Post buckling modelling and optimization of sandwich panels with corrugated cores

H.Asadi, M.Gorji, D.Ashouri, A.Khalkhali

Engineering Faculty
The University of Guilan
P.O. Box 3756, Rasht
IRAN

Abstract: In the present study, analytical expressions for critical loads are derived and eigenvalue and post buckling analysis are performed on the panels, using the finite element method, using the commercial software ABAQUS. Genetic algorithm is used to optimization of sandwich panels with periodic, open-cell cores, observing yielding and buckling in components as optimization constraint and then minimum weights for some load capacities are determined. Design variables and cost function are defined as non-dimensional and optimal results are shown by diagrams.

Key-Words: - Sandwich panel, corrugated core, finite element method, genetic algorithm, optimization

1 Introduction

Sandwich panels with open and continuous cells are modern and important structures that are produced by new methods [1]. Having low weight with specific strength is important property that leads to increase of this panel application. Also, other properties such as energy absorption, acoustic, thermal and cooling, durability lead to increase of panel application [2]. Finding geometrical structures to achieve low weight and high load capacity are the scope of the optimization of these panels.

Genetic algorithm (GA) is a search technique used to find exact or approximate solutions to optimization problems. Genetic algorithm is categorized as global search heuristics. This method is a particular class of evolutionary algorithms that use techniques inspired by evolutionary biology such as inheritance, mutation, selection, and crossover [7, 8].

In this paper, equations of sandwich panel with corrugated core are provided and buckling and yielding load is determined then the finite element method, using the commercial software, is apply and obtained results of analysis methods and software are compared.

By Using genetic algorithm minimum value of structure weight based on buckling and yield constraints, against specific loading, is determined.

2 Analytical modeling

As you see in Fig.1, sandwich panel with corrugated core includes of two face sheets and some core members.

Fig.1- A sandwich panel with corrugated core
Design variables that have shown in Fig.2, include \( d \) as face thickness, \( d_c \) as core member thickness, \( H \) as distance between face sheets, \( n \) as core member folds and \( \theta \) as angle between face sheet and core members.

Various types of loading have shown in Fig.3. To various moods of loading, parameter \( l \) is defined as \( l = \frac{M}{V} \) and \( M, V \) is maximum torque and maximum shear force on defined cross-section.

According to values of \( M, V \) non-dimensional loading on panel is shown with equation (1) [2]:

\[
\Pi = \frac{V}{\sqrt{E}M}
\]  

(1)

Where \( \Pi \) is load index, \( E \) is elasticity module. The weight of unit width, \( W \), can be shown in the non-dimensional form:

\[
\psi = \frac{W}{\rho l^2}
\]  

(2)

Where \( \rho \) is the density of the constituent material. The direction of loading is shown in Fig.4.

2.1 Yielding stress

To evaluate panel behaviours under various loading, one cell of panel with core and face members, according to Fig.5, is considered and the results would be extended. The width of a cell is determined with equation (3):

\[
B_n = \frac{2(H_d)}{n \tan \theta}
\]  

(3)
Maximum normal stress is defined:

\[ \sigma = \frac{MB_y}{I_i} \]  

(4)

Where \( I_i \) is the moment of area of section that is determined with equation (5):

\[ I_i = \frac{1}{2} B_n d(H - d)^2 + \frac{1}{6} \frac{d_c}{\sin \theta} (H - d)^3 \]  

(5)

And shear stress is calculated with equation (6):

\[ \tau = \frac{VB}{I_j b} \int_{y_0}^{(H - d)/2} ydA \]  

(6)

In which \( y_0 \) is stress calculation point and \( b \) is width of face and member sheets. According to equation (7), shear face stress is linear in \( z \) direction, and shear stress of core is parabolic in \( y \) direction.

\[ \tau_f = \frac{V(\frac{1}{\tan \theta} - \frac{z}{\sqrt{(H - d)}})}{d + \frac{n}{6 \cos \theta} d_c} \]  

(7)

\[ \tau_c = \frac{V[\frac{1}{2 \tan \theta} \frac{d}{d_c} + \frac{1}{2 \sin \theta} \frac{1}{4} - \left(\frac{y}{H - d}\right)^2]}{\frac{1}{2} + \frac{n}{12 \cos d_c}} \]  

(8)

In face sheets, the normal stress \( \sigma_f \) is uniform along \( z \) direction and the maximum shear stress occur in \( z = 0 \). Stresses on face are defined as (9, 10):

\[ \sigma_f = \frac{V l}{d(H - d) + \frac{n}{6 \cos \theta} d_c (H - d)} \]  

(9)

\[ \tau_f = \frac{V}{n \tan \theta + \frac{n^2 \tan \theta}{6 \cos \theta} d_c} = \sigma_f \frac{1}{n \tan \theta} \frac{H - d}{l} \]  

(10)

In the core members, normal stress is maximum at the points of contact with the face sheets (\( y = \frac{H}{2} - d \)), and the shear stress is maximum in neutral axis (\( y = 0 \)). Stress in all points of core is defined as (11):

\[ \sigma_c = \frac{2 V l}{d + \frac{n}{6 \cos \theta} d_c} \frac{(H - d)^2}{y} \]  

(11)

\[ \tau_c = \frac{V[\frac{1}{2 n \tan \theta} \frac{d}{d_c} + \frac{1}{2 \sin \theta} \frac{1}{4} - \left(\frac{y}{H - d}\right)^2]}{\frac{1}{2} + \frac{n}{12 \cos d_c}} \]  

(12)

Where \( y \) is defined among 0 to \( y = \frac{H}{2} - d \), Replacing equations 9–12 in Mises relation, panel yielding would be determined:

\[ \sigma^2 + 3 \tau^2 = \sigma_y^2 \]  

(13)

Face yielding condition:

\[ \frac{V^2}{EM} = \frac{\sigma_c (H - d) [n \frac{d}{l} + \frac{n^2}{6 \cos \theta} \frac{d_c}{l}]}{\sqrt{n^2 + \frac{3}{\tan \theta} \frac{H - d}{l}^2}} \]  

(14)

Core yielding condition:

\[ \frac{V^2}{EM} = \min_{y \in \left(0, \frac{H}{2} - d\right)} \left\{ \frac{\sigma_c (H - d) [n \frac{d}{l} + \frac{n^2}{6 \cos \theta} \frac{d_c}{l}]}{\sqrt{4 \left(\frac{y}{H - d}\right)^2 + \frac{3}{n \tan \theta} \frac{H - d}{l} \frac{d}{l} + \frac{n^2}{6 \cos \theta} \frac{d_c}{l} + \cdots}} \right\} \]  

(15)

\[ \frac{1}{\sin \theta} \left(\frac{H}{l} - \frac{d}{l}\right) \left[\frac{1}{4} - \left(\frac{y}{H - d}\right)^2\right] \]

2.2 Buckling load

To estimate buckling load, two supports are considered, simply and fully clamped. Normal stress in face buckling is defined as (16) [3]:

\[ \sigma_{f\text{crit}} = \frac{K \pi^2 E}{12(1 - v^2)} \left(\frac{d}{\lambda_f}\right)^2 \]  

(16)

Where coefficients of \( K \) become following, also we have:

\[ \lambda_f = 2(H - d) / n \tan \theta \]  

(17)
To core member, shear and normal stress must be assumed. For panel with Core fold \( n = 1 \), critical shear stress is calculated as (18) [3]:

\[
\tau_{c}^{\text{cri}} = \frac{K_{c} \pi^{2} E}{12(1-\nu^{2})} \left( \frac{d_{x}}{\lambda_{c}} \right)^{2}
\]  

(18)

The critical normal stress is calculated as (19) [3]:

\[
\sigma_{c}^{\text{cri}} = \frac{K_{c} \pi^{2} E}{12(1-\nu^{2})} \left( \frac{d_{x}}{\lambda_{c}} \right)^{2}
\]  

(19)

Also we have:

\[
\lambda_{c} = (H - d) / n \sin \theta
\]  

(20)

To panel with \( n = 2 \), critical normal stress is calculated as [3]:

\[
\sigma_{c}^{\text{cri}} = \frac{K_{c} \pi^{2} E}{12(1-\nu^{2})} \left( \frac{d_{x}}{\lambda_{c}} \right)^{2}
\]  

(21)

And critical shear stress calculated by (18).

For panel with \( n = 4 \), two cases are possible:

Buckling of internal core and buckling of external core. In both cases critical shear stress would be calculated by equation (18), and for critical normal stress we have [3]:

\[
\sigma_{c}^{\text{cri}} = \frac{K_{c} \pi^{2} E}{12(1-\nu^{2})} \left( \frac{d_{x}}{\lambda_{c}} \right)^{2}
\]  

(22)

As you see, it should be achieved the coefficients of \( K_{c}, K_{bc}, K_{b}, \) and \( K_{s} \) to gain the shear and normal stresses of buckling. We act in following arrangement to achieve the coefficient of \( K_{s} \) in simply support and fully clamped. The differential equation of plate buckling is achieved equation (23) [10].

\[
\nabla^{4} w = \frac{1}{D} \left( 2N_{wz} \frac{\partial^{2} w}{\partial x^{2}} \right)
\]  

(23)

Where, \( D \) is a flexural rigidity and is gained from the equation (24):

\[
D = \frac{Ed^{3}}{12(1-\nu^{2})}
\]  

(24)

\( N_{wz} \) is an external fixed shear load, and \( W \) is the displacement resulting from buckling. One of the answers of this equation is:

\[
w(x, z) = F(z)e^{\frac{i \theta}{\pi}}
\]  

(25)

Where, in above relation \( \theta \) is a parameter related to wavelength of buckling mode in longitudinal direction of plate, and \( B \) is the width of plate. If we choose the function

\[
F(z) = c_{m} e^{\frac{imz}{\pi}}
\]  

(26)

Where \( C_{m} \) is a constant, by replacing it in the first equation, we have

\[
m^{4} + 2\theta^{2}m^{2} + \frac{2N_{wz}}{D} \theta m + \theta^{4} = 0
\]  

(27)

Equation above has four roots in exchange for every specific value of \( \theta \) which we present them with \( m_{1}, m_{2}, m_{3}, \) and \( m_{4} \). So, the answer of the Equation will be as below:

\[
w(x, z) = \left( c_{1} e^{\frac{im_{1}z}{\pi}} + c_{2} e^{\frac{im_{2}z}{\pi}} + c_{3} e^{\frac{im_{3}z}{\pi}} + c_{4} e^{\frac{im_{4}z}{\pi}} \right) e^{\frac{i \theta}{\pi}}
\]  

(28)

If we replace the answer equation in for equations related to boundary conditions and assume that of determinant of stability is equal to zero, it is to achieve the critical load \( F_{\text{cri}} \). If it is to choose the boundary conditions related to edges of \( x = l, x = 0 \) with very much distance from each other, the buckling coefficients related to simply supports and fully clamped and gained in this form.

\( K_{s} = 5.35 \) for the simply supports and \( K_{s} = 8.98 \) for fully clamped. Other buckling coefficients also gain in same method. Also we can gain these from experimental relations. The tables and the curves which has represented for buckling coefficient of \( K \) are exist.

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We can obtain easily the coefficient of buckling of plate with different boundary conditions and different loadings using the mentioned tables [10]. In these tables, it has presented the side ratio of \( \alpha = \frac{l}{B} \), in which \( B \) side is always a side that the loading is applied on it. The value \( \psi \) also is the ratio of the loading intensity of bottom fibre of plate of the loading
intensity of top fibre of that, which in here \( \psi = 1 \) and \( \alpha \geq 1 \)

<table>
<thead>
<tr>
<th>simply supports</th>
<th>fully clamped</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_c = \frac{8.4}{1.1+\psi} = 4 )</td>
<td>( K_c = 6.97 )</td>
</tr>
<tr>
<td>( K_b = 23.9 )</td>
<td>( K_b = 41.5 )</td>
</tr>
<tr>
<td>( K_{bc} = 7.81 )</td>
<td>( K_{bc} = 13.6 )</td>
</tr>
<tr>
<td>( K_s = \frac{5.34 + \frac{4}{\alpha^2}}{\alpha^2} = 5.35 )</td>
<td>( K_s = 8.98 )</td>
</tr>
</tbody>
</table>

\( \alpha \geq 40 \)

Table1. The coefficients of buckling for different statuses

Replacing value from equation (16) to (22) in equation (29), critical face and core buckling, for panels with \( n=1, 2, 4 \) would be determined [3]:

\[
\left( \frac{\sigma}{\sigma_{cri}} \right) + \left( \frac{\tau}{\tau_{cri}} \right) = 1 \tag{29}
\]

Face buckling:

\[
\frac{V^2}{EM} = \frac{K_c \pi^2 \sin^2 \theta}{24(l-1) \tan \theta} \left( \frac{H}{l} + \frac{d}{l} \right) \left( \frac{d}{l} \right)^2 \left( \frac{1}{n \tan \theta} + \frac{d}{l} \right)^2
\tag{30}
\]

Core buckling with \( n=1 \):

\[
\frac{V^2}{EM} = \frac{\pi^2 \sin^2 \theta}{12(l-1)} \left( \frac{H}{l} + \frac{d}{l} \right) \left( \frac{d}{l} \right)^2 \left( \frac{1}{n \tan \theta} + \frac{d}{l} \right)^2
\tag{31}
\]

Core buckling with \( n=2 \):

\[
\frac{V^2}{EM} = \frac{\pi^2 \sin^2 \theta}{24(l-1)} \left( \frac{H}{l} + \frac{d}{l} \right) \left( \frac{d}{l} \right)^2 \left( \frac{1}{n \tan \theta} + \frac{d}{l} \right)^2
\tag{32}
\]

Core buckling with \( n \geq 4 \):

\[
\frac{V^2}{EM} = \min \left\{ \frac{V^1_{\text{mode}}}{EM}, \frac{V^1_{\text{mode}}}{EM} \right\}
\tag{33}
\]

Internal core buckling with \( n \geq 4 \):

\[
\frac{V^2}{EM} = \frac{\pi^2 \sin^2 \theta}{12(l-1)} \left( \frac{H}{l} + \frac{d}{l} \right) \left( \frac{d}{l} \right)^2 \left( \frac{1}{n \tan \theta} + \frac{d}{l} \right)^2
\tag{34}
\]

External core buckling with \( n \geq 4 \):

\[
\frac{V^2}{EM} = \frac{\pi^2 \sin^2 \theta}{12(l-1)} \left( \frac{H}{l} + \frac{d}{l} \right) \left( \frac{d}{l} \right)^2 \left( \frac{1}{n \tan \theta} + \frac{d}{l} \right)^2
\tag{35}
\]

3 Finite element modeling

In this part, the accuracy of analytical results is compared by ABAQUS software. For this purpose critical load would be determined using finite element method, using commercial software ABAQUS and the result would be compared with analytical method's result. The panels are modelled by ABAQUS software and buckling analysis is performed by eigenvalue and post buckling methods and yielding analysis is performed by statically method. To perform this, we assume three panels with non-dimensional weight \( \psi = 0.01 \) with length of 1m and \( n=1, 2, 4 \) and cell width of \( B \) that are determined by equation (3).

Geometrical variables of \( H, d_c, d \) are defined based on figures (12) to (15). Quadratic thin shell element is used to core and face modelling. During modelling, all freedom degrees of nodes of one end of panel are limited to zero and freedom degrees of nodes of other end of panel would be moved in above and below directions. Deformation of panel with \( n=2 \) is shown in Fig.6.
Core and face buckling have occurred in this case. Post buckling diagram of this case is shown by Fig.7.

![Post buckling diagram](image1)

**Fig.7- Post buckling diagram**

According to this diagram, critical load is 195KN that is in conformity to analysis load. Results of modelling finite elements for \( n = 1, 4 \) are shown in Fig.8.

![Comparison of the FEM buckling analysis and the analytical method for the buckling critical loads](image2)

**Fig.8- Comparison of the FEM buckling analysis and the analytical method for the buckling critical loads**

Also analytical yielding loads are comparing with FEM method.

![Comparison of the FEM yielding analysis and the analytical method for the yielding critical loads](image3)

**Fig.9- Comparison of the FEM yielding analysis and the analytical method for the yielding critical loads**

4 Genetic algorithm

During recent years, genetic algorithm is used to solve science and engineering complex problems \([5, 6, \text{ and } 7]\). In contrast to gradient base optimization methods, the GA method works with global search methods. This helps significantly to avoid being trapped in local optima. The other advantage of this algorithm is that to calculate optimized points, there is no need to function derivation or gradient. Therefore there is no limitation for cost function as derivation or continuity. Therefore, genetic algorithm would be used to linear or nonlinear and continuous or discrete problems \([7]\).

In this method a population of abstract representations (called chromosomes or the genotype or the genome) of candidate solutions (called individuals, creatures, or phenotypes) to an optimization problem evolves toward better solutions. The evolution usually starts from a population of randomly generated individuals and happens in generations. In each generation, the fitness of every individual in the population is evaluated, multiple individuals are stochastically selected from the current population (based on their fitness), and modified (recombined and possibly randomly mutated) to form a new population. The new population is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population. Traditionally, solutions are represented in binary as strings of 0s and 1s, but other encodings are also possible. In this paper real number encoding is used. In the crossover operation shown in Fig.6, firstly crossover function operates on only two genes by equal number that would be selected as random. Then, the right hand genes of two mentioned genes in two selected strings would be replaced. Crossover function is as below.

![Crossover operation on two chromosomes](image4)

**Fig.10- Crossover operation on two chromosomes**
Two vectors, $C_1$ and $C_2$ would be selected as two strings to perform crossover.

$C^2 = (C_1^2, C_2^2, ... C_n^2)$

$C^1 = (C_1^1, C_2^1, ... C_n^1)$

Related gene would be selected as random as below:

$[C_{min} - 1α → C_{max} + 1α]$  

Where:

$c_{min} = \min \{C^1 \cap C^2\}$ and

$c_{max} = \max \{C^1 \cap C^2\}$

$l = c_{max} - c_{min}$

Mutation operand will operate as random variation in one of the genes. If characters are continuous numbers, mutation would be performed as positive or negative random performance around previous value. This type of mutation is creeping mutation [8].

5 Sandwich panel optimization, using genetic algorithm

In this paper, the purpose of optimization is the minimization of the weight of structure that subjected to the loading which is specified with the specific load index ($I_l$). Therefore structure weight is cost function that is calculated as (36):

$$w = 2 pdl + npd_c l / \cos \theta$$  \hspace{1cm} (36)$$

This equation is as below as non-dimensional.

$$\psi = \frac{w}{pl^2} = 2 \frac{d}{l} + \frac{n}{\cos \theta} \frac{dc}{l}$$  \hspace{1cm} (37)$$

The constraint that related to the face and core yielding is as below based on equation (14) and (15):

Face yielding constraint is:

$$\frac{V^2}{EM} \left[ \sigma_y \left( \frac{H}{l} \frac{d}{l} \left( \frac{d}{l} + \frac{n}{6 \cos \theta} \frac{d_c}{l} \right) \right) \right] \leq 1$$  \hspace{1cm} (38)$$

And core yielding constraint is (39):

$$\frac{V^2}{EM} \left[ \min \{V_{core buckling} \} \left( \frac{\sigma_y}{E} \left( \frac{H}{l} \frac{d}{l} \frac{d}{l} + \frac{n}{6 \cos \theta} \frac{d_c}{l} \right) \right) \right] \leq 1$$  \hspace{1cm} (39)$$

The constraint that related to the face and core buckling is as below based on equation (29) and (35):

$$\frac{V^2}{EM} \left( \frac{V^2}{EM} \right)_{core buckling} \leq 1$$  \hspace{1cm} (40)$$

Core buckling constraint is:

$$\frac{V^2}{EM} \left( \frac{V^2}{EM} \right)_{core buckling} \leq 1$$  \hspace{1cm} (41)$$

Where value of $(\frac{V^2}{EM})_{core buckling}$ in (41) is calculated by equation (31) to (35), to perform constraint in cost function, penalty function is used. In this method, large weight coefficients would be added to cost function and new cost function is defined as below:

$$f(x) = \tau(x) + 10^{10} G(x)$$  \hspace{1cm} (42)$$

Where:

$$G(x) = \sum_{i=1}^{k} w_i \langle g_i(x) \rangle$$  \hspace{1cm} (43)$$

Where:

$$\langle g(x) \rangle = 1 \text{ If principles denied}$$

$$\langle g(x) \rangle = 0 \text{ If principles not denied}$$

Design variables are $\frac{H}{L}$, $\frac{d}{l}$ and $\frac{d_c}{l}$. The angle between face sheet and core members, $\theta$, is fixed as $\theta = \tan^{-1} \sqrt{2} = 54.7^\circ$ because according this angle core shear stiffness is maximum [2].

Number of initial population, number of generations, crossover probability and mutation probability are 30, 500, 0/9, and 0/1 respectively.
Sandwich panels with corrugated core with various core fold \( n \) are considered under various loads and optimized weight would be determined. Therefore, 4 load index \( \Pi \) considered as 0/002, 0/0015, 0/001, 0/0005. Applied alloy is aluminium with \( E=70 \) Gpa and \( G_s=490Mpa \). Evolutionary diagram of genetic solving of problem which is provided to load index \( V/\sqrt{EM} = 0.002 \) and \( n=1 \) is shown in Fig.7.

Fig.11- Evolutionary diagram of genetic algorithm for \( V/\sqrt{EM}=0.002 \) and \( n=1 \)

The results of optimization are shown in Fig.12 to Fig.15. Fig.12 shows minimum weight diagram in relation with load index in various core fold \( (n=1, 2, 4) \) for fully clamped.

Fig.12- Minimum weight related to load index

Fig.13- Optimized non-dimensional distance between sheet faces

Fig.14- Optimized non-dimensional core thickness

Fig.15- Optimized non-dimensional face sheet thickness

It is seen that among core fold \( n=1, 2 \), there is no sensible difference based on minimum weight index but for \( n>2 \), by increasing number of core folds, optimized weight would be increased. Fig.13 to Fig.15 show changes of non-dimensional design parameters \( \frac{H}{l} \), \( \frac{d_c}{l} \) and \( \frac{d}{l} \) in accordance with minimum weight. Therefore the designer can provide optimized estimation of design parameters based on enforced load and mentioned features.

6 Conclusion

In this paper sandwich panels with corrugated core are modelled by ABAQUS software and eigenvalue and post buckling analysis are performed. Optimization results of analysis method and results of finite element analysis show accuracy of applied method. Optimization of sandwich panel with corrugated core is
performed, using genetic algorithm. By using genetic algorithm to perform crossover, specific method is used that is effective in rapid convergence of optimized point. Based on results, by increase of number of core folds, the rate of weight increase would be increased, by load increasing. So, by increasing of number of folds, gives no advantage. Therefore it is recommended to perform multi-objective optimization based on other cost functions as deflection, heat transfer coefficient and sound transfer coefficient with weight cost function.

References:


