In-depth Analogies among Entropy, Information and Sensation The concept of Time in Thermodynamics

Georges Maniatis, Eustathios Reppas & Vassilis Gekas

Department of Environmental Engineering, Technical University of Crete, Polytechnioupolis, Chania 73100, Crete, GREECE

Abstract: In this paper the analogies between properties such as entropy, information, sensation are highlited. Time seems to belong to the same type of properties. Time behaves as entropy as is depicted in diagrams of the growth of living species and in other examples such it is the respiratory ability as a function of the age. The challenge, in general, is to modify the Boltzmann equation to take into account the deviations from the continuous increasing trend of the classic Boltzmann expression.

Keywords: Entropy, Information, Sensation , Perception, Time, Boltzmann equation, bits, digits, nats, Boltzmann's constant

1. Introduction

Time does not belong to the parameters of Thermodynamics. At least this is the case in the Classical Thermodynamics of equilibrium where enough time should be given to the systems during a change towards equilibrium. As Ilya Prigogine puts it, a process is reversible only in the limit of infinite slowness[1].

Only, in the field of the Irreversible Thermodynamics, time is first introduced, although indirectly, and namely in the thermodynamic fluxes. It is known from the Entropy production theorem, which also can be derived from the entropy ensemble and also from the 1st thermodynamic law, a differential change of entropy dS is given by:

$$TdS = dU + PdV + \sum_k \mu_k dN_k + \sum_r A_r d\xi_r + \gamma dA + .. \quad (1)$$

The term in the left hand of equation is recognizable as the heat supplied to the system and in the right hand we have the change of the Internal energy of the system and a sum of possible work terms, as for example, volume change work, mass transfer type of work, chemical reactions associated work , surface tension work and so on.

In the general case the equation is

$$T\,dS = dU + \sum_i X_i dYi \quad (2)$$

X_i is thermodynamic force and Y_i the conjugative thermodynamic flux.

In any irreversible process of a closed system, $TdS > 0$.

Contrary to the reversible processes which take an eternity in order to be accomplished, the irreversible processes occur in a finite amount of time. So that we could make the following statement

"Time is not ab initio a parameter in Thermodynamics but its closest relative is Time".

A challenge is to consider the entropy change in an open system such as it is a living system. In a living system Time doesn't only bring decay.

Flourishing should be first occurring, then decay.

Therefore Entropy cannot only be the Law of Downhill. In this paper the analogies among the properties of Entropy, Information and Sensation are explored and the concept of Time is added to the category of properties which are described by a similar equation, the equation Boltzmann.

2. The equation of Boltzmann

The following properties: Entropy, Information

Sensation, Perception are expressed by the same equation:

$$S=k *\ln W \qquad (3)$$

Sometimes the symbols are different but , for the sake of this paper's discussion, we denote by S the above mentioned properties and by W the property whose the logarithm is taken. Also the kind of the logarithm can vary, of course. In Figure 1 the decadic logarithm is used. In the Information equation sometimes the logarithm of base 2 is convenient, anyway the reader should consider the logarithm in equation 3 in the general case i.e. the logarithm of any possible base, either 10 or e or 2.

Figure 1.The equation Boltzmann is the only one appeared in a tomb. In this case in the tomb of a Great Man.

For Information the equation is known as the Shannon equation and the Information is called the Hausdorf property symbolized ny H. For Sensation or more generally Conception, the equation is known as the Weber-Heffner Law.

3. The concept of W

W, is the first letter of the German word Warcheinlichkeit. Sometimes instead of W the Greek symbol Ω is used. This corresponds to the English multiplicity, not to the probability. Multiplicity is the number of possible events or arrangements or microstates. Probability of each of those events is thus the opposite of W. The multiplicity covers the cases of entropy and Information. In the case of sensation, W is the stimulus, for example a weight, the brightness of a star, the hydrophobicity of the part of a molecule, or the loudness.

In all cases W is an accumulation term, something quantitative, a quantity whose logarithmic change gives an Intensity, S, thus becoming logarithmically dependend on W.

4. The constant k.

The constant k depends on the pair of the two properties S-W:

If S is the Entropy then it is known that the Boltzmann constant is the ratio of the universal constant of the ideal gases R and the Avogadro number A and equal to

$$k_B= R/N_A = 1.381x10^{-23} \text{ J/K} \qquad (4)$$

If S is the Information , then the value of the constant depends on the kind of logarithm used and the units of Information. S is the number of questions, to be answered by yes or no, that are required in order to distinguish a certain microstate from a set of microstates that define the macrostate or an event from a possible set of events (equiprobable). The following kinds of logarithmic function are used

Kind of logarithm	Information units
• $\ln_2$	bits
• ln (the natural)	nats
• log	digits

Under this use of the above units for the Information, the k constant is equal to one.

If the natural logarithm is used and we wish the Information in bits then k=1/ln2. If the decadic logarithm is used and we wish the Information in bits the constant k=1/log2, and so on.

If S is Sensation we distinguish various cases.
In the case of acoustic sensation the pair of properties is decibel scale (S) vs sound level (W).
The threshold of hearing is 10^{-12} W/m^2 corresponding to 0 dB . The threshold of pain is 10 W/m2, that is 10^{13} higher than the level of TOH. This corresponds to 130 dB for the S property. Therefore the value of the constant k is 130 dB/W/m^2
In the case of stellar brightness the pair is
S: Stellar magnitude W: star's brightness
Hipparchus, with the aid of the naked eye, was the first to create the scale considering the faintest stars to be of 6th magnitude ,and the brightest stars to be 1st. In the centuries to come the scale has been extended towards to even brighter and to eveb more faint stars. The magnitudes s1 and s2 for two stars are related to the corresponding brightnesses w1 and w2 through the equation

$$s1-s2 = 2.5 [\log(w1)-\log(w2)]. \quad (5)$$

the constant obviously being 2.5.

5. The perception of sweetness

- Recent findings relate the sweetness of food substances to a geometrical structure, in the form of a certain triangle (see Figures 2 & 3). AH is a donor of protons, B a reciver of protons and X is a hydrophobic group.[2]

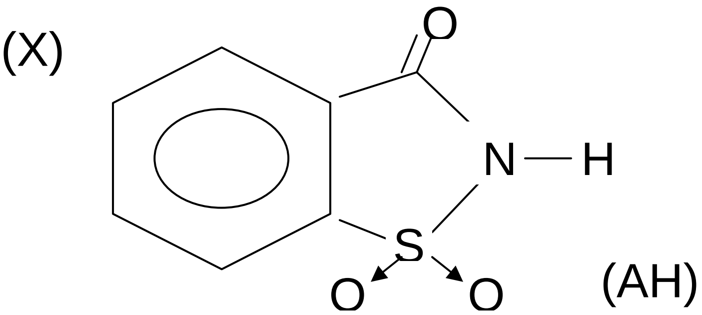

Fig. 2. A sweet molecule with the triangle detail A-BH-X.

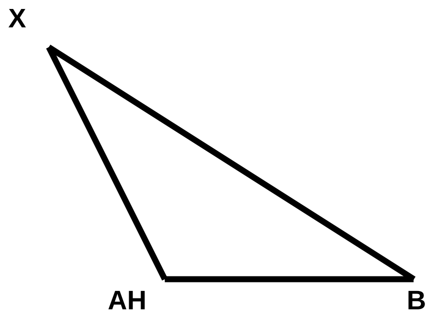

Figure 3. The triangle of sweetness

As X increases the sweteness also increases. The relationship is according to Weber-Heffner logarithmic. However exceeding a value of X there is a discontinuity: the sweteness is converted to bitterness! A way to modify the Boltzmann equation in that case is the one provided in the Fig. 4 below

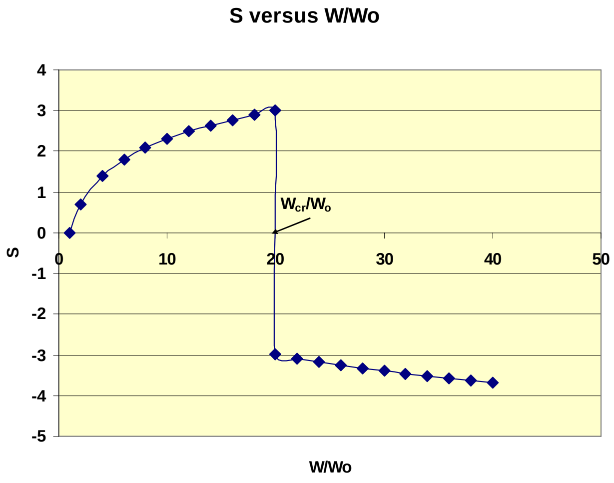

Figure 4 Swetness as a function of hydrophobicity of the X group.

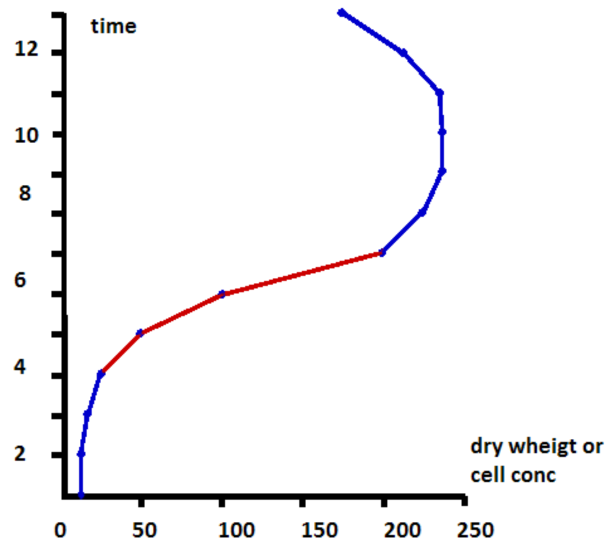

Figure 5. Time in arbitrary units plotted as a function of a growth parameter (dry weight or cell concentration)

6. The perception of Time

Our perception of time, is strongly dependent on the time scale. As for an example, we tend to extrapolate linearly from observations in a short time scale, because in a logarithmic scale, we feel the time period in which we live as a huge time interval. Expanding those ideas to the concept of Time, we distinguish between the astronomical or objective time and the perceived kairos or “Temps vaicu”.The two notions of time are most probably logarithmically connected in a relationship analogous the Boltzman equation.

7. Time vs Growth in living species

It is known [3] that during an initial lag phase the rate of growth or cell division is very slow. Growth or cell division then starts to accelerate into the exponential phase - when, for example, with a unicellular organism (e.g.yeast species) any 1 cell produces 2 in a given period of time, those 2 produce 4, the 4 produce 8, 8 produce 16 and so on . This exponential phase in a diagramm growth parameter vs time in a time vs growth parameter diagram will be represented by a logarithmic part of the curve (central red region in Fig.5) This shows te period when the fungus is growing or multiplying most rapidly. This phase will continue until one or more nutrients become limiting, oxygen becomes depleted and/or metabolic by-products accumulate to toxic levels, (decline). This may be followed by a stationary phase, during which there is no discernible change in cell concentration or biomass. Finally, we may observe a phase of cell death and lysis - which results in a decrease in cell number and/or biomass (Figures 5).

If the X axis becomes logarithmic i.e. time vs plot $\log_e$ (dry mass or cell concentration) the exponential phase of growth is now represented by the linear (straight line) red region. The slope of this red region is now constant and represents the inverse of the specific growth rate (or relative growth rate) of the fungus = $1/\mu$. μ is a measure of the rate of change in biomass or cell concentration relative to the biomass or cell concentration already present.

So we are not just measuring the rate of change in biomass or cell concentration (i.e. dN/dT, change in biomass divided by change in time), but the rate of change relative to the biomass or cell concentration already present (i.e. $(dN/dT)/N = \mu$). If all the conditions are optimal for growth of the fungus then the maximum specific growth rate (μ_{max}) is obtained - this is characteristic for any particular organism .

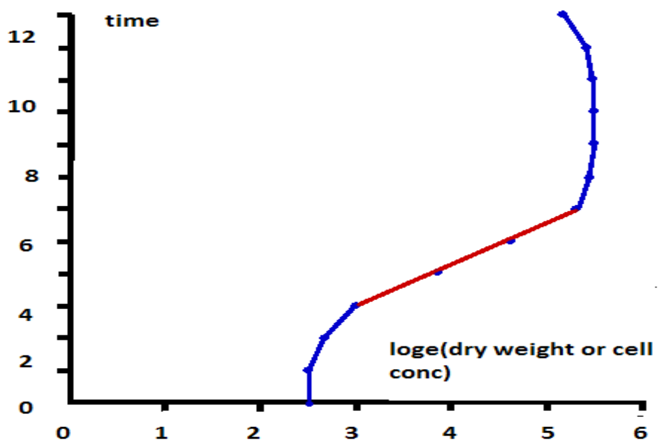

Figure 6. Time as a function of the logarithmic growth parameter

8. Respiratory Ability

After the age of 25 years an exponential decay is observed in the respiratory ability of Man, measured by the volume of exhaled air or CO_2, [4]. The equation describing this decay is

$$w = w_{max} - Be^{t} \quad (6)$$

If we consider w/w max as the relative respiratory ability we get Fig. showing a natural exponential decay. For smokers the B constant is of course higher. Plotting time vs the relative respiratory ability Fig.8 is obtained showing a logarithmic, again, relation of the Time.

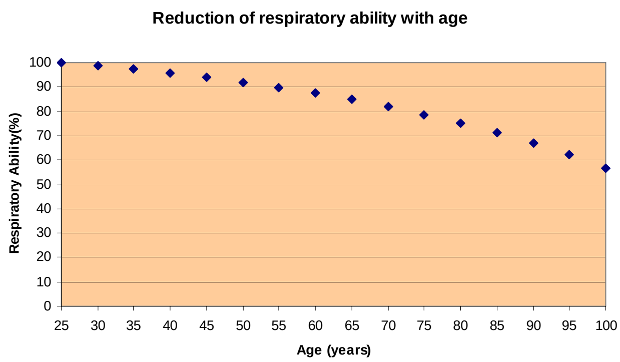

Figure 7. Relative respiratory ability vs age

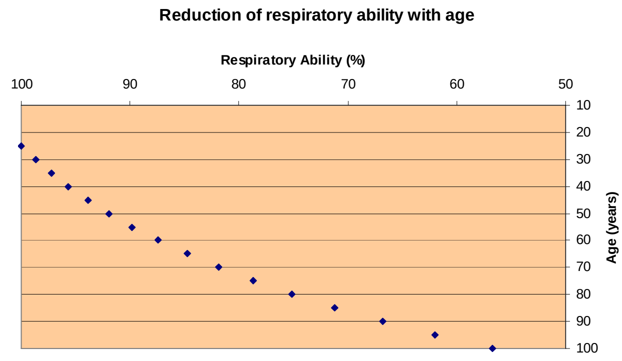

Figure 8. Time vs realative respiratory ability

9. Concluding Remarks

In this paper the analogies between properties such as entropy, information, sensation have been highlited. Time seems to belong to the same type of properties. Time behaves as entropy in the figures 5-8 at least for an interval of the W function. The challenge is to modify the Boltzmann equation to take into account the deviations from the continuous increasing trend of the classic Boltzmann expression.

References

[1]Dilip Kondepudi, Ilya Prigogine, Modern Thermodynamics, John Wiley & Sons, England, 1998

[2] H.D Belitz, W Grosh, P. Scieberle, Food Chemistry, Springer Verlag, Berlin, 2004

[3] Introduction to Microbiology : A Case-History Study Approach by John L. Ingraham, Catherine A. Ingraham, Publisher: Brooks Cole, 2000

[4] Frank H. Netter, The CIBA collection of Medical Illustrations, Vol.7 The Respiratory System, 1992, 3d ed., CIBA-GEIGY Corporation publications