

Teaching Linear Algebra, Analytic Geometry and Basic Vector Calculus with Mathematica at Riga Technical University

I. VOLODKO

Department of Engineering Mathematics
Riga Technical University
1 Meza street, Riga
LATVIA

V. KOLISKINA

Department of Engineering Mathematics
Riga Technical University
1 Meza street, Riga
LATVIA

Abstract: - In the present paper we discuss a new teaching module used at Riga Technical University to teach first year linear algebra, basic vector calculus and analytic geometry. The module consists of three components: lectures, tutorials and computer labs. Lab sessions run every second week and cover (at least partially) the material which is discussed during lectures and tutorials. The results of students' survey related to the use of Mathematica labs during the course are presented.

Key-Words: - undergraduate mathematics, Mathematica package, linear algebra

1 Introduction

Riga Technical University (RTU) is the largest engineering school in the Baltic States. It was founded in 1862. At present about 1800 first year students study mathematics at RTU. Department of Engineering Mathematics (together with the Department of Probability Theory and Statistics) is involved in teaching basic mathematical disciplines such as calculus or analytic geometry to all engineering students at RTU. It should be pointed out that these disciplines are not taught as separate courses like Calculus or Linear Algebra but are integrated in one course ("Higher Mathematics"). This is two or three-semester course (depending on the program). The first semester covers linear algebra, basic vector calculus, analytic geometry and differential calculus of one and several variables. The teaching module used two years ago had the following characteristics: three hours of lectures and three hours of tutorials each week (the length of each semester at RTU is 16 weeks).

In order to make mathematical courses more attractive to engineering students and to bring new technology to the classroom the Department of Engineering Mathematics came up with the initiative to change the way mathematics is taught at RTU. In particular, it was suggested to re-distribute the teaching load in accordance with the following

schedule: three hours of lectures, two hours of tutorials and one hour of labs. Labs run eight weeks per semester (two hours every second week). The package "Mathematica" is used to illustrate basic concepts of linear algebra, analytic geometry and vector calculus.

2 The content of lab sessions

Research was done prior to implementation of the new teaching strategy with the aim to understand basic principles that are used to teach students mathematical concepts with the help of packages like Mathematica, Matlab or Maple.

In 1990 the Curriculum Study Group was formed in the USA to work on teaching reform in linear algebra [1]. Five major points are suggested in the report: (1) The syllabus should respond to the client disciplines, (2) The first linear algebra course should be matrix-oriented, (3) Faculty should consider the needs and interests of students, (4) Technology should be used in the classroom, (5) At least one follow-up course is required.

It is recognized nowadays that methods of linear algebra (such as eigenvalue analysis) are extremely important in other engineering disciplines. On the other hand, "classical" linear algebra courses usually suggest methods which are hardly used in

applications (for example, the approach for eigenvalue calculations is based on a characteristic polynomial). In practice, however, eigenvalues are calculated by matrix algorithms such as QR algorithm [2].

The use of Mathematica allows the instructor to show the way real problems are solved in engineering applications. There are several textbooks that are devoted to teaching linear algebra, analytic geometry and basic vector calculus with Mathematica [3] – [7]. The ideas discussed in [3] – [7] are used to develop lab sessions for Higher Mathematics course at RTU. All the materials needed for lab sessions are available to students through **Blackboard** system. A brief description of the topics covered in the lab is presented below.

First, a short introduction to Mathematica is given. In practice, one two-hour session is devoted to basic principles of Mathematica. During this session students learn how to enter Mathematica commands and how to use graphical tools to display the results. No previous knowledge of programming is expected. We found that for some students it was difficult to understand that, for example, all the commands in Mathematica must start with capital letter. Another difficult task was to keep in mind that all the arguments in any built-in Mathematica function must appear in square brackets (not curly brackets or parentheses). Our experience has shown that students need more time to understand the concepts. Therefore, some time in the end of each session is devoted to individual work

Second session is devoted to matrix operations and solutions of systems of linear equations. In the beginning of this session the instructor explains how to enter matrices in Mathematica (using the command **Input** \Rightarrow **Create Table/Matrix/Palette**). Then basic rules of matrix algebra are explained. For example, students are asked to enter two matrices,

say, $a = \begin{pmatrix} 2 & 5 \\ -1 & 4 \end{pmatrix}$ and $b = \begin{pmatrix} 1 & -3 \\ 2 & 7 \end{pmatrix}$, and

calculate $a + b$, $a - 2b$ and ab . Next, the following commands are explained in detail: **Det[a]**, **Transpose[a]**, **Inverse[a]** and **MatrixRank[a]**, which are used to calculate the determinant of a matrix, find the transpose and the inverse of the given matrix and compute the rank of the matrix. As an exercise students are given the following

problem: Find the inverse of $A = \begin{pmatrix} 1 & 4 & 2 \\ -3 & 6 & 5 \\ 0 & 1 & 8 \end{pmatrix}$ and

check the obtained result using the definition of the

inverse matrix. It should be pointed out at this stage that there are a few individual homework assignments in higher mathematics course that cover different topics such as linear algebra and analytic geometry. Students have to submit their solutions by doing calculations on a paper. The use of Mathematica allows them to check the results and in addition, sends the message that there are useful commands in Mathematica that can be used to solve different mathematical problems. The last part of the second session is devoted to the solution of linear systems using commands **LinearSolve[a,b]** and **RowReduce[c]**. The instructor should pay students' attention to the fact that the command **LinearSolve** should be preferably used in cases where it is known that the solution is unique. Despite the fact that **LinearSolve** can also give an answer in the case where the corresponding linear system has infinitely many solutions, it should be pointed out by the instructor that the command does not give the complete solution – only one solution is given by Mathematica. In order to obtain the general solution to a linear system one should use the command **RowReduce** which presents the reduced row-echelon form of the augmented matrix of the system.

Two-dimensional graphs (in Cartesian coordinates, in polar coordinates and parametric plots) are discussed during the third session. Different options of the **Plot** command are discussed in detail. In addition, three-dimensional graphics is considered as well. There are no lectures on three-dimensional surfaces in the course so that this topic is discussed only during the lab session. However, additional materials that are available online help students to understand the ideas better.

The first part of the last session in this module (linear algebra, analytic geometry and basic vector calculus) is devoted to algebraic transformations. Our experience shows that first year students have, in general, rather poor knowledge of algebraic transformations such as polynomial factorization, addition of two fractions or simplification by cancellation. Therefore, several commands such as **Expand**, **ExpandAll**, **Simplify** and **FullSimplify** are discussed to improve students' problem-solving skills. The second part of the last session deals with basic vector calculus. Students learn how to use the commands **Norm** (to calculate the modulus of a vector), **Dot** (to calculate the dot product of two vectors) and **Cross** (to calculate the cross product of two vectors). A typical problem which covers all the topics discussed (which is also included as a problem for one of the homework assignments) is the following.

Given two vectors $\vec{a} = (2, -3, 1)$ and $\vec{b} = (4, -1, 2)$ find the modulus of \vec{a} , the dot product of \vec{a} and \vec{b} , the cross product of \vec{a} and \vec{b} and the angle between the vectors \vec{a} and \vec{b} .

The other lab sessions are devoted to differential calculus. The last lab session in the semester is reserved for the test. The test is on a pass/fail system. Duration of the test (in computer lab) is 45 minutes, six questions are asked in the test. An example of a test is presented below.

Test questions

1. Solve the system of linear equations using the inverse of a coefficient matrix:

$$\begin{cases} x - 2y + 3z - 4w = 4 \\ x + y - z + w + 3 = 0 \\ x + 3y + 2z - 3w = 1 \\ w - 7y + 3z = -3 \end{cases}$$

2. Find all first-order partial derivatives of the function $f(x, y) = \ln \tan(x - 2y)$, and simplify the result.

3. Plot the graph of the function $f(x) = \frac{3}{2x^2} \ln(x + \sqrt{x^2 + 4})$ on the interval $(-7, 5)$.

4. Find the angle between the planes

$$\begin{cases} 3x - 2y + 6z - 12 = 0 \\ -3x - 5y + 2z + 4 = 0 \end{cases}$$

5. Plot the three-dimensional graph of the function given in the parametric form by the relations $x = \cos t(3 + \cos u)$, $y = \sin t(3 + \cos u)$, $z = \sin u$, $t \in (0, 2\pi)$, $u \in (0, 2\pi)$.

The result should be displayed from the point $M(-1; 2; 5)$.

6. Find the first derivative of the function $y = y(x)$ which is given (implicitly) in the form $x^3 y - 3x^2 y^2 + 5y^3 - 3x + 4 = 0$.

The maximum test score is 18 points. Student passes the test if he/she scores at least 12 points. During the test students can use all the information available (lecture notes, practical examples solved in class, Mathematica help system). Students can have a few attempts to pass the test. The number of attempts is limited only by the time between the date of the first test (the last week of the semester) and the date of the final exam. Large data bank is used to select test problems.

It can be seen from the test questions shown above that some of the questions require mathematical knowledge of the subject. For example, in order to solve Problem 1 correctly one has to rewrite the last equation in the system (in order to obtain the correct form of the coefficient matrix). Formula for the calculation of the angle between planes is needed to solve Problem 4. Thus, the test is used (at least partially) to force students to understand mathematical concepts; the knowledge of Mathematica functions may not be enough to pass the test.

3 Statistical survey

In the end of the first semester we have conducted a statistical survey in order to assess students' reaction to the use of Mathematica in first year course on higher mathematics. A survey was conducted among 89 first year students studying computer science at RTU. Two questions assumed "Yes" or "No" answer. In particular, 81% of the surveyed students agreed that the package Mathematica should be taught to RTU students studying computer science. In addition, 73% of all students from the sample used Mathematica to solve problems which are not directly related to the course in higher mathematics. The results show that (1) students reaction to Mathematica was quite positive and (2) about 3/4 of them are using Mathematica to solve other problems.

The following three questions were also asked in the survey:

1. The use of Mathematica helped me to prepare for the final exam in mathematics.
2. The use of Mathematica helped me to understand better mathematical concepts.
3. Package Mathematica should be used in other courses that are taught at RTU.

The answers to the three questions were given on a 5-point scale (one, two, three, four and five points were given to the answers "strongly disagree", "disagree", "neither agree nor disagree", "agree" and "strongly agree", respectively). The following scores were obtained:

Scores	1	2	3	4	5
Q1	15	19	26	22	7
Q2	13	24	35	15	2
Q3	12	16	29	19	13

Table 1. Scores on the three questions in the survey.

As can be seen from Table 1, 33% of all students think that the use of Mathematica helped them (to a certain extent) to prepare for the final exam in mathematics. On the other hand, large proportion of students did not have a definite opinion with respect to the three questions (29%, 39% and 33% answered “Neither agree nor disagree” to questions Q1, Q2 and Q3, respectively).

We have also compared the final exam marks in 2006 (before the new module with Mathematica was introduced) and in 2007 (the first year when Mathematica was used). Two samples among students studying computer science were selected. The mean final exam score and variance were calculated for both samples. The measures of descriptive statistics are summarized in Table 2.

	2006	2007
Sample size	$n_1 = 39$	$n_2 = 29$
Mean score	$\bar{x}_1 = 6.51$	$\bar{x}_2 = 5.90$
Variance	$s_1^2 = 3.94$	$s_2^2 = 4.10$

Table 2. The mean exam score and variance before and after the new teaching module was implemented.

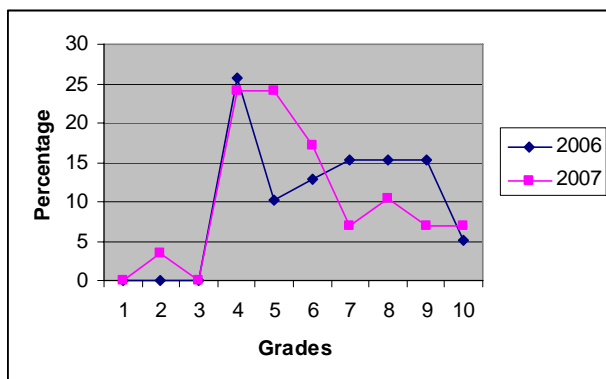


Figure 1. Distribution of final exam grades in 2006 and 2007.

The distribution of final exam grades in the analyzed sample is shown in Fig. 1. It should be pointed out that 10 point grading scale is currently used in Latvia; the minimal passing grade is “4”. As can be seen from Fig.1, the grades in 2006 were, in general, also higher. One possible reason is that the admission grades to the university in 2006 were also higher (at least in the Faculty of Computer Science and Information Technology).

In order to assess whether there is any significant difference between the mean exam scores in 2006 and 2007 (before and after the new teaching module with Mathematica labs was implemented) we have conducted hypothesis testing for the difference between population means. The hypotheses were defined as follows:

$$H_0 : \mu_1 = \mu_2,$$

$$H_1 : \mu_1 \neq \mu_2.$$

We have used the pooled-variance test [8] with the additional assumption that the variances of the two populations are equal (the corresponding test for the difference between the variances was performed and it was shown that there was no enough evidence to conclude that the variances were different). The test statistic is given by the Student’s distribution with $n_1 + n_2 - 2$ degrees of freedom and has the form

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

where

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}.$$

The calculated value of the test statistic using the data from Table 2 is $t = 1.24$. The critical t -values for different level of significance are show in Table 3.

Level of significance (α)	0.01	0.05	0.10
Critical points	± 2.6524	± 1.9966	± 1.6683

Table 3. Critical points for different significance levels.

As can be seen from Table 3, in all three cases ($\alpha = 0.01$, $\alpha = 0.05$ and $\alpha = 0.1$) the calculated point ($t = 1.24$) is located in the region of non-rejection. Thus, the null hypothesis $H_0 : \mu_1 = \mu_2$ is not rejected. Hence, there is no enough evidence to conclude that the mean final exam scores were different in 2006 and 2007.

The results of the test are consistent with the results of the survey (see Table 1). In fact, the majority of students were not sure whether the knowledge of Mathematica helped them to pass the exam or to understand better mathematical concepts. From our point of view poor performance on the exam is mainly related to weak knowledge of elementary mathematics. Strong students will

benefit from the use of Mathematica more than weak students. We believe that the assessment of the new teaching module with Mathematica labs cannot be done solely on the basis of the performance on the final exam. Other factors (like poor knowledge of elementary mathematics) have to be taken into account as well.

4 Conclusions

The new teaching module which includes Mathematica labs for the first year course in higher mathematics at Riga Technical University is described in the paper. Mathematica labs are designed in such a way that basic concepts of linear algebra, analytic geometry and vector calculus are covered in parallel with lectures and tutorials in higher mathematics course at RTU. The results of students' survey show that (1) students' reaction to the use of Mathematica is quite positive; (2) about 2/3 of all students surveyed use Mathematica for the solution of other problems that are not directly related to the course; (3) there is no statistical difference between the mean final exam scores before and after Mathematica labs were implemented. Despite the fact that exam scores are not affected by implementation of Mathematica labs we believe that the new teaching module has overall positive impact on the course. First, it brings new technology to the classroom. Second, the implementation of the new module is positively evaluated by students. Third, the knowledge of Mathematica allows students to use the package for the solution of other problems. Fourth, the introduction of Mathematica labs also changed instructors' attitude towards students and new technology in general.

5 Acknowledgments

The authors wish to thank the Latvian Council of Science for financial support under the Project No. 04.1239.

References:

- [1] Carlson, D., Johnson, C. R., Lay, D. C., and Porter, A. D. The Linear Algebra Curriculum Study Group Recommendations for the First Course in Linear Algebra, College Mathematics Journal, Vol. 24, 1993, pp. 41-46.
- [2] Golub, G.H., and van Loan, C. Matrix computations, The Johns Hopkins University Press, 1996.
- [3] Szabo, F. Linear algebra with Mathematica, Elsevier, 2000.
- [4] Wicks, J.R. Linear algebra: an interactive laboratory approach with Mathematica, Addison – Wesley, 1996.
- [5] Anton, H. Contemporary linear algebra with Mathematica manual set, Wiley, 2003.
- [6] Herman, E.A., and Pepe, M.D., Visual linear algebra (with Maple and Mathematica tutorials), Wiley, 2005.
- [7] Vossler, D.L. Exploring analytic geometry with Mathematica, Academic Press, 1999.
- [8] Levine, D.M., Krehbiel, T.C., and Berenson, M.L., Basic business statistics: concepts and applications, Prentice Hall, 10th edition, 2006.