Performance Analysis of Multi-User DS-UWB System under Multipath Effects

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Abstract- For the multi-user application the multiple access interference (MAI) due to imperfect orthogonality between spreading codes induced by UWB dense multi-path significantly degrades system performance. Over the past few years, performance analyses for various UWB systems in an AWGN environment have been presented. However, those works did not consider UWB multipath effect; thus leading to an unacceptable analytical result. To compensate for the shortage of previous works, this research aims to theoretically derive accurate performance of DS-UWB system in the presence of AWGN, MAI, and multipath effect. The BER performance will be analyzed in terms of the number of active users. An upper bound on the maximum number of users supported by the system will be derived. In addition, the allowed multiple access transmission rate will also be evaluated.

Key-Words: - Multiple Access Interference (MAI), Multipath Channel, Direct Sequence Ultra-Wideband (DS-UWB)

1 Introduction
Recently, UWB systems have attracted much attention in short-range wireless communications due to its many advantages, such as high bit rate, low power, robust to interference, and so on [1, 2]. Currently, direct sequence code division multiple access (DS-CDMA) [3] and multi-band orthogonal frequency division multiplexing (MB-OFDM) [4] are two main techniques employed in UWB.

DS-CDMA UWB is an impulse-based system where the spreading codes corresponding to each BPSK-modulated bit are transmitted using very short monocycle impulse. It can achieve a high data rate of up to 1 Gbps with relatively low complexity. However, the multiple access interference (MAI) due to imperfect orthogonality between spreading codes induced by UWB dense multipath effect significantly degrades system performance for the multi-user application. To address the multipath effect, the RAKE receivers are commonly used in DS-CDMA UWB by taking the advantages of high multipath resolvability of UWB channels.

Over the past few years, research on performance of various UWB systems has received much attention [5]-[11]. In [5]-[8], performances of several UWB systems have been evaluated over an AWGN channel. The effects of MAI and narrowband interference on DS-UWB performance are investigated via simulation [9]. Performances of time-hopping (TH) and DS-UWB systems with multi-user interference in an AWGN environment are derived [10]. An analysis is derived for the performances of time-hopping and direct-sequence modulation schemes with different numbers of users and frame widths in an AWGN channel [11]. It is shown that direct-sequence binary phase-shift keying outperforms time-hopping binary phase-shift keying for medium and large SNRs. Unfortunately, those results would be unsatisfied since they did not consider the interpulse and intersymbol interferences of UWB channel, which can seriously degrade system performance due to multipath effect. While RAKE receivers are effective in dealing with the multipath effect, a typical UWB channel has numerous resolvable paths and therefore a large number of fingers would be necessary to capture all these energies. In practice, only RAKE receiver with limited number of fingers is allowed due to power consumption and channel estimation issues. Theoretically, the performance that employs RAKE receiver with multiple correlators (fingers) is absolutely better than that without using RAKE receiver. Therefore, the receiver is simplified in this paper to represent an important intermediate result in looking for some bounds of exact analysis in UWB channel with multipath effect.

To compensate for the weakness of previous works, this research aims to theoretically derive accurate performance of DS-UWB system in the presence of not only AWGN and MAI, but multipath effect. The BER performance will be analyzed in terms of the number of active users. In addition, an upper bound on the maximum number of users supported by the
Fig. 1: The modified SV channel model

2 Multi-User DS-UWB System Under Multipath Effects

2.1 System Model

A direct sequence spread spectrum (DSSS) UWB system with \( N_u \) users is modeled as (1). Assume that each user has a pseudo-noise (PN) sequence with \( N_c \) chips per data symbol period \( T_f \), such that \( N_c T_f = T_u \), where \( N_c \) is the spread spectrum processing gain. The transmitted signal of the \( k \)th user can be expressed as

\[
s^{(k)}(t) = \sum_{j=-\infty}^{\infty} \sum_{n=0}^{N_c-1} d^{(k)}_j p^{(k)}_n(t-jT_f-nT_u)
\]

where \( w(t) \) represents the transmitted monocycle waveform, \( \{d^{(k)}_j\} \) are the BPSK-modulated data symbols for the \( k \)th user, \( \{p^{(k)}_n\} \) are the spreading codes of the \( k \)th user.

The modified SV channel model [12] is employed to represent the UWB multipath effect. As shown in Fig.1, the arrival time for each \( l \)th cluster is denoted by \( T_{l} \), \( l=0,1,2,\ldots \). Moreover, let the arrival time of the \( m \)th ray within the \( l \)th cluster be denoted by \( \tau_{m,l} \), \( m=0,1,2,\ldots \). By definition, for the first cluster, \( T_0 = 0 \), and for the first ray within the \( l \)th cluster, \( \tau_{0,l} = 0 \).

The probability distributions of arrival time for each cluster and each ray are respectively given as follows

\[
P(T_l | T_{l-1}) = \Lambda \exp[-\Lambda(T_l - T_{l-1})], \quad l > 0
\]

\[
P(\tau_{m,l} | \tau_{(m-1),l}) = \lambda \exp[-\lambda(\tau_{m,l} - \tau_{(m-1),l})], \quad m > 0
\]

where \( \Lambda \) and \( \lambda \) are respectively the cluster arrival rate, and the ray arrival rate. Both \( \Lambda \) and \( \lambda \) are statistical parameters. Let the gain of the \( m \)th ray of the \( l \)th cluster be \( h_m \), then the impulse response of UWB multipath channel for \( k \)th user can be represented as below

\[
h_k(t) = \sum_{l=0}^{\infty} \sum_{m=0}^{M} \alpha^{(k)}_{m,l} \delta(t - T^{(k)}_{l} - \tau^{(k)}_{m,l})
\]

Though cluster and ray extend over an infinite time, as signified by the double infinite sum in (3), in fact, those terms can be ignored when \( \exp(-T_l/\Gamma) \ll 1 \) and \( \exp(-\tau_{m,l}/\Upsilon) \ll 1 \), since their impulse responses rapidly decrease to zero. Thus, the channel impulse response for \( k \)th user can be simplified into

\[
h_{k}(t) = \sum_{l=0}^{L} \sum_{m=0}^{M} \alpha^{(k)}_{m,l} \delta(t - T^{(k)}_{l} - \tau^{(k)}_{m,l})
\]

where \( L+1 \) and \( M+1 \) represent significant terms of cluster and ray, respectively.

The \( \{\alpha_{m,l}\} \) are statistically independent positive random variables with Rayleigh distribution as below:

\[
p(\alpha_{m,l}) = (2\alpha_{m,l}/\alpha_{m,l}^2) \exp(-\alpha_{m,l}^2/\alpha_{m,l}^2)
\]

and whose mean square values \( \{\alpha_{m,l}^2\} \) follow the probability model given below, which is a monotonically decreasing function of \( \{T_l\} \) and \( \{\tau_{m,l}\} \)

\[
\alpha_{m,l}^2 = \alpha^2(T_l, \tau_{m,l}) = \alpha^2(0,0) e^{-\tau_{m,l}/\Upsilon} e^{-\tau_{m,l}/\Upsilon}
\]

where \( \alpha^2(0,0) = \alpha_{0,0}^2 \) is the average power gain of the first ray of the first cluster, and \( \Gamma \) and \( \Upsilon \) are power-delay time constants for the clusters and the rays, respectively.
2.2 Receiver Signal Processing

For \( N_u \) active users, the signal received by the receiver can be expressed by

\[
    r(t) = \sum_{k=1}^{N_u} A_k s^{(k)}(t - \Delta_k) * h_k(t) + n(t)
\]

in which \( A_k \) represents the attenuation over the propagation path of the signal \( s^{(k)}(t - \Delta_k) \) received from the \( k \)th user. The random variable \( \Delta_k \) represents the time asynchronism between the signal received from user \( k \) and the receiver, and \( n(t) \) is additive white Gaussian noise (AWGN) with two-sided power spectral density of \( N_0/2 \). The system model from user to receiver is shown in Fig. 2.

The receiver is a form of correlator-filtered matched filter, which is based on the theory of hypothesis testing for fully coherent data detection. The receiver decides whether \( d_t^{(j)} \) is -1 or 1, based on an observation of the received signal depicted as below:

\[
    r(t) = A, s^{(0)}(t - \Delta_1) * h_1(t) + \sum_{k=2}^{N_u} A_k s^{(k)}(t - \Delta_k) * h_k(t) + n(t)
\]

\[
= A, \sum_{i=1}^{N_u} \alpha_{i,0}^{(k)} d_{i,0}^{(k)} p_{u,i}^{(k)} w(t - \Delta_i - j_i T_f - n T_r) + n(t)
\]

\[
+ A, \sum_{a=0}^{N_u} \sum_{b=1}^{M} \alpha_{a,b}^{(k)} d_{a,b}^{(k)} p_{u,a}^{(k)} w(t - \Delta_i - j_i T_f - n T_r - \tau_{a,b}^{(k)})
\]

\[
+ A, \sum_{a=0}^{N_u} \sum_{b=1}^{M} \sum_{c=1}^{M} \alpha_{a,b,c}^{(k)} d_{a,b,c}^{(k)} p_{u,a}^{(k)} w(t - \Delta_i - j_i T_f - n T_r - \tau_{a,b,c}^{(k)})
\]

\[
+ A, \sum_{a=0}^{N_u} \sum_{b=1}^{M} \sum_{c=1}^{M} \sum_{d=1}^{M} \alpha_{a,b,c,d}^{(k)} d_{a,b,c,d}^{(k)} p_{u,a}^{(k)} w(t - \Delta_i - j_i T_f - n T_r - \tau_{a,b,c,d}^{(k)})
\]

\[
+ n(t)
\]

where \( d_{t,0}^{(j)} \) represents the current bit to be detected.

According to the measurement result of [13], the average power behind the second cluster is much smaller than the first one. Further, without loss of generality, we could suppose that \( 1/\Lambda > 2T_f \) , and further simplify \( r(t) \) as

\[
r(t) = A, \sum_{i=1}^{N_u} \alpha_{i,0}^{(k)} d_{i,0}^{(k)} p_{u,i}^{(k)} w(t - \Delta_i - j_i T_f - n T_r) + n(t)
\]

\[
+ A, \sum_{a=0}^{N_u} \sum_{b=1}^{M} \alpha_{a,b}^{(k)} d_{a,b}^{(k)} p_{u,a}^{(k)} w(t - \Delta_i - j_i T_f - n T_r - \tau_{a,b}^{(k)})
\]

\[
+ \sum_{a=0}^{N_u} \sum_{b=1}^{M} \sum_{c=1}^{M} \alpha_{a,b,c}^{(k)} d_{a,b,c}^{(k)} p_{u,a}^{(k)} w(t - \Delta_i - j_i T_f - n T_r - \tau_{a,b,c}^{(k)})
\]

\[
+ n(t)
\]

It is assumed that the receiver can achieve perfect clock and sequence synchronization for the signal transmitted by the first user. The decision statistic consists of

\[
d_{j}^{(1)} = \begin{cases} 1 & \text{if } y > 0 \\ -1 & \text{if } y < 0 \end{cases}
\]

where

\[
y = \int_{A_{k,1}^{(j)} T_f}^{A_{k,2}^{(j)} T_f} r(t) \sum_{n=0}^{N_u-1} p_{u,n}^{(k)} w(t - \Delta_i - j_i T_f - n T_r) \, dt
\]

Using a simple mathematical derivation, the test statistic can be rewritten as

\[
y = m + I
\]

where

\[
I_t = I_t + \sum_{k=2}^{N_u} A_k r_{u,k}^{(k)} + I_n
\]

and

\[
m = \int_{A_{k,1}^{(j)} T_f}^{A_{k,2}^{(j)} T_f} r(t) \sum_{n=0}^{N_u-1} p_{u,n}^{(k)} w(t - \Delta_i - j_i T_f - n T_r) \, dt
\]

\[
= \alpha_{0,0} N_u A J d_{j}^{(1)} E_u
\]

where \( E_u = \int_{-\infty}^{\infty} w^2(u) \, du \)

\( I_1 \) is caused by the rest of rays in the first cluster of 1st user, which can be expressed as

\[
I_1 = \int_{A_{k,1}^{(j)} T_f}^{A_{k,2}^{(j)} T_f} r(t) \sum_{n=0}^{N_u-1} p_{u,n}^{(k)} w(t - \Delta_i - j_i T_f - n T_r) \, dt
\]

where

\[
r_2(t) = A, \sum_{a=0}^{N_u} \sum_{b=1}^{M} \alpha_{a,b}^{(k)} d_{a,b}^{(k)} p_{u,a}^{(k)} w(t - \Delta_i - j_i T_f - n T_r - \tau_{a,b}^{(k)})
\]

where \( \tau_{a,b}^{(k)} \) is caused by multiple-access interferences from undesired users with multipath effect, which can be represented as

\[
f^{(1)}_{u} = \int_{A_{k,1}^{(j)} T_f}^{A_{k,2}^{(j)} T_f} r(t) \sum_{n=0}^{N_u-1} p_{u,n}^{(k)} w(t - \Delta_i - j_i T_f - n T_r) \, dt
\]

where

\[
t_1(t) = \sum_{a=0}^{N_u} \sum_{b=1}^{M} \sum_{c=1}^{M} \alpha_{a,b,c}^{(k)} d_{a,b,c}^{(k)} p_{u,a}^{(k)} w(t - \Delta_i - j_i T_f - n T_r - \tau_{a,b,c}^{(k)})
\]

and \( I_1 \) is caused due to AWGN , i.e

\[
I_n = \int_{A_{k,1}^{(j)} T_f}^{A_{k,2}^{(j)} T_f} n(t) \sum_{n=0}^{N_u-1} p_{u,n}^{(k)} w(t - \Delta_i - j_i T_f - n T_r) \, dt
\]

2.3 SNR of the Receiver

In order to analyze the BER, we first define the SNR at correlator output. The receiver output signal to noise ratio (SNR) is defined as

\[
\text{SNR}_{s_n} = \frac{E\{m^2\}}{E\{|h|^2\}}
\]

where the numerator of this expression is given by

\[
E\{m^2\} = \alpha_{0,0}^2 N_u^2 A^4 E_u^2
\]

Since all random variables defined in (13) are independent to each other and are with zero mean, the
quantity $E\{ |I|^2 \}$ becomes

$$E\{ |I|^2 \} = E\{ |I_1|^2 \} + \sum_{i=2}^{N} A_i E\{ |I_i|^2 \} + E\{ |I_s|^2 \}$$  \hspace{1cm} (23)

$E\{ |I_s|^2 \}$ can be obtained from (16):

$$E\{ |I_s|^2 \} = \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} (u-v)w(u-v)du \right] dv$$

where

$$\sigma^2 = \int_{-\infty}^{\infty} \lambda e^{-\lambda} \left[ \int_{-\infty}^{\infty} w(u-v)w(u)du \right] dv$$  \hspace{1cm} (26)

Using (18), we get

$$E\{ |I_s|^2 \} = N_0 \sigma^2 \sum_{m=0}^{M} a_{m,0}^2$$  \hspace{1cm} (27)

Similarly, we have

$$E\{ |I_s|^2 \} = N_s \sigma^2 \sum_{m=0}^{M} a_{m,0}^2$$  \hspace{1cm} (28)

where

$$\sigma^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \lambda e^{-\lambda} \left[ \int_{-\infty}^{\infty} (u-v)w(u-v)w(u)du \right] dv$$  \hspace{1cm} (29)

Furthermore, $E\{ |I_s|^2 \}$ can be evaluated from (20) and expressed as below

$$E\{ |I_s|^2 \} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \lambda e^{-\lambda} \left[ \int_{-\infty}^{\infty} w(u-v)w(u)du \right] dv$$  \hspace{1cm} (30)

Similarly, we have

$$E\{ |I_s|^2 \} = N_s E_u N_0/2$$  \hspace{1cm} (31)

Thus, we have

$$E\{ |I|^2 \} = N_s \sigma^2 A_s E_s^2$$  \hspace{1cm} (32)

Then, the receiver output SNR can be expressed as

$$SNR_u = \frac{\alpha_u^2 N_s E_s^2}{N_s \sigma^2 A_s E_s^2 + N_s \sigma^2 \sum_{k=0}^{M} a_{k,0}^2 + N_s E_s N_0/2}$$

$$= \left[ \frac{N_s E_s^2}{\sigma^2} \sum_{k=0}^{M} (\frac{a_{k,0}^2}{\sigma^2}) + \frac{N_s E_s^2}{\sigma^2} \sum_{k=0}^{M} \frac{a_{k,0}^2}{\sigma^2} + \frac{N_s E_s}{N_0/2} \right]^{-1}$$

$$= \left[ \frac{\alpha_u^2 N_s E_s^2}{N_0/2} \right]^{-1}$$

which can be reduced to the following single user output SNR when $N_u = 1$

$$SNR_u = \left[ \frac{N_s E_s^2}{\sigma^2} \sum_{k=0}^{M} e^{-\tau_u s / \gamma} + \frac{\alpha_u^2 N_s E_s^2}{N_0/2} \right]^{-1}$$  \hspace{1cm} (34)

### 3 Multiple Access Performance

Given (33), the BER can be obtained by

$$P_b(k)= \frac{1}{\sqrt{N_0/2 \sigma^2}} \left[ \frac{N_s E_s^2}{\sigma^2} \sum_{k=0}^{M} e^{-\tau_u s / \gamma} + \frac{\alpha_u^2 N_s E_s^2}{N_0/2} \right]^{-1/2}$$

With perfect power control assumptions (i.e., $A_k = A_1$ for all $k$) we have

$$SNR_u = \left[ \frac{N_s E_s^2}{\sigma^2} \sum_{k=0}^{M} e^{-\tau_u s / \gamma} + \frac{\alpha_u^2 N_s E_s^2}{N_0/2} \right]^{-1}$$

and (35) can be simplified into

$$E\{ |I|^2 \} = \frac{1}{\sqrt{N_0/2 \sigma^2}} \left[ \frac{N_s E_s^2}{\sigma^2} \sum_{k=0}^{M} e^{-\tau_u s / \gamma} + \frac{\alpha_u^2 N_s E_s^2}{N_0/2} \right]^{-1/2}$$

Further, assuming perfect power control, the excess single link SNR, is defined as $\Delta P = 10 \log_{10} (SNR_u / SNR_{u_0})$, which indicates excess SNR required for supporting $N_u$ active users, where the modulation rate $R_s = (N_u T_s)^{-1}$. Under this assumption, the number of active users supported by this system is

$$N_u (\Delta P) = \frac{E_s^2 (1 - 10^{-\Delta P/10})}{SNR_u \sigma^2 R_s T_s \sum_{k=0}^{M} e^{-\tau_u s / \gamma}} + 1$$

In addition, we define system data rate that can be achieved with $\Delta P$, where $N_u$ is fixed.

$$R_s(\Delta P) = \frac{E_s^2 (1 - 10^{-\Delta P/10})}{SNR_u \sigma^2 (N_u - 1) T_s \sum_{k=0}^{M} e^{-\tau_u s / \gamma}}$$

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Therefore, we can see that both $N_u$ and $R_s$ are monotonically increasing functions of $\Delta P$. The ultimate limits to multiple access communication with DS-UWB receiver occur when $\Delta P$ is increased without bound. That is,

$$N_{\text{max}} = \left[ \frac{E_w^2}{\text{SNR}_N \sigma_o^2 R_s T_c \sum_{m=0}^{M} e^{-\gamma_{m,0} / Y}} \right] + 1 \quad (40)$$

and

$$R_{\text{max}} \equiv \left[ \frac{E_w^2}{\text{SNR}_N \sigma_o^2 (N_u - 1) T_c \sum_{m=0}^{M} e^{-\gamma_{m,0} / Y}} \right] \quad (41)$$

Hence, for a specific level of performance as embodied in $\text{SNR}_{N_u}$, there are upper bounds on the modulation rate for a given number of users, and on the number of users for a given modulation rate.

The value $N_{\text{max}}$ is the largest value that $N_u$ can attain when the performances determined by multipath environment and the amount of multiple-access interference produced by the other active users. $N_{\text{max}}$ can be used to define $C_{\text{max}}$, the largest value that the total combined bit transmission rate $N_u R_s$ can attain,

$$C_{\text{max}} = N_{\text{max}} R_s \equiv \frac{E_w^2}{\text{SNR}_N \sigma_o^2 T_c \sum_{m=0}^{M} e^{-\gamma_{m,0} / Y}} \quad (42)$$

$$= N_u R_{\text{max}} \quad \text{(when } N_u \gg 1)$$

Therefore, we find a bound, the transmission capacity, for the multiple access transmission in a UWB multipath environment.

4 Numerical Results

This section illustrates numerical results of (37), (38), and (39) based on statistical parameters of CM1, CM2, CM3, and CM4. In this study, the impulse signal expressed below is employed, where its duration (D) is set to 0.2877 ns.

$$w(t) = \left[ 1 - 4\pi \left( \frac{t}{D} \right)^2 \right] \exp \left[ -2\pi \left( \frac{t}{D} \right)^2 \right] \quad (43)$$

Several parameters in (37) including $E_w$, $\gamma_{m,0}$, and $\sigma_o^2$ are calculated based on the $w(t)$. The BER performance, number of users supported by the system, and system data rate for various multipath channels are respectively shown in Fig. 3, Fig.4, and Fig. 5. For the evaluations of BER performance and number of users supported by the system, the system bit rate $R_s$ is set to 100 Kbps. In Fig. 3 a single user $\text{SNR} \ (\text{SNR}_1)$ is assigned to ensure a desired single user BER of $P_e \ (N_u = 1) = 10^{-5}$. The result shows that when the number of active users increases from $N_u = 1$ to $N_u = 10000$, the bit error rate degrades from $10^{-2}$ to $10^{-4}$ for the case of CM1. This indicates that for CM1 environment the BER performance remains at an acceptable level even when the number of active users is increased to a very large value. However, the BER performance significantly degrades to $10^{-2}$ for the case of CM4 due to severe multipath effect, resulting in an unacceptable performance. It also demonstrates that the BER performances of CM2 and CM3 are between CM1 and CM4.

Fig. 4 presents the result of the number of active users versus single link excess SNR ($\Delta P$). Results show that the total number of active users supported by the system saturates as $\Delta P \geq 10$ dB. For the case of CM1, the maximum number of users is around 17,000. For the case of CM4, the maximum number of users is reduced to about 2800 only. It also demonstrates that the results of CM2 and CM3 are between CM1 and CM4.

Fig. 5 demonstrates the allowed system data rate versus single link excess SNR ($\Delta P$). The result indicates that system data rate saturates as $\Delta P \geq 15$ dB. The system data rates of CM1 and CM4 are respectively around 60 Mbps and 9 Mbps as $\Delta P \geq 15$ dB. Again, the results of CM2 and CM3 are between CM1 and CM4.

5 Conclusions

This research analyzes the performance of multi-user DS-UWB system in the presence of AWGN, MAI, and multipath effect. The simplified receiver represents a lower bound of multiple access performance for UWB transmissions with multipath effect but permits exact analytical solutions. Theoretical expressions for BER performance, the upper bound of active users supported by the system, and the allowed multiple access transmission rate are derived and numerical results are demonstrated. The analysis also represents an important intermediate result in looking for a solution to the more general case that employs common RAKE receivers, or even more complex receiver, such as MMSE-RAKE receiver. To verify the derived results, simulations will also be conducted in the future work.
References:


