2\left(\frac{\left|\hat{h}_k^{n,n}\right|^2 s_k^n}{\Gamma\sigma_k^n + \sum_{m\neq n}\left|\hat{h}_k^{n,m}\right|^2 s_k^m}\right)\right.\right. \\ &\left.\left. + v_k^n\right) + \lambda_n s_k^n + \beta\sum_{m=1}^N c_k^{n,m}\right) \quad \text{where } -J_k \leq (J_k)_{relaxed}. \end{aligned} \tag{10}$$

The obtained relaxed objective function is still non-convex. However, a transformation $\tilde{s}_k^n = \ln(s_k^n)$ converts it to a convex objective function [9]. Fortunately, the constraints are also convex leading to a convex optimization problem which can be solved efficiently.

$$(J_k)_{convex} = \sum_{n=1}^{N} \Big(-\alpha_n f_s \big(u_k^n \log_2 \big(\frac{|\hat{h}_k^{n,n}|^2 e^{\tilde{s}_k^n}}{\Gamma\sigma_k^n + \sum_{m\neq n} |\hat{h}_k^{n,m}|^2 e^{\tilde{s}_k^m}}\big) + v_k^n\big) + \lambda_n e^{\tilde{s}_k^n} + \beta \sum_{m=1}^{N} c_k^{n,m}\Big) \qquad (11)$$

The solution of this convex relaxation forms an upper bound for the global minimum. Using the obtained upper bound as a new point of approximation (see algorithm 2 adopted from [9]), it can be proven that the sequence of relaxations produces a monotonically decreasing objective value and will always converge. Upon convergence it can be proven that the obtained solution is a local optimum. Although there is no theoretical proof for global optimality, simulation results are very promising showing global optimality for very different multi-user scenarios.

Algorithm 2 Successive convex relaxation

1: Initialize iteration counter $z = 0$;
2. Initialize $\hat{\mathbf{s}}_k$ and $\hat{\mathbf{c}}_k$;
3. Initialize $u_k^n(z), v_k^n(z)\ \forall n$;
4: **loop**
5: *tighten:* Compute $u_k^n(z), v_k^n(z)\ \forall n$ at $\hat{\mathbf{s}}_k(z), \hat{\mathbf{c}}_k(z)$;
6: *minimize:* $\hat{\mathbf{s}}_k(z+1), \hat{\mathbf{c}}_k(z+1)$ = Solve the (6) with the objective function $(J_k)_{convex}$;
7. increment *z;*
8: **until** convergence

4.2 Iterative efficient low complexity algorithm

We can solve the convex relaxed problem (6) with the convex objective function $(J_k)_{convex}$ in (11) by means of standard convex software; however we can also use an efficient low complexity distributed algorithm [9]. With fixed value of Lagrange multipliers $\alpha_n, \lambda_n, \beta$, then the global minimum point of (11) can be obtained by taking partial derivative with respect to $\tilde{\mathbf{s}}_k, \mathbf{c}_k$. This leads to the following fixed point equations:

$$\frac{\delta (J_k)_{convex}}{\delta \tilde{s}_k^n} = 0 \Rightarrow s_k^n = w_k^n(s_k^n) = \left(\frac{\alpha_n f_s u_k^n}{\lambda_n \ln(2) + \sum_{p\neq n} \alpha_p f_s u_k^p \frac{|\hat{h}_k^{p,n}|^2}{\sum_{m\neq p} |\hat{h}_k^{p,m}|^2 s_k^m + \Gamma\sigma_k^p}}\right) \qquad (12)$$

$$\frac{\delta (J_k)_{convex}}{\delta c_k^{n,m}} = 0 \Rightarrow c_k^{n,m} = y_k^{n,m}(c_k^{n,m}) = 1 + \frac{\sum_{p\neq n,m} |\hat{h}_k^{n,p}|^2 s_k^p + \Gamma\sigma_k^n}{\Gamma |h_k^{n,m}|^2 s_k^m} - \frac{\alpha_n f_s u_k^n}{\beta \ln(2)}.$$

By iteratively updating the transmit powers s_k^n and cancellation taps $c_k^{n,m}$ using (12), convergence to the global minimum point can be achieved due to convexity. Moreover, the derivative of $w_k^n(s_k^n)$ and $y_k^{n,m}(c_k^{n,m})$ is typically much smaller than one for all points s_k^n and $c_k^{n,m}$. In order to keep within the spectral mask constraints, the spectra have to be bounded. Moreover, the cancellation taps must be binary value, which leads to the following update formulas:

$$\begin{aligned} s_k^n(z+1) &= \max(0, \min(w_k^n(s_k^n(z)), s_k^{n,mask})) \\ c_k^{n,m}(z+1) &= \max(0, \min(y_k^{n,m}(c_k^{n,m}(z)), 1)), \end{aligned} \qquad (13)$$

where *z* is the iteration number (see algorithm 3).

Algorithm 3 efficient low complexity algorithm

1: Initialize transmit spectra and cancellation taps $\mathbf{s}_k, \mathbf{c}_k$;
2: **loop**
3: calculate new $\mathbf{s}_k, \mathbf{c}_k$ using (13)
4: **until** convergence

Now, the per-tone optimization problem (6) can be solved via joint use of algorithm 2 and 3 instead of exhaustive search proposed in line 13 of algorithm 1. This provides a significantly lower complexity than exhaustive search. However, the above solution only can be used as upper bound on the optimal solution since the cancellation taps are naturally binary value. In the next section we propose an efficient branch and bound method to solve optimally the per-tone optimization problem (6).

4.3 Efficient branch and bound method

The per-tone optimization problem (6) has now reformulated as a mixed binary-convex problem based on successive convex relaxation approach, where $\mathbf{s}_k$ (continuous values) and $\mathbf{c}_k$ (binary values) are the optimization variables. We denote the optimal value of this problem as p^*. One way to solve this problem is by exhaustive search. That is, we must solve $2^{(N-1)^2}$ convex optimization problems, one for each possible value of the binary matrix $\mathbf{c}_k$, and then choose the smallest of these optimal values. This involves solving enormous number of convex problems that is exponential in the size of the cancellation taps. For $(N-1)^2$ more than 30 or so, this is clearly not possible.

We will use an efficient branch and bound method to solve this problem [10]. In the worst case, we end up solving the $2^{(N-1)^2}$ convex problems, i.e., carrying an exhaustive search. But with luck, this does not occur. Fortunately, the convex relaxation

$$\mathbf{s}_k^{opt}, \mathbf{c}_k^{opt} = \arg\min_{\mathbf{s}_k, \mathbf{c}_k} (J_k)_{convex}$$
$$\text{subject to} \quad 0 \le s_k^n \le s_k^{n,mask} \quad \forall n \qquad (14)$$
$$\alpha_n \ge 0\,,\ \lambda_n \ge 0, \beta \ge 0 \quad \forall n$$
$$c_k^{n,m} \in [0,1] \quad \forall n, \forall m$$

with continuous variables $\mathbf{s}_k$ and $\mathbf{c}_k$, is convex, and so easily solved by use of algorithm 3. Its optimal value, which we denote L_0, is a lower bound on the optimal value of (6). This lower bound can be $+\infty$ (in which case the original problem is surely infeasible) or $-\infty$. We can also get an upper bound on p^* using this relaxation. For example, we can take the solution of the relaxed problem, and then round each of the variables $c_k^{n,m}$ to 0 or 1. This upper bound can be $+\infty$, if the rounded solution isn't feasible. We'll denote this upper bound by U_0. Of course, if we have $U_0 - L_0 \le \varepsilon$, the required tolerance, we can quit.

Now we are going to branch. Pick any cancellation tap (binary value) $c_k^{p,q}$, and form two problems. We fix the value of $c_k^{p,q}$ to 0 in the first problem, and 1 in the second. We call these sub-problems of the original, since they can be thought of as the same problem, with one variable eliminated or fixed. Each of these sub-problems is also a mixed binary-convex problem but with $(N-1)^2 - 1$ binary variables. The optimal value of the original problem is clearly the smaller of the optimal values of these two sub-problems.

We now solve the two convex relaxations of these sub-problems with $c_k^{p,q} = 0$ and $c_k^{p,q} = 1$ by joint use of algorithm 2, 3. Thus we can obtain a lower and upper bound on the optimal value of each sub-problem. We'll denote these as $\tilde{L}, \tilde{U}$ (for $c_k^{p,q} = 0$) and $\bar{L}, \bar{U}$ (for $c_k^{p,q} = 1$). Each of these two lower bounds must be larger than L_0, i.e., $\min(\tilde{L}, \bar{L}) \ge L_0$. We can also assume, without loss of generality, that $\min(\tilde{U}, \bar{U}) \ge U_0$. From these two sets of bounds, we obtain the following bounds on p^*: $L_1 = \min(\tilde{L}, \bar{L}) \le p^* \le U_1 = \min(\tilde{U}, \bar{U})$. By the inequalities above, we have $U_1 - L_1 \le U_0 - L_0$.

At the next step, we choose either of the sub-problems, and then split it, by choosing another indexes (not equal to *p*, *q*, the indexes used to split the original problem). We solve the convex relaxations for the split sub-problems (which have $(N-1)^2 - 2$ binary variables), and obtain lower and upper bounds for each.

At this point we have formed a partial binary tree of sub-problems. The root is the original problem; the first split yields two children sub-problems, one with $c_k^{p,q} = 0$ and one with $c_k^{p,q} = 1$. The second iteration yields another two children of one of the original children. We continue in this way. At each iteration, we choose a leaf node (which corresponds to a sub-problem, with some of the binary variables fixed to particular values), and split it, by fixing a variable that is not fixed in the parent problem. An edge in the tree corresponds to the value 0 or 1 of a particular variable $c_k^{n,m}$. At the root node of the tree, the values of none of the $c_k^{n,m}$ are specified. A node at depth *i* in the tree corresponds to a sub-problem in which *i* of the binary variables have fixed values. For each node, we have an upper and lower bound on the optimal value of the sub-problem.

The minimum of the lower bounds, over all the leaf nodes, gives a lower bound on the optimal value p^*; similarly, the minimum of the upper bounds, over all the leaf nodes, gives an upper bound on the optimal value p^*. We refer to these lower and upper bounds as L_k and U_k, respectively. We always have $U_{k+1} - L_{k+1} \le U_k - L_k$; we can terminate the algorithm when $U_k - L_k \le \varepsilon$.

Proving convergence of the algorithm is trivial: it must terminate in fewer than $2^{(N-1)^2}$ steps. To see this, note that if a leaf has depth $(N-1)^2$, it means that all the binary variables are fixed in the sub-problem, so by solving the convex relaxation we get the exact solution. In other words, for any leaf of depth $(N-1)^2$, we have $U = L$. The worst thing that can happen is that we develop a complete binary tree which requires $2^{(N-1)^2}$ steps, at which point every sub-problem lower and upper bound is equal, and therefore the algorithm terminates. This is nothing more than exhaustive search.

At any point in the algorithm we have an upper bound U on p^*. If any node has a lower bound that is more than U, we can prune it, i.e., remove it from the tree. The choice of which leaf to split at a given stage in the algorithm is arbitrary. Several heuristics are used. One is to split the leaf corresponding to the smallest lower bound, since we might expect to find the optimal value in that sub-tree, and in any case splitting this leaf will lead to an improvement in the global lower bound. The same can be said for the choice of index to use to split a leaf. It can be any index corresponding to a binary variable that has not yet been fixed. One heuristic is to split along a binary variable in the relaxation; the hope is that this might allow us to discard the whole sub-tree starting from the other value. If many of the relaxed values are 0 or 1, we can choose the one with the largest associated Lagrange multiplier. (The idea here is that this is the relaxed variable that is 0 and 1, and has the highest pressure to stay there.)

For this specific problem we pick the leaf of the tree with the lowest lower bound to compute the next iteration of the algorithm. Based on the solution of the relaxation at that node we select the variable we fix at the next iteration: We fix the variable whose solution in the relaxed problem is closest to either 0 or 1. If more than one of the variables is equal to 0 or 1, we select among these one whose associated dual variable is largest. The idea behind this heuristic is to fix the variable that, based on the solution of the relaxation, seems to be more likely equal to be 0 or 1 for the optimal solution. The complete efficient branch and bound procedure for one tone of optimization problem (6) is presented in algorithm 4.

Algorithm 4 Efficient branch and bound method

1: *Branching:* choose one of the binary values of cancellation taps ($c_k^{p,q}$) which has not been used so far
2: Solve two convex relaxed problems with objective function $(J_k)_{convex}(11)$ based on algorithm 3 where $c_k^{p,q} = 0$ and $c_k^{p,q} = 1$
3: *Bounding:* determine the lower and upper bound of the two sub-problems
4: Consider minimum of lower bounds as current lower bound on the optimal value of the optimization problem
5: Consider minimum of upper bounds as current upper bound on the optimal value of the optimization problem
6: *Pruning:* Prune the sub-problem which has lower bound that is more than current upper bound
7: **If** desired accuracy achieved **then** quit the algorithm
8: **Else** select the sub-tree (sub-problem) which has the smallest lower bound to split
9: go to step 1

In summary, the per-tone optimization problem (6) in line 11 of algorithm 1 can be solved by successive convex relaxation approach presented in algorithm 2. Algorithm 4 can be used in line 6 of algorithm 2 and algorithm 3 is used in line 2 of algorithm 4.

5 Computational Complexity Comparsion

The overall complexity of the joint multi-user power control and partial crosstalk cancellation problem is $O(K(B2^{N-1})^N)$ in the case of per-tone exhaustive search [8]. In [8], they have used the sub-optimal ON/OFF power loading joint with the line selection and user independence observation and thus reduce this overall complexity to $O(K2^N N^2)$. However, it has still exponential complexity respect to the number of users. In our proposed branch and bound procedure, at any node of the tree, we prune one of leaves (sub-problems). Thus, in the worst case we must solve this optimization problem through $O(N^2)$ runs of convex sub-optimization problem. Algorithm 3 can be used to solve the relaxed mixed binary-convex sub-optimization problem with complexity of $O(N^2)$. Hence, $O(KN^4)$ executions are required to find the optimal solution of the per-tone problem as well. Analytical and numerical studies have shown that algorithms 2 and 3 converge to optimal values with just 2-3 iterations. Thus, they have much low computational complexity and very high convergence speed. Consequently, the overall complexity of the proposed branch and bound procedure is $O(KN^4)$ in the case of per-tone problem. This shows that our proposed solution has much lower pre-processing complexity than the current proposed algorithm in [8] particularly where $N > 4$ and results in optimal power loading and crosstalk cancellation taps allocation.

6 Numerical Results and Discussion

This section provides some simulation results generated by using proposed joint multi-user power control and partial crosstalk cancellation problem. We compare the performance of the joint solution and the independent solution obtained by independently solving the multi-user power control problem (with convex relaxation procedure) and the partial crosstalk cancellation problem.

Our simulations consider a distributed upstream VDSL scenario, with strong crosstalk, that consists of 8 VDSL users, split into two equal groups of 4 users ($4\times600\,m, 4\times1200\,m$) with full signal-level coordination. A line diameter of 0.5 mm (24 AWG) is used and the maximum transmit power is 11.5 dBm. The SNR gap Γ is set to 12.9 dB, corresponding to a target symbol error probability of 10^{-7}, coding gain of 3 dB and a noise margin of 6 dB. The tone spacing is Δ_f = 4.3125 kHz and the DMT symbol rate f_s = 4 kHz. The simulations are performed in Matlab on a $2\times$Intel Xeon 2.4 GHz with 4GB RAM. Due to the inherent symmetry in the channel model [1], the resulting bit-rates for users having equal loop lengths end up the same.

The simulations are performed for the first two-user case (one from each user group) up to the eight-user case (see Table 1). Table 1 shows enormous pre-processing complexity reductions compared to per-tone exhaustive search. For example, an exhaustive search for two user case require 4 hours for simulation time whereas the proposed method only requires 10 seconds with similar resulting bit loadings and identical set of dominant cross-talkers.

Figures 1 and 2 show the resulting bit-rate of near-users (600m) and far-users (1200m) versus online complexity budget where bit-rate of far and near users are set to 2Mbps and 20Mbps, respectively. The figures show significant performance gains (bit-rate improvement) of the joint solution compared to the independent solution. The figures shows that no performance is gained by increasing the crosstalk cancellation tap budget beyond 30% of full crosstalk cancellation. When the multi-user power control problem and the partial crosstalk cancellation problem are solved jointly, transmit spectra are chosen such that only crosstalk that cannot be cancelled is avoided. Depending on the crosstalk cancellation tap budget, transmit spectra can overlap on frequencies with the highest capacity, resulting in significant performance gains.

Table 1. Comparison of simulation times

Users	Exhaustive search (Estimated time)	Our approach
2	4 hours	10 seconds
4	10 days	40 seconds
6	2 years	4 minuets
8	20 years	10 minuets

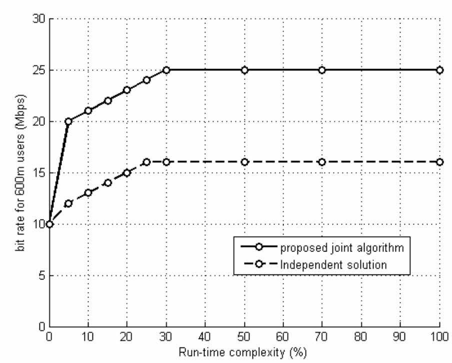

Fig. 1. Near users bit-rate versus online complexity budget where far users bit-rates are set to 2 Mbps

It is also seen that with 30% complexity, our algorithm achieves 99% of the performance gain on near users and achieves 97% of performance gain on far users with 30% complexity budget. With a target rate 15 Mbps on near users and 2 Mbps on far users; the required complexity budget is 20% for independent solution and 4% for joint solution. These show a significant gain of 500% in online complexity reduction as is observed in the figures. It is noted that the joint solution operates much closer to the full crosstalk canceller with much low online complexity in particularly near-far scenarios. This is due that the crosstalk coupling of the dominant cross-talkers are a few order of magnitude greater than those of the non-dominant cross-talkers in near-far scenarios. This near-far effect particularly is seen in upstream scenarios or complex central office and remote terminal based scenarios.

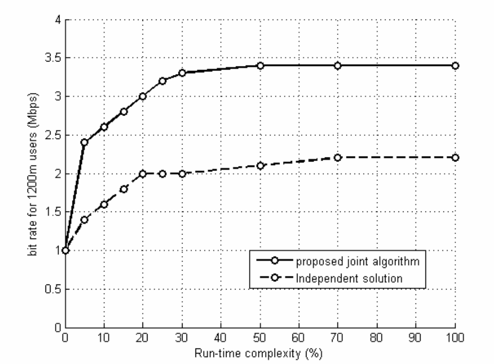

Fig. 2. Far users bit-rate versus online complexity budget where near users bit-rates are set to 20 Mbps

7 Conclusion

The DSL channel is essentially stationary. However, the crosstalk profile can change in time for short-term stationary DSL systems. Therefore, it is crucial that the partial crosstalk canceller has low online complexity. It can be achieved thorough joint use of multi-user power control and partial crosstalk cancellation techniques. However, these joint solutions suffer from pre-processing complexity. In this paper, an efficient low complexity joint multi-user power control and partial crosstalk cancellation solution was introduced. We consider this problem as a mixed binary-non-convex optimization problem. The Lagrange multipliers are used to decouple the constrained optimization problem into a series of per-tone unconstrained optimization problems. However, this problem still suffers from per-tone exhaustive search because of non-convexity of its per-tone optimization problem. Therefore, we reformulated the non-convex per-tone optimization problem into a mixed binary-convex problem based on a successive convex relaxation technique. Then we solved it by using an efficient branch and bound approach keeping the pre-processing complexity significantly low. Analytical and numerical studies have shown that our convex relaxation method have very high convergence speed. Thus, the simulation times are reduced significantly, e.g. from 20 years down to only a few minuets for an eight-user scenario. Moreover, it was shown that network capacity can be increased with this joint solution by efficient crosstalk cancellation taps allocation. Therefore, the pre-processing complexity will be reduced extremely due to our proposed low complexity algorithms. In addition, the online complexity will be decreased significantly by this joint optimal solution as shown by our simulation results presented in this paper and other simulation results not reported here.

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