Alternative Methods of Propagating Contradictory Evidence

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Abstract: This paper examines two alternative methods of propagating contradictory evidence in intuitionistic fuzzy sets using mass assignment techniques as the underlying semantics, and proposes a third. Earlier papers assigned contradictory evidence arising from a combinator by considering that contradictory evidence from either operand should result in a contradictory result. The monotonic increase means that a small amount of contradiction results in a very large contradiction component even after a relatively short reasoning path. Adopting an alternative approach that assigns contradictory evidence only where both operands contain contradictory evidence results in monotonically decreasing contradictory components. This approach would eliminate contradiction in the conclusion giving a specious validity to the result. It is proposed that, ideally a satisfactory propagation method should result in a contradictory measure that is independent of the length of the chain of reasoning. A final algorithm constructed from the earlier two is presented, which is independent of the length of the chain of reasoning. The level of contradiction in the final answer depends only on the contradictory evidence introduced during the reasoning, any contradictions arising directly from the combinations, and not on the length of the chain of reasoning.


1 Introduction

This work begins with Hinde’s [Hinde and Patching(2007)] approach to the propagation of contradiction in intuitionistic fuzzy sets [Atanassov(1983), Atanassov(1986), Atanassov(1999)]. The paper made no distinction between inconsistency [Patching et al.(2006)Patching, Hinde, and McCoy] and contradiction and this distinction was made in a later paper [Hinde et al.(2008a)Hinde, Patching, and McCoy], the terminology there will be adopted for in this paper. Another paper [Hinde et al.(2007)Hinde, Patching, and McCoy] linked the fields of mass assignment and intuitionistic fuzzy set theory generalising both to accommodate contradictory evidence and inconsistency [Patching et al.(2006)Patching, Hinde, and McCoy]. The monotonic increase in the contradiction measure means that a small amount of contradiction results, in a fairly short reasoning path, in a very large contradiction component. By adopting an alternative approach that results in monotonically decreasing contradiction a system would eliminate contradiction in the conclusion giving a specious validity to the result. A final system is proposed that has more useful properties.

An intuitionistic fuzzy set with a set of support $\mathbb{U}$ [Atanassov(1983)] is:

$$A^* = \{ (x, \mu_A(x), \nu_A(x)) \mid x \in \mathbb{U} \}$$

Subject to the constraint:

$$\mu_A(x) + \nu_A(x) \leq 1$$

1.1 Evidence

Although it is relevant to distinguish between positive and negative evidence, in this paper the evidences are distinguished by their degree of membership in the mass assignment triple and so we use the neutral notation $\{T, F\}$.

An intuitionistic fuzzy set with a set of support $\mathbb{U}$ that has contradictory evidence is represented as:

$$A^{**} = \{ (x, \mu_A(x), \nu_A(x), \iota_A(x)) \mid x \in \mathbb{U} \}$$

Subject to the constraint:

$$\mu_A(x) + \nu_A(x) + \iota_A(x) \leq 1$$
or to mimic Zadeh’s [Zadeh(1965)] the following notation will also be used:

\[ A^x = \{ (\mu_A(x), \nu_A(x), \iota_A(x)) \mid x \in U \} \]

An initial exploration of the relationship to mass assignment to intuitionistic fuzzy sets is given in \[\text{Hinde et al.}(2007)\text{Hinde, Patching, and McCoy}\]. The main contribution which shall be used here is the representation of mass as a triple:

\[ m_A(X) = (\mu_A(X), \nu_A(X), \iota_A(X)) \]

Note that the notation here uses \( X \) as \( X \subseteq A \) rather than \( x \) as \( x \in A \).

The elements are as follows:

\( \mu_A(X) \) the mass assigned to the membership function of \( X \subseteq A \)

\( \nu_A(X) \) the mass assigned to the non-membership function of \( X \subseteq A \)

\( \iota_A(X) \) the mass assigned to the contradictory votes of \( X \subseteq A \)

We need a set of selection functions to extract sub masses so the equations can be sensibly expressed, these are defined in Equation 1.

\[ m_A(X) = (\mu_A(X), \nu_A(X), \iota_A(X)) \]

The notation here has used the quantities \( \mu, \nu \) and \( \iota \) as projection operators to extract the values \( \mu, \nu \) and \( \iota \) from the mass triples. Although this is overloading we believe this is better as it is clear what they are extracting.

\[ m_A(\{\}) = (0.0, 0.7, 0.0) \]

Note that the notation here uses \( x \) as \( x \in U \) and are votes both for and against the set. These mass assignment triples give rise directly to an intuitionistic fuzzy set. Any intuitionistic set may be broken down into a mass assignment triple but the mapping is not unique.

\[ m_A(\{\}) = 0.7 \]

The intuitionistic set resulting from this assignment is shown in Equation 5.

\[ A = \{ (0.9, 0.0, 0.1) \mid a, \}
\[ \{0.8, 0.0, 0.0 \mid b, (0.6, 0.2, 0.0) \mid c \} \]

2 Propagation.

Propagation of contradictory evidence was explored in \[\text{Hinde and Patching}(2007)\] and \[\text{Hinde et al.}(2007)\text{Hinde, Patching, and McCoy}\] drew parallels with mass assignment but essentially used the same basis as \[\text{Hinde and Patching}(2007)\] used in Intuitionistic Fuzzy Sets for propagating contradiction. This paper explores three choices for the propagation of contradiction using the similarity function defined in \[\text{Hinde et al.}(2008b)\text{Hinde, Patching, and McCoy}\]. This requires computation of sub functions and so is a suitable vehicle for exploring propagation through several combinators. This requires some explanation to make this paper self contained.

2.1 Similarity.

The similarity function of \( A \sim B \) involves calculating the conditional quantity \( A \mid B, B \mid A \) and then the conjunction of these to form \( A \mid B \land B \mid A \) giving a definition of similarity as shown in Equation 6.

\[ A \sim B \triangleleft A \mid B \land B \mid A \]

The valuation of \( (A \mid B) \) involves generalising the semantic unification operator described in Patching et
\[ \mu(M(A \otimes B)) = \mu(M(A)) \odot_\mu \mu(M(B)) \quad (7) \]
\[ \nu(M(A \otimes B)) = \nu(M(A)) \odot_\nu \nu(M(B)) \quad (8) \]
\[ \mu'(M(A \otimes B)) = \{ \mu( m_{\frac{A}{B}}(X)) - \mu( m_{\frac{A}{B}}(X)) | X \in A \otimes B \} \]
\[ \nu'(M(A \otimes B)) = \{ \nu( m_{\frac{A}{B}}(X)) - \nu( m_{\frac{A}{B}}(X)) | X \in A \otimes B \} \]
\[ \iota(M(A \otimes B)) = SUP(\mu'(M(A \otimes B)), \nu'(M(A \otimes B))) \quad (9) \]

This approach maximises the amount of contradiction propagated. Essentially the result is contradictory if either operand has a contradictory element.

Another viewpoint is to take the minimum amount of contradiction implied by the operation. This is given by the Equations 10, 11 and 12.

\[ \iota(M(A \otimes B)) = \{ \iota( m_{\frac{A}{B}}(X)) \}
\]
\[ \mu( m_{\frac{A}{B}}(X)) | X \in A \otimes B \} \quad (10) \]
\[ \mu(M(A \otimes B)) = \{ \mu( m_{\frac{A}{B}}(X)) - \iota( m_{\frac{A}{B}}(X)) | X \in A \otimes B \} \quad (11) \]
\[ \nu(M(A \otimes B)) = \{ \nu( m_{\frac{A}{B}}(X)) - \iota( m_{\frac{A}{B}}(X)) | X \in A \otimes B \} \quad (12) \]

This approach minimises the amount of contradiction propagated. Essentially the result is contradictory only if both operands have a contradictory element.

Equations 7, 8 and 9 result in a monotonically increasing quantity of contradiction in a chain of reasoning, whereas Equations 10, 11 and 12 result in a monotonically decreasing quantity of contradiction in a chain of reasoning. This would result in a low level of contradiction for any reasonably sized chain of reasoning resulting on a false level of confidence about the result. In each case the increase and decrease is given that no new contradictory evidence is introduced.

It would be reasonable to ask for a level of contradiction in the result that depended on the contradiction in the initial operands rather than the chain of reasoning.

Surprisingly simply, the average of the two measures above results in the required result. If the amount of mass assigned to contradiction in both the initial operands is equal to \( C \) then the final contradiction in Equation 9 is \[ 1 - (1 - C)^2 = 2 \ast C - C^2, \]
whereas the final contradiction in Equation 10 is \( C^2 \). So the mean is \( \frac{2C - C^2 + C^2}{2} = C \), which is the original amount in both operands. If the initial quantities are \( C_1 \) and \( C_2 \) then the final amount is \( \frac{C_1 + C_2}{2} \). These are the choices if the operations are performed multiplicatively using mass assignment techniques as the underlying semantics.

3 Examples

The examples section will illustrate the propagation of contradiction using the similarity function derived in [Hinde et al. (2008b); Hinde, Patching, and McCoy].

3.1 Detailed Example

Starting with the mass assignment in Equation 13 we evaluate how similar this is to the mass assignment in Equation 15.

\[
\begin{align*}
\mu_A(\{\}) & = (0.1, 0.6, 0.0) \quad (13) \\
\mu_A(\{a\}) & = (0.2, 0.0, 0.1) \\
\mu_A(\{a, b\}) & = (0.1, 0.0, 0.0) \\
\mu_A(\{c\}) & = (0.0, 0.1, 0.0) \\
\mu_A(\{b, c\}) & = (0.0, 0.2, 0.0) \\
\mu_A(\{a, b, c\}) & = (0.5, 0.0, 0.0)
\end{align*}
\]

The corresponding intuitionistic set is given in Equation 14.

\[
\begin{align*}
A & = \{ (0.8, 0.0, 0.1) \mid a, \\
& (0.6, 0.2, 0.0) \mid b, (0.5, 0.3, 0.0) \mid c \}
\end{align*}
\]

\[
\begin{align*}
\mu_B(\{\}) & = (0.1, 0.6, 0.0) \quad (14) \\
\mu_B(\{a\}) & = (0.2, 0.0, 0.1) \\
\mu_B(\{a, b\}) & = (0.1, 0.0, 0.0) \\
\mu_B(\{c\}) & = (0.1, 0.1, 0.0) \\
\mu_B(\{b, c\}) & = (0.0, 0.2, 0.0) \\
\mu_B(\{a, b, c\}) & = (0.4, 0.0, 0.0)
\end{align*}
\]

3.1.1 Maximum contradiction

The similarity of the mass assignments in Equations 13 and 15 with propagation according to Equations 7, 8 and 9 results in the triple mass assignment in Equation 17.

\[
\begin{align*}
\mu_{A[B \land B]}(\{\}) & = (0.077, 0.284, 0.021) \quad (17) \\
\mu_{A[B \land B]}(\{F\}) & = (0.089, 0.372, 0.012) \\
\mu_{A[B \land B]}(\{T\}) & = (0.192, 0.0, 0.147) \\
\mu_{A[B \land B]}(\{T, F\}) & = (0.298, 0.0, 0.164)
\end{align*}
\]

The corresponding intuitionistic set is given in Equation 18.

\[
\begin{align*}
A \mid B & = \{ (0.49, 0.0, 0.3111) \mid T, \\
& (0.3873, 0.3721, 0.1762) \mid F \}
\end{align*}
\]

The total amount of mass contradiction in this example is 0.3439. This is given starting values of 0.1 for the two sets \( A \) and \( B \).

3.1.2 Minimum contradiction

The similarity of the mass assignments in Equations 13 and 15 with propagation according to Equations 11, 12 and 10 results in the triple mass assignment in Equation 19.

\[
\begin{align*}
\mu_{A[B \land B]}(\{\}) & = (0.1062, 0.4592, 0.0) \quad (19) \\
\mu_{A[B \land B]}(\{F\}) & = (0.1369, 0.4624, 0.0) \\
\mu_{A[B \land B]}(\{T\}) & = (0.2949, 0.0783, 0.0001) \\
\mu_{A[B \land B]}(\{T, F\}) & = (0.4619, 0.0, 0.0)
\end{align*}
\]

The corresponding intuitionistic set is given in Equation 20.

\[
\begin{align*}
A \mid B & = \{ (0.757, 0.078, 0.0001) \mid T, \\
& (0.599, 0.462, 0.0) \mid F \}
\end{align*}
\]

The total amount of mass contradiction in this example is 0.0001. This is given starting values of 0.1 for the two sets \( A \) and \( B \). However, the membership value plus the non-membership value for this set totals 0.061 and so the resolution of this assigning the excess to contradiction,
see [Hinde et al.(2008b)Hinde, Patching, and McCoy],
is shown in Equation 21:

\[ A \mid B = \{0.757, 0.078, 0.0001\} \mid T, \quad 0.538, 0.401, 0.061\} \mid F \} \] (21)

3.1.3 Average contradiction

The similarity of the mass assignments in Equations 13 and 15 with propagation according to the average of the two methods results in the intuitionistic fuzzy set 23.

The corresponding intuitionistic set is given in Equation 23.

\[ A \mid B = \{0.6786, 0.108825, 0.09015\} \mid T, \quad 0.538375, 0.437863, 0.0509625\} \mid F \} \] (22)

The total amount of mass contradiction in this example is 0.1. This is given starting values of 0.1 for the two sets A and B.

4 Conclusions

A similarity measure has been defined in terms of the conjoined degree of mutual support using a semantic unification, or conditionalisation, measure. The measures of similarity behave intuitively as they should. Based on the underlying semantics represented by mass assignment triples taking into account inconsistent evidence as well as contradictory evidence the measure propagates contradictory evidence strongly. There is insufficient space to explore the properties and draw comparisons between other similarity measures in this paper but this work will follow.

References:


