# Sliding Mode Technique in the Task of the Drive Control 

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#### Abstract

The paper aims to explain the basic ideas related to the use of sliding-mode technique for the control design for an electromechanical drive system containing an AC motor and a converter and for the observation of the mechanical coordinates. A comprehensive investigation of possible AC motors, converters and control plant structures was carried out. Based on this analysis different original design procedures for control and observer are presented. It is show the possibility to use the "classical" result of sliding-mode theory for the real drive systems with more then $2^{\mathrm{m}}$ switched structures ( m is the control space order).


Key-Words: - AC machines, control, converter control, drives, motion control, sliding mode control, variable speed drives, voltage source inverter, vector control.

## 1 Introduction

Progress in the area of electric energy conversion, especially in the construction and the topology of the AC converters during the last quarter century, namely the transition from thyristors to GTO thyristors and power transistors and from analog to digital technology, opens new possibilities thanks to the complete controllability of the power flux, quick control computation, big memory and the possibility to integrate into networks. However for using all these possibilities in the AC drive, new advanced control techniques are needed. A standard item of modern drives is the AC machine (ACM) fed by converters [2, 7]. There are many types of such converters, but the main feature of them is that are equipped with power semiconductor switches as Insulated Gate-Commutated Thyristors (IGCTs) or high-voltage Insulated-Gate Bipolar Transistors (IGBTs) [2], operating with distinctly higher switching frequency, than known from forcecommutated thyristors. The AC motor behavior can be described using different non-linear differential equations which form depended on the used reference frame and the space variables [7]. Due to the complexity of the AC drive that is switched nonlinear control plant having a series coupling of two non-linear objects different by nature, many different control techniques were proposed, such as e.g. FieldOriented Control [1], Direct Self Control [4, 5] and Indirect Stator-Quantity Control [5] or Direct Torque

Control [3, 6]. But nearly all these techniques have in common the decomposition of the main high-order control task to several separated lower-order tasks, using linear control technique and heuristic solutions.
The switching nature of the AC drive with converter opens the possibility to use the principal operational mode of this class of control systems - sliding mode - for solving the control task. Due to its property of order reduction and its robustness against disturbances and plant parameter variations, Sliding-Mode Control (SMC) is an efficient tool to control electromechanical systems, as it is well known $[6,9,10]$. The primary aim of this paper is to show the possibility to use the sliding-mode technique for the control design of the electromechanical system that can have a different topology and consist of different types of AC motor and converters. The main problem is that in contrast to the "classical" sliding-mode theory the real switched system has an order of more than $2^{\mathrm{m}}$ ( m is the control space order).
The paper can be outlined as follows. Section 2 gives a brief introduction of sliding-mode control background. A presentation and discussion of the used AC motors and converters models is given in Section 3. Section 4 presents the formulation of the control task and considers a using of the sliding-mode control technique for the different AC drives. The design result is the control for the above-mentioned drive system. The example of the using the sliding mode technique for the observer design is presented in Section 5 followed by conclusions.

## 2 Sliding-Mode Control Background

Sliding mode is a special type of behavior of a control plant with switched structures. None of the used general switched structures can realize such behavior, which is a result of the used special switched structures. The switching function $F$ that controls the switching of the structures is a function of the system variables $x(t)$ and is usually an error function that must be led to zero. Using a switching of the structures with high frequency attains this condition.

Formally, the control aim is the following: the system state $x(t)$ must come to the manifold

$$
\begin{equation*}
F=0 \tag{1}
\end{equation*}
$$

and "slides" on this manifold to the reference point, independently of the system dynamic.
It must be noted, that depending on the used control structure there are two types of variable structure systems, for which SMC has been considered: Variable-structure control systems with switching of the feedback gains, and systems with discontinuous control or "relay" systems with switching of the control outputs. From the viewpoint of the switch mode of the semiconductor switches the above-mentioned ACM drive belongs to the second type.

In this case a typical sliding mode control $\mathrm{u}\left(u \in R^{m}\right)$ has the form

$$
\begin{equation*}
u=-U(x) \operatorname{sgn}(F), \tag{2}
\end{equation*}
$$

where $x \in R^{n}, n \geq m ; \mathrm{U}(\mathrm{x})$ is the square diagonal matrix of the control magnitude; $\operatorname{sgn}(F)$ is the vector of the signs of the switching functions, $\operatorname{sgn}(F) \in R^{m}$. It guarantees that the system state will reach the sliding manifold in finite time from the initial condition, which has been bounded by the value of the constituent of the matrix $U(x)$, and will keep to it. This magnitude bounds the uncertainty of the system, the load value unto which the system is commonly robust.
The motion on the sliding manifold can be described by using the equivalent control $u_{e q}(x)$ [9]. It is calculated from the condition that the time-derivative of the switching function $F$ on the system trajectories is equal to zero

$$
\begin{equation*}
d F / d t=0 . \tag{3}
\end{equation*}
$$

The equivalent control $u_{e q}(x)$ is a continuous control that would guarantee the same motion, if all needed
information about the load and the system uncertainty were available. In this case the system behavior can be written with a vector equation of reduced order. The full order is reduced to the order of the sliding-mode manifolds.

## 3 Used Mathematical Models

### 3.1 AC machines

It's known the mathematics model of three phases AC machine can be divided in two parts:

- mechanical part, which is equal for all electric machines:

$$
\begin{align*}
& (d \Gamma / d t)=\Omega \\
& (d \Omega / d t)=\left(T-T_{L}\right) / J \tag{4}
\end{align*}
$$

where $J$ is the inertial of the AC machine and load, $\Gamma$ is a angle position of the motor shaft; $\Omega$ is motor shaft speed; $T$ is electromagnetic torque produced in the machine; $T_{L}$ is the load torque.

- electrical part that depends on the type of AC machine and can be described by using of the physical variables as a following matrix equation:

$$
\begin{equation*}
d \tilde{x}(t) / d t=\tilde{f}(x, t)+\tilde{B}(t) \tilde{u}(t) \tag{5}
\end{equation*}
$$

where $\tilde{x}(t)$ is a state vector, $x \in R^{k} ; \tilde{f}(x, t)$ is a vector-column describing behavior of the AC motor; $\tilde{B}(t)$ is a control matrix with periodic coefficients, $\tilde{B}(\omega t)=\tilde{B}(\omega t+2 \pi), \omega$ is an electrical rotation frequency of AC motor; $\tilde{u}(t)$ is a control vector, i.e. voltage space vector, $\tilde{u} \in R^{l}, l \leq k$. The features of various types of AC motors find their reflection in dimension and components of vectors, and also in a rate of the matrix elements. (Table 1)

These two parts is connected by algebraic equation of electromagnetic torque, which depends on the type of AC machine too.

The classical technique that allows avoiding using of the special methods for decision of the nonlinear differential equations with periodic coefficients is wellknown Park's transformation. In this case the description of the electrical part of the real three-phase AC machine is transformed in the description of the ideal two-phase
one in rotating frame $(d, q)$ and separate equation of the zero-sequence. It's well known the best rotating frames for AC machines: connected with rotor field for induction machine and with rotor for synchronous one.

Table 1. Main AC motors features

| Motor | State <br> space <br> order | Control <br> space <br> order | Control <br> number |
| :---: | :---: | :---: | :---: |
| Induction | 4 | 2 | 3 |
| Synchronous <br> with exciter | 3 | 3 | 4 |
| Permanent <br> magnet <br> synchronous <br> motor | 2 | 2 | 3 |
| Synchronous <br> reluctance | 3 | 2 | 3 |

In this case electric part of AC machine is described already by nonlinear differential equations with new state vector and control one. The coefficients are constant and controls entries linear in the equation. Usually the stator windings are connected as "star" or "delta" and the zerosequence current is automatically equal zero. In this case the system and control orders are on one lower and the control has the same number as the order of the control space:

$$
\begin{equation*}
\frac{d x(t)}{d t}=f(\tilde{x}, t)+B(t) u(t) \tag{6}
\end{equation*}
$$

where $x(t)$ is a new state vector, $x \in R^{n}, n=k-1$; $f(x, t)$ is a new vector-column describing behavior of the AC drive in the new frame; $B(t)$ is a new control matrix with periodic coefficients; $u(t)$ is a new control vector, $u \in R^{m}, m=l-1$.

It's important too that from the control viewpoint the new ACM model (6) is more simple as (5) and allows more easy deciding the control task.

### 3.2 Converters

Electrical power converter produce from the input three-phase voltage $U_{i}(i=A, B, C)$ with constant in time frequency and magnitude the output three-phase voltage $U_{j}(j=R, S, T)$ with constant or variable in time frequency and/or magnitude which is needed for solving of drive problem. The switched mode is very efficient means of controlling the power flow in converter. However, in this case there are two types
of output signals: voltage pulse sequence with the input magnitude $U_{\text {in }}$ and the average value of this pulse sequence during any one time period $\mathrm{U}_{\text {eq. }}$. From the viewpoint of the control only the second signal is needed. Thus, the faster that the switches operate, the better the approximation of the desired voltage form across the ACM stator is. Today the various converter circuits are available for use in variable frequency AC drives. Ones of the most used are voltage source inverter (VSI). Two type of them classical two-level VSI and tree-level VSI, which are standard solution for high- and medium power drives will be examined for realization of sliding mode control.

### 3.2.1 VSI

As known the input voltage for VSI is DC voltage, which is producing from input AC voltage. The standard power circuit of a three-phase bridge VSI is shown in Fig.1.


Fig. 1. Three-phase bridge VSI
Output phases voltages $\mathrm{U}_{\mathrm{j} 0}$ are equal to the DC supply voltage $\mathrm{U}_{\text {in }}$ or 0 in accordance with the switch $\mathrm{K}_{\mathrm{j}}$ state $\mathrm{p}_{\mathrm{j}}$ ( $p_{j} \in\{0,1\}$ ), which can be ON (lower) or OFF (upper). The output voltage space vector $\mathrm{U}^{\mathrm{T}}=\left(\mathrm{U}_{\alpha}, \mathrm{U}_{\beta}\right)$ is defined as

$$
\left|\begin{array}{l}
U_{\alpha}  \tag{7}\\
U_{\beta}
\end{array}\right|=\frac{2}{3}\left|\begin{array}{lll}
0 & -1 / 2 & -1 / 2 \\
1 & -\sqrt{3} / 2 & -\sqrt{3} / 2
\end{array}\right|\left|\begin{array}{l}
U_{R O} \\
U_{S O} \\
U_{T O}
\end{array}\right|,
$$

where columns numerical coefficients in the right part (7) can be considered the directional vectors of
the phases $R, S$, T respectively. There are only seven different momentary output voltage vectors $U_{i}$ $(\mathrm{i}=0, \ldots 6)$ : one zero vector ( $\mathrm{i}=0$ ) and six non-zero vectors $(i=1, \ldots, 6)$, which are located at $\pi / 3$ angle to one another and shown in Fig.2. Near each vector there are the combinations of the power switch state $\left(p_{R}, p_{s}, p_{T}\right)$. The magnitudes of the non-zero vectors are expressed in relation to the DC supply voltage of the inverter $\mathrm{U}_{\mathrm{in}}$ and equal to $2 \mathrm{U}_{\mathrm{in}} / 3$. By the way the zero vector $U_{0}$ arises when either the upper (111) or the lower (000) switches are chosen.


Fig. 2. Diagram of output voltage vectors of VSI

### 3.2.2 LVSI

The model of the NPC-type 3LVSI (Fig. 3, [8]) has three input rails: the positive ( $\mathrm{L}+$ ) one, the negative (L-) one of the dc link and the middle potential (M) between positive and negative potential (neutral point). The output terminals $a, b, c$ can be connected to each of them by using semiconductor switches (S1, S2, S3).


Fig. 3 Canonical schema of neutral-point-clamped 3LVSI

The switch has three positions: " + " means the connection with the positive rail, "."" with the negative one and " 0 " with the neutral point. In this case there are 27 possible switching combinations. By assumption that the middle potential is balanced ( $\mathrm{E} 1=\mathrm{E} 2$ ) they make up 19 different output voltage space vectors with four different magnitudes, parts of them multiply redundant. Fig 4 gives an overview.

There are six full-voltage space vectors (A) with magnitude $B_{A}=2 U_{\text {in }} / 3$, where $U_{\text {in }}=E_{1}+E_{2} ;$ six intermediate-voltage space vectors $(\mathrm{Z})$ with magnitude $B_{Z}=\sqrt{3} U_{\text {in }} / 3$; six half-voltage space vectors with magnitude $B_{H}=U_{i n} / 3$, attained each by two switch combinations (there is a control redundancy), and the zero-voltage space vector ( N ), with three possible switch combinations. The positions of the switch [S1, S2, S3] producing the space vectors are entered in Fig. 4 near the space vectors.


Fig. 4. Voltage space-vector diagram of 3LVSI

## 4 Control Strategy

The main goal of the AC machine control is satisfied that the average value of the mechanical output variable (for example rotor speed or position) will be equaled the prescribed value one. In depend on the number of the control loops all known solutions can be dividend in two group. The drives of the first group have only one control loop; opposite ones of the second group have the two loops that simplicities the control design. Somehow or other the control goal can be interpreted as a task to select the switching sequence for every switch of converter so
the average value of the mechanical output variable satisfies prescribed or reference value. Clearly the realized average value of the output variable must be done with using of the momentary voltage vectors. From this viewpoint all modulation methods, which used for design prescribed value of output voltage, can be divided in two large parts depending of the control techniques:

- feedforward pulse width modulation (PWM), based on the Space Vector Modulation [2, 7];
- feedback modulation, based on the Sliding mode control [9-11].
The first PWM technique is characterized by formation of output voltage with open-loop structure of control system. In this case the dynamic behavior of the drive is bounded by the PWM frequency, and only help of the mechanical control loop can reduce the disturbances, which there are always in the system. The PWM frequency can be constant or separately from control changed. The pulse width is calculated in the controller.

The second modulation allows on-line realization of switching pattern based on the sliding mode or bang-bang feedback control. This control technique ensures that the frequency and pulse width are generated automatically together with the solving of the drive control aim. Such system has a good dynamic behavior, it's used all feedback control possibility for the current error decreasing, it will be decreasing as quickly as possible and influence of disturbances is reduced. Unfortunately, in this case the switching frequency of each VSI phase switches is sensitive depended on the initial current error conditions and the switching frequencies are variable.

In the case of the two control loops there are the cascade control. The outside controller is mechanical output variable one that used only the mechanical equation (4). His output signal is the reference signal for the inner control loop that controls the converter. From the viewpoint of the inner control loop this control task is reduced to a task of maintenance on the shaft of the motor of the given meaning (importance) of the target torque or stator current.

In this framework it's described two possibility of the sliding mode control design: one loop and two loops control.

As known the control strategy is more comfortable to design by using of the ideal two-phase machine model in rotating frame.

### 4.1 One loop control

As the control number is equal the order of the control space and higher as one there is a possibility
to control addition to the mechanical variable another variable(s), which describes electrical or power AC machine requirements. E.g. for the induction machine it could be a magnetization current $i_{\mu}$ or flow $\psi$, orthogonal active current, for synchronous machine - the current component $i_{d}$ and of the excitation current $i_{f}$, if it's present.

The control aim is to do equal the reference value and the real one of the control variables. By using SMC technique so-called vector switchover function must be formed. Its dimension is equal control dimension and its components are the functions of control errors. One of such functions is the function of a mechanical variable error, for example rotation speed:

$$
\begin{equation*}
F_{1}=d\left(\Omega_{z}-\Omega\right) / d t+C\left(\Omega_{z}-\Omega\right), \tag{8}
\end{equation*}
$$

where $z$ is an index of the reference.
As above indicated the next switchover function can provide performance of any electrical or power requirements. The reference value of the control variable can be calculated from this requirement. E.g. in the induction machine as such control variable could be used a magnetization current $i_{\mu}$

$$
\begin{equation*}
F_{2}=i_{\mu z}-i_{\mu}, \tag{9}
\end{equation*}
$$

or flow $\psi$

$$
\begin{equation*}
F_{2}=\psi_{z}-\psi . \tag{10}
\end{equation*}
$$

If there is the sliding movement on these surfaces the control error is equal to zero in conditions of the inexact information about AC drive parameters and external disturbances.
The main problem using the sliding-mode technique for this control task is that all "classic" results have been received for the case that the number of discontinuous controls is equal to the control order. In the electrical system here, as shown in the Section 3, the situation is different: the number of discontinuous controls is bigger than the control order. But they have constant directions that cannot be changed; it is only possible to change the control magnitude. The main problem of control design is the synthesis of the switching law that produces the sliding motion on the manifold (1), (8), (9).

In this case a two-step design technique [9] is fruitful. It allows us to solve the control design task for the converter that has more as 4 different output-voltage space vectors in a two-dimensional control space. The
main idea is the decomposition of the control task, taking separately into account the nonlinearities of AC motor and converter. At the first step only the two-phase equivalent model of the AC motor has been used. In this case the number of discontinuous controls is equal to the order of the voltage plane. The sliding mode can be designed by using the standard technique. As design results the magnitudes of the needed stator-voltage components and the switching law are obtained.

At second step the real discontinuous output voltage of converter has to be taken into account, and the realization task of the above-mentioned sliding mode has to be solved with them.

### 4.1.1 SMC design for two phase motor

In this case the design task is carried out with using of known standard methods as the number of discontinuous controls is equal to the order of the voltage plane [10 ]. As design results there are needed magnitudes of the voltage stator components in the rotating frame $U_{q 0}$ and $U_{d o}$ and for the synchronous motor with the excitation $U_{f 0}$ that bound the initial condition, from which the state will reach the sliding manifold in finite time, and the uncertainly of the system and load value, to which the system is robust in general,

$$
\begin{align*}
& U_{q 0} \geq u_{q e q},  \tag{11}\\
& U_{d 0} \geq u_{d e q},  \tag{12}\\
& U_{f 0} \geq u_{f e q}, \tag{13}
\end{align*}
$$

and the control law in such form:

$$
\begin{align*}
u_{q} & =U_{q 0} \operatorname{sgn} F_{1},  \tag{14}\\
u_{d} & =U_{d 0} \operatorname{sgn} F_{2},  \tag{15}\\
u_{f} & =U_{f 0} \operatorname{sgn} F_{3} . \tag{16}
\end{align*}
$$

### 4.1.2 Transition to the converter control

However in reality there are only a set of the different output-voltage space vectors produced by the discontinuous controls (switching) of the converters and described in the Section 3.2. The question is whether it is possible to attain the above designed sliding motion with these output vectors.
There are two possibilities. Using the filter could be produced the equivalent controls $u_{q e q}$ and $u_{d e q}$ that are the continuous equivalents of the discontinuous controls (14), (15). They can be transformed from the rotating frame to three-phase voltage frame and used as the references for the converter with the feed-forward PWM.

The sine-wave voltages would be the mean values of the output voltages of the semiconductor converter with high-frequency switches. However for the calculation of PWM information as the average value of the output voltage space vector $\mathrm{U}_{\mathrm{eq}}$ during any time period would be needed, the switching time for each switch and the sequence of their switching. Such an approach requires additional calculations and does not reach any of the basic properties of sliding mode, namely simplicity of realization.
The alternative approach to design of the converter output voltages control, i.e. transferring the twodimensional control (14), (15) to the converter control, is based on the fact, that the selection conditions of the amplitudes of the formally entered controls in the reference frame rotating uses the inequalities (11) and (12). In this case there is the area of allowable controls $U^{*}$ in the space of the formally entered controls $u_{d}, u_{q}$.
It is obvious, if we design the real discontinuous voltages thus that their projections on suitable axes of the rotating frame have their marks and sizes, which are needed by the control algorithm with the formally entered controls, the sliding mode on crossing before the chosen surfaces will take place. Of course, the sizes of the formally entered controls will change during work.
Inequalities (11), (12) and control (14), (15) determine in the control space rotating with the stator-flux four control areas $U_{1}^{*}, U_{2}^{*}, U_{3}^{*}, U_{4}^{*}$ that guarantee sliding motion (Tab. 2): $U^{*}=\left\{U^{*}\right\}=U_{1}^{*} \cup U_{2}^{*} \cup U_{3}^{*} \cup U_{4}^{*}$, $U_{1}^{*} \cap U_{2}^{*}=0, \quad U_{1}^{*} \cap U_{3}^{*}=0, \quad U_{1}^{*} \cap U_{4}^{*}=0$, $U_{2}^{*} \cap U_{3}^{*}=0, U_{2}^{*} \cap U_{4}^{*}=0, U_{3}^{*} \cap U_{4}^{*}=0$.

Table 2. Sliding-mode control areas

|  | $U_{1}^{*}$ | $U_{2}^{*}$ | $U_{3}^{*}$ | $U_{4}^{*}$ |
| :--- | :---: | :---: | :---: | :---: |
| $\operatorname{sgn} F_{1}$ | 1 | -1 | 1 | $\sim_{1}-1$ |
| $\operatorname{sgn} F_{2}$ | 1 | ${ }^{2}$ | -1 | $\sim-1$ |

These four control areas $U_{1}^{*}, U_{2}^{*}, U_{3}^{*}, U_{4}^{*}$ can be transformed to the three-phase stator-winding-fixed frame ( $a, b, c$ ). In this frame they have the same form, but move with the velocity of the rotating frame.

The sliding mode behavior, which has been synthesized in the stator-flux-rotating frame, will be secured by using the discontinuous voltages of the converter, if each control area has at any time at minimum one of the converter output voltage vectors. This condition is fulfilled by the calculation of the converter input dc voltage, the design of the transition law between the designed controls (14), (15) and the
switching control of 3LVSI switches. Of course it depends on the converter topology.

The above-mentioned control design procedure will be presented for the 3LVS by the following assumptions:

- The used 3LVSI output-voltage space vectors are full ones and intermediate ones.
- The value of the 3LVSI dc-link voltage must be the minimal possible.
- The value of the 3LVSI dc-link voltage has been calculated, using the magnitude of the inter-mediate voltage space vectors. In this case the supply voltage is:

$$
\begin{equation*}
B_{A}-B_{Z}=(2-\sqrt{3}) U_{i n} / 3 \tag{17}
\end{equation*}
$$

The positions of the four control areas and 19 different output-voltage space vectors with four different magnitudes, produced by discontinuous control (switching) of the 3LVSI are presented in Fig.5.


Fig.5. Sliding mode control areas and voltage space vectors of 3LVSI

By the above assumptions the selection condition of the 3LVSI input dc-voltage value is, for geometrical reasons:

$$
\begin{equation*}
\arcsin \left(u_{d e q} / B_{z}\right)+\arcsin \left(u_{q e q} / B_{z}\right) \leq \pi / 3 \tag{18}
\end{equation*}
$$

and the 3LVSI dc-link voltage value can be calculated as

$$
\begin{equation*}
U_{i n} \geq 2 \sqrt{U_{d e q}^{2}+U_{q e q}^{2}+U_{d e q} U_{q e q}} \tag{19}
\end{equation*}
$$

The strategy to transfer the controls $\operatorname{sgn} F_{1}$ and $\operatorname{sgn} F_{2}$ to controls $S_{a}, S_{b}, S_{c}$ of the switches $\mathrm{S} 1, \mathrm{~S} 2$ and S 3 is presented in the Tables 3 and 4 . There are 12 angle zones, where switch control is equal to one of the controls $\operatorname{sgn} F_{1}$ and $\operatorname{sgn} F_{2}$ or their opposite values. The positions of the $(\pi / 6)$ zones depend upon the values of the selected magnitudes $u_{d e q}$ and $u_{q e q}$ and are connected with the rotating frame angle $\rho$ via the angle $\xi$

$$
\begin{equation*}
\xi=\pi / 6-\arcsin \left(u_{d e q} / B_{z}\right) \tag{16}
\end{equation*}
$$

Table 3. Switch controls

| $\rho$ | $S_{a}$ | $S_{b}$ | $S_{c}$ |
| :---: | :---: | :---: | :---: |
| $(-\xi \ldots \pi / 6-\xi)$ | $\operatorname{sgn~} \mathrm{F}_{2}$ | $\operatorname{sgn~} \mathrm{~F}_{1}$ | C |
| $(\pi / 6-\xi \ldots \pi / 3-\xi)$ | D | $\operatorname{sgn~} \mathrm{F}_{1}$ | $-\operatorname{sgn} \mathrm{F}_{2}$ |
| $(\pi / 3-\xi \ldots \pi / 2-\xi)$ | $-\operatorname{sgn} \mathrm{F}_{1}$ | A | $-\operatorname{sgn} \mathrm{F}_{2}$ |
| $(\pi / 2-\xi \ldots 2 \pi / 3-\xi)$ | $-\operatorname{sgn} \mathrm{F}_{1}$ | $\operatorname{sgn~} \mathrm{~F}_{2}$ | B |
| $(2 \pi / 3-\xi \ldots 5 \pi /-\xi 6)$ | C | $\operatorname{sgn~} \mathrm{F}_{2}$ | $\operatorname{sgn} \mathrm{~F}_{1}$ |
| $(5 \pi / 6-\xi \ldots \pi-\xi)$ | $-\operatorname{sgn} \mathrm{F}_{2}$ | D | $\operatorname{sgn} \mathrm{F}_{1}$ |
| $(\pi-\xi . .7 \pi / 6-\xi)$ | $-\operatorname{sgn~} \mathrm{F}_{2}$ | $-\operatorname{sgn} \mathrm{F}_{1}$ | A |
| $(7 \pi / 6-\xi \ldots 4 \pi / 3-\xi)$ | B | $-\operatorname{sgn} \mathrm{F}_{1}$ | $\operatorname{sgn~} \mathrm{~F}_{2}$ |
| $(4 \pi / 3-\xi \ldots 3 \pi / 2-\xi)$ | $\operatorname{sgn~} \mathrm{F}_{1}$ | C | $\operatorname{sgn} \mathrm{F}_{2}$ |
| $(3 \pi / 3-\xi \ldots 5 \pi / 3-\xi)$ | $\operatorname{sgn~} \mathrm{F}_{1}$ | $-\operatorname{sgn} \mathrm{F}_{2}$ | D |
| $(5 \pi / 3-\xi \ldots 11 \pi / 6-\xi)$ | A | $-\operatorname{sgn} \mathrm{F}_{2}$ | $-\operatorname{sgn} \mathrm{F}_{1}$ |
| $(11 \pi / 6-\xi \ldots 2 \pi-\xi)$ | $\operatorname{sgn~} \mathrm{F}_{2}$ | B | $-\operatorname{sgn} \mathrm{F}_{1}$ |

Table 4. Quantities A...D for Table 3

| $\operatorname{sgn} \mathrm{F}_{1}$ | 1 | -1 | 1 | -1 |
| :---: | :---: | :---: | :---: | :---: |
| $\operatorname{sgn} \mathrm{~F}_{2}$ | 1 | 1 | -1 | -1 |
| A | 1 | 0 | 0 | -1 |
| B | 0 | -1 | 1 | 0 |
| C | -1 | 0 | 0 | 1 |
| D | 0 | 1 | -1 | 0 |

This 3LVSI dc-link voltage value and the control table guarantee that from initial condition selected in (11) - (13) the system will reach the sliding manifold (1), (8), (9) in finite time, remain on it and will be robust against the load value, the selected uncertainty of the system parameters.

It must be mentioned, that the AC drive is robust against the uncertainty of the 3LVSI dc-link voltage and the estimation of the angle position, too. If
condition (18) is satisfied, sliding motion is guaranteed. But the dc voltage must not to be constant; it may change and is bounded only to its lowest value. In this case the value of the dc-link capacitor and thus its size can be small.

The same remark can be made to the definition of the angle-sectors table. If the dc-link voltage value is higher than the minimum value from (18), the request upon the measuring of the angle sectors is lower.

As design result the SMC algorithms (11) - (13), (14) - (16), (18) and tables 3, 4 are obtained, that guarantee zero torque and flux-modulus errors, caused by variation of drive parameters, load and inverter dc-link voltage.

### 4.2 Two loops control

From the control viewpoint the control design task is divided in two ones: the design of the mechanical coordinate control by using the mechanical equation (4), where the control is a motor torque, and the converter control that must guarantee the producing of the needed torque. Opposite to the one loop control for the SMC design for the converters is used only the equations of the electrical part (6). The switchover function (8) must be written for the new control variable: motor torque or a stator current. This function has a reduced order. However the all SMC design is the same, as in the case of the direct converter control. It must be mentioned that the VSI works a a current source inverter.

## 5. Sliding Mode Observer

During the last time there are the tendency to construct the adjustable AC drive without the mechanical coordinates sensors [2, 7]. The mechanical coordinates estimation methods use only terminal current and voltage sensors. One of the important approaches is based on the using the observer technique that depends on the used plant model. The main difficulty is the non-linearity of the drive. In this case SMC is very useful. However the structure of the sliding mode observer depends on both the motor type and its mathematical model. Because the individual design procedure must be done. The procedure for the permanent magnet exterior synchronous motor with one pole pair will be presented below. It will be used the mathematical model in the stationary frame $(\alpha, \beta)$. The mathematical model of the motor electrical part is

$$
\begin{align*}
& d i_{\alpha} / d t=\left(-r i_{\alpha}+\Omega \psi \operatorname{Sin} \Gamma+u_{\alpha}\right) / L \\
& d i_{\beta} / d t=\left(-r i_{\beta}-\Omega \psi \operatorname{Cos} \Gamma+u_{\beta}\right) / L \tag{17}
\end{align*}
$$

where $I^{T}=\left(i_{\alpha}, i_{\beta}\right)$ and $U^{T}=\left(u_{\alpha}, u_{\beta}\right)$ are the space vectors of the stator current and voltage; $r$ and $L$ are the stator resistance and inductance; $\psi$ is no-load magnet flux linkage.

The dynamic system that included a structure, which is similar to one of the observed motor (17)

$$
\begin{align*}
& d \hat{i}_{\alpha} / d t=\left(-r \hat{i}_{\alpha}+u_{\alpha}\right) / L+u_{1} \\
& d \hat{i}_{\beta} / d t=\left(-r \hat{i}_{\beta}+u_{\beta}\right) / L+u_{2} \tag{18}
\end{align*}
$$

where $u_{1}, u_{2}$ are the action controls, the upper index " $\wedge$ " have the current and voltage of the model. The input signals are electrical current and voltage measured in the stator windings, and the output signals must be a rotational speed $\Omega$ and position $\Gamma$ of the motor shaft. The model motion will be equal to the motor motion, if the components of the model current $\hat{i}_{\alpha}, \hat{i}_{\beta}$ will be equal to the same components of the motor current $i_{\alpha}, i_{\beta}$. It may be designed by intentional introduction of the sliding mode on the intersection of two sliding surfaces $F_{\alpha}=0$ and $F_{\beta}=0$, where $F_{\alpha}$ and $F_{\beta}$ are the switchover functions:

$$
\begin{align*}
& F_{\alpha}=i_{\alpha}-\hat{i}_{\alpha} \\
& F_{\beta}=i_{\beta}-\hat{i}_{\beta} \tag{19}
\end{align*}
$$

by using the action control $u_{1}, u_{2}$.
In this case the projection of the systems (17) and (18) on the subspaces $F_{\alpha}$ and $F_{\beta}$ that is used for the sliding mode design is

$$
\begin{align*}
& \left.\frac{d}{d t}\left|\begin{array}{l}
S_{\alpha} \\
S_{\beta}
\end{array}\right|=-\frac{r}{L}\left\|\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right\|| | \begin{array}{l}
i_{\alpha}-\hat{i}_{\alpha} \\
i_{\beta}-\hat{i}_{\beta}
\end{array}\left|+\frac{\Omega \psi}{L}\right| \begin{array}{c}
\operatorname{Sin} \Gamma \\
-\operatorname{Cos} \Gamma
\end{array} \right\rvert\,-  \tag{20}\\
& -\left\|\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right\|\left|\begin{array}{l}
u_{1} \\
u_{2}
\end{array}\right|
\end{align*}
$$

The matrix before the vector control is a identity one. As a result the problem of sliding domain is reduced to a sequential analysis of two scalar cases. The scalar sliding mode condition is

$$
\begin{equation*}
\lim (d F / d t)<0 \& \lim (d F / d t)>0 \tag{21}
\end{equation*}
$$

The action controls $u_{1}, u_{2}$ are selected depending upon the sign of the components of the switchover function $F_{\alpha}$ and $F_{\beta}$

$$
\begin{align*}
& u_{1}=U_{1} \operatorname{sgn} F_{\alpha} \\
& u_{2}=U_{2} \operatorname{sgn} F_{\beta} . \tag{22}
\end{align*}
$$

Their magnitudes $U_{1}$ and $U_{2}$ can be selected using the following inequalities

$$
\begin{align*}
& U_{1} \geq \frac{1}{L}\left|-r\left(i_{\alpha}-\hat{i}_{\alpha}\right)+\Omega \psi \sin \Gamma\right| \\
& U_{2} \geq \frac{1}{L}\left|-r\left(i_{\beta}-\hat{i}_{\beta}\right)-\Omega \psi \cos \Gamma\right| \tag{23}
\end{align*}
$$

If sliding mode on the intersection of the two sliding surface $F_{\alpha}=0$ and $F_{\beta}=0$ is generated, the equivalent controls exist

$$
\begin{align*}
& u_{1 e q}=-\Omega \psi \operatorname{Sin} \Gamma \\
& u_{2 e q}=\psi \Omega \operatorname{Cos} \Gamma \tag{24}
\end{align*}
$$

that have an information about the rotational speed $\Omega$ and position $\Gamma$ of the motor shaft. These equivalent controls may be used for calculating of the rotational speed $\Omega$ and position $\Gamma$ получения требуемой информации об угловом положении и частоте вращения ротора:

$$
\begin{align*}
& \Gamma=-\operatorname{arctg}\left(u_{1 e q} / u_{2 e q}\right),  \tag{25}\\
& \Omega=\sqrt{u_{1 e q}^{2}+u_{2 e q}^{2}} / \psi . \tag{26}
\end{align*}
$$

Hence for the estimation of the rotational speed $\Omega$ and position $\Gamma$ of the motor shaft first of all the equivalent action controls $u_{1 \text { eq }}, u_{2 \text { eq }}$ must be received, e.g. with the help of the low-pass filters, and then the rotational speed $\Omega$ and position $\Gamma$ must be computed in accordance with (25), (26).

## 5 Conclusion

The application of the sliding mode technique to the control and observer design of the AC drives on the base of the induction motor or synchronous one and the different converters with high-frequency power semiconductor devices has been presented. Two different control strategies have been analyzed (one
loop and two loops). It has been shown that there are typical methods for design of SMC. For a correct design of the sliding mode control system the use of such methods is suggested. Using a two-step technique decomposes the control design task and allows analyzing separately the nonlinearities of the AC motors and the switching character of the converters. It makes the control design more simple and graphical. The drive control, that has been designed by using the SMC technique would be able to guarantee all advantageous characteristics of the control plant with such type of control as high dynamic, low sensitivity to disturbance of both the load and the input dc-link voltage and to the plant parameters variations.

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