Applying Mathematical Programming and GRA Technique to Optimize E-Learning Based Educational Systems: Implementation and Teaching Skills

IRAJ MAHDAVI*1, HAMED FAZLOLLAHTABAR2, NARGES YOUSEFPPOOR1
1Department of Industrial Engineering, Mazandaran University of Science & Technology, Babol, Iran
2Young Researchers Club, Islamic Azad University, Babol Branch, Iran

Abstract: - The increasing use of an internet improved internet technologies as well as web-based applications. Also, increasing effectiveness of the e-Learning has become one of the most practically and theoretically important issues in both educational engineering and information system fields. The online training environment enables learners to undertake customized training at any time and any place. Therefore the costs and benefits in implementation is of significance. There is a competence set consisting of ideas, knowledge, information, and skills for solving a decision problem. In order to effectively acquire the needed skills in the competence set to solve the problem, finding appropriate learning sequences of the needed skills for decision makers should be taken into account. This paper concerns with identification of varied cost elements in e-learning educational system and optimization by the means of mathematical programming. Then an effective method to estimate the learning cost between any two skills by using the grey relational analysis is proposed.

Key-Words: E-learning systems; Mathematical programming; Grey relational analysis; Learning sequences

1 Introduction

E-Learning system is an internet based service like the application system or the internet based virtual course study service [1]. This system is able to be interpreted in various ways such as “computer based, education delivery system which is provided through the Internet”, or “an educational method that is able to provide opportunities for the needed people, at the right place, with the right contents, and the right time” [2]. The e-Learning system is one of many methods of the education (the teaching and learning procedure) that allows flexible learner-centered education. It is an information system based on the World Wide Web [3]. E-Learning provides an interdisciplinary approach to information technology and educational engineering, and an assessment of e-Learning effectiveness could also be achieved [4]. As of IT, the end user assessment, the quality of the information system, and the system’s user satisfaction could be measured. As of educational engineering, however, the learner’s academic achievement or the degrees of self-study ability could be measured. The academic achievement is an assessment of the learner’s e-Learning environment, while self-study ability is an assessment of one’s aptitude regarding his or her self-study [5]. This approach reveals the extensive and effective trends resulting from an e-Learning research [6]. Many researchers are quite divided over the various views regarding educational engineering and information systems. Many researchers are on exploratory level trying to get explanations regarding the variations of e-Learning effectiveness i.e., [7].

The tendency of educational engineering to introduce theoretical variables in order to explain e-Learning effectiveness is insufficient except for limited numbers of information systems i.e., [8]. Moreover, this approach of putting together information systems and educational engineering is rarely observed.

There is a competence set consisting of ideas, knowledge, information, and skills for solving a decision problem [9]. When decision makers have acquired the needed competence set and are proficient in it, they will be comfortable and confident in making decisions [10-11]. Otherwise, they must acquire the needed competence set to solve the problem. In order to acquire a needed set of skills to cope with a decision they face, finding appropriate learning sequences of acquiring needed skills is very necessary.
Skills can be roughly classified into two types: one is single skills, and the other is compound skills. A compound skill represents a collection of single skills that might be acquired by decision makers. A compound skill may facilitate the acquisition of other single skills [12-13]. For instance, the courses “marketing management”, “financial management” and “business policy” are needed single skills for obtaining a bachelor’s degree of business administration, and are denoted by \( m, f \) and \( b \), respectively. When both “marketing management” and “financial management” have been acquired or learnt, these two single skills form a compound skill, denoted by \( m \land f \). Usually, whether \( m \) or \( f \) has been acquired or not for an undergraduate student can be determined by considering the corresponding grade that can be got at the end of a semester. It seems to be easier for us to learn “business policy” after the aforementioned compound skill (i.e., \( m \land f \)) has been already acquired in comparison with the situation when only “marketing management” or “financial management” has been acquired. It is reasonable to assume that the determination of which compound skills are useful for facilitating the acquisition of single skills can be subjectively pre-specified by users.

The total cost of a learning sequence only consisting of single skills would be above that of a learning sequence consisting of single and useful compound skills. In many methods regarding the generation of learning sequences, such as the deduction graph with an integer programming method [13], the minimum spanning table method [14] and the stage expansion method [11], the learning costs are assumed to be known. Sometimes, the cost required for learning directly from one skill to another skill is measured by time or money. But, how much money or time will be spent is too subjective to be approved for all users. It is reasonably considered that, the larger the interrelationship between two skills, the smaller is the learning cost between these two skills. In fact, the skills in the competence set are strongly interrelated [12]. Another aim of this paper is to propose a relational analysis technique, the grey relational analysis (GRA) proposed by Deng [15], to determine the learning costs between two skills.

2 Costs Factor Identification for Implementation of an e-Learning System

Definitely cost analysis and budget allocation is a complicated task for e-learning [16]. Because of the pace of thrive of technologies and innovations and also the increase of functionality and effectiveness [17] for the time factors of new technologies in establishing e-learning has its own cost affects [18]. While you can expect e-learning to deliver an attractive return on investment, start-up costs for an enterprise-wide implementation are significant, and so is the cost of developing custom content. Cost issues can be mitigated, for example, by a phased implementation, using a hosted e-learning application instead of building one inside the firewall, developing a curriculum based on generic content with only a limited number of essential custom-built courses.

To find out whether the monetary value of the results exceed the cost of the training or not we need to know three items:

- The cost of both developing and delivering e-learning.
- The return or benefits.
- The period over which benefits accrue.

We analyze learning development and delivery costs under five headings: direct, indirect, opportunity, fixed and variable.

2.1 Cost Optimization for Implementation

Considering the stated costs, to make the implementation and administration of an e-learning system of education more economic an optimization is essential. In this paper this optimization is done by mathematical programming. The notations are as follows:

**Notations:**

- \( C_F \): Fixed costs
- \( C_V \): Variable costs
- \( C_D \): Direct costs
- \( C_I \): Indirect costs
- \( C_O \): Opportunity costs
- \( C_{Development} \): Cost of development
- \( C_{Purchase Material} \): Cost of purchasing material
- \( C_{Hardware} \): Cost of hardwares
- \( T \): Time unit (Hour)
- \( t' \): Period of time per month unit
- \( E_1 \): Initial Expense of development
- \( E_2 \): Expense of Purchasing materials
- \( E_3 \): Expense of Hardwares
- \( R \): Revenue of a user per hour
- \( F \): Facility payment to students
- \( \phi \): User’s ability level
- \( I \): Interest rate
- \( X_V \): Variable cost per each user limited to \( B_2 \)
- \( X_F \): Fixed cost per an appropriate number of user limited to \( B_3 \)
- \( N \): Total number of users \( N=1,2,3,...,N \)
- \( M \): Subset of users \( M=1,2,3,...,S \)

Number of subsets is defined in levels as follows:
Let $K$ and $f_p$ ($1 \leq p \leq K$) denote the number of teachable single skills, which are truly needed for solving a problem by a student, and the $p$th single skill, respectively. Also, let $c(f_i, f_j)$ denote the learning cost of a student from a teacher for directly learning $f_j$ from $f_i$. It is considered that if the interrelationship that exists between two single skills, say $f_1$ and $f_3$, are much larger than another two skills, say $f_2$ and $f_3$, then it is more practical to acquire $f_3$ from $f_1$ instead of from $f_2$. In other words, $c(f_1, f_3)$ and $c(f_2, f_3)$ should reflect grades of interrelationships such that $c(f_1, f_3) < c(f_2, f_3)$. In this paper, $c(f_i, f_j)$ is interpreted to be unhelpful grades for acquiring $f_j$ from $f_i$. In the proposed method, the learning costs between any two single skills are first obtained by the GRA.

GRA is a relational analysis technique that can measure the degree of interrelationships between one major sequence and the other sequences in a given system [19]. The use of the GRA aims to find the learning cost between any two single skills. Then, a learning cost table is constructed.

### 3.1 Grey relational analysis

As mentioned above, there is an interrelationship between any two single skills. Thus, it is necessary to find the learning costs that can reflect the interrelationships. It is pertinent to treat each needed skill $f_s$ as a subsystem, and its finite output data series is evaluated by $n$ different criteria as $(f_{p_1}, f_{p_2}, \ldots, f_{p_n})$, where $n$ is the number of criteria and $f_{p_j}$ denotes the performance value for the $j$th criterion. Hence, let $(f_{p_1}, f_{p_2}, \ldots, f_{p_n})$ denote $f_{p_j}$.

Given one reference sequence, say $f_{p_j}$ ($1 \leq j \leq K$), and some comparative sequences, say $f_i$ ($1 \leq i \leq K$), we can easily obtain the grey relation between $f_{p_j}$ and $f_i$ by viewing $f_{p_j}$ as a desired goal [20]. Formally, given the reference sequence $f_{p_j}$ and the comparative sequences $f_i$ with the normalized form, the grey relational coefficient (GRC) $\xi(f_{p_j}, f_i)$ between $f_{p_j}$ and $f_i$ ($1 \leq j \leq n$) is able to be computed as follows [19,21]:

$$
\xi(f_{p_j}, f_i) = \frac{\Delta_{\min} + \rho\Delta_{\max}}{\Delta_{\min} + \rho\Delta_{\max}},
$$

where $\rho$ is the discriminative coefficient $(0 \leq \rho \leq 1)$, and usually $\rho = 0.5$ [19]. It should be noted that the appropriate value of $\rho$ is dependent on requirements of individual applications. Moreover,
\[ \Delta_{\text{min}} = \min_j \min_i \left| f_{p_i} - f_{i_j} \right|, \quad 1 \leq i \leq k, \quad 1 \leq j \leq n \]  
(8)

\[ \Delta_{\text{max}} = \max_j \max_i \left| f_{p_i} - f_{i_j} \right|, \quad 1 \leq i \leq k, \quad 1 \leq j \leq n \]  
(9)

\[ \Delta_{ij} = \left| f_{p_i} - f_{i_j} \right| \]  
(10)

where \[ \cdot \] denotes the absolute value. Clearly, \( \xi(f_{i_j}, f_{p_i}) \) is between zero and one. Then, the grey relational grade (GRG) denoted by \( \gamma(f_{i}, f_{p}) \) can be computed as follows:

\[ \gamma(f_{i}, f_{p}) = \frac{1}{n} \sum_{j=1}^{n} \xi(f_{i_j}, f_{p_j}), \]  
(11)

\[ 0 \leq \gamma(f_{i}, f_{p}) \leq 1 \]  
thus holds. Let \( c(f_{i}, f_{j}) \) denote the learning cost for directly learning \( f_{j} \) from \( f_{i} \). \( c(f_{i}, f_{j}) \) is further defined as follows:

\[ c(f_{i}, f_{p}) = 1 - \gamma(f_{i}, f_{p}), \quad 1 \leq i, p \leq K. \]  
(12)

It can be seen that, the larger the relationship that exists between two skills, the smaller is learning cost between these two skills. When \( c(f_{i}, f_{p}) \) has been determined by the GRA, an initial learning cost table like Table 1 can be generated, in which it can be seen that there are \( K \) single skills. In Table 1, only \( K^2 - K \) learning costs are taken into account. That is, it is not necessary to consider \( c(f_{p}, f_{p}) \) since it is impossible to learn a single skill, say \( f_{p} \), from \( f_{p} \). It should be noted that, if a learning cost exists between any two skills in a competence set, then the expansion of this competence set can be categorized as a cyclic expansion problem [12].

4 Discussion and Conclusions

The initial purpose of this paper is the cost optimization in e-learning system of education. To do that, different cost factors have been discussed. The elements of cost are substantial in implementing e-learning systems; therefore identifying them and trying to minimize them lead to advantages in enforcement of educational organizations. The approach which has applied in this paper for cost optimization is mathematical programming. By mathematical programming, the optimal values of decision variables would be achieved that are helpful tools in decision making for now and future of educational organizations.

The main aim of this paper is to propose a novel method to estimate the costs of learning of a student from one skill to the other skill by a teacher. Furthermore, each skill has its synthetic performance value by evaluating on different criteria. The GRA is first employed to derive the learning cost between two single skills. Then, a learning cost table is generated. The reason for not considering the compound skills at the same time is to keep the cost of learning one single skill from the other single skill fixed, and to consider the variability of the compound skills. In comparison with the single skills, which compound skills is more useful are more dependent on the subjective thinking and perception of users. For the future study, subsequently, a RBFN is trained by using the learning cost table to realize the interrelationship between two single skills and the corresponding learning cost. By presenting an input–output pair consisting of one compound skill and one single skill to the trained network, the corresponding cost of acquiring the single skill from the compound skill can be obtained. The advantage of the proposed method is to provide a reasonable way to estimate the learning costs by measuring the grade of the relationship between any two single skills instead of using money or time.

| Table 1. Learning cost table with \( K \) single skills |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|                  | \( f_1 \)         | \( f_2 \)         | \( f_3 \)         | \( f_4 \)         | \( f_5 \)         |
| Single skill     |                  |                  |                  |                  |                  |
| \( f_1 \)        | -                | \( c(f_1,f_2) \) | \( c(f_1,f_3) \) | \( c(f_1,f_4) \) | \( c(f_1,f_5) \) |
| \( f_2 \)        | \( c(f_2,f_1) \) | -                | \( c(f_2,f_3) \) | \( c(f_2,f_4) \) | \( c(f_2,f_5) \) |
| \( f_3 \)        | \( c(f_3,f_1) \) | \( c(f_3,f_2) \) | -                | \( c(f_3,f_4) \) | \( c(f_3,f_5) \) |
| \( \ldots \)     | \( \ldots \)     | \( \ldots \)     | \( \ldots \)     | -                | \( \ldots \)     |
| \( f_{k-1} \)    | \( c(f_{k-1},f_1) \) | \( c(f_{k-1},f_2) \) | \( c(f_{k-1},f_3) \) | \( \ldots \) | \( c(f_{k-1},f_5) \) |
| \( f_k \)        | \( c(f_k,f_1) \) | \( c(f_k,f_2) \) | \( c(f_k,f_3) \) | \( c(f_k,f_4) \) | -                |
References