Systematic Non-Linearity for Multiple Distributed Illumination Units for Time-of-Flight (PMD) Cameras

O. Lottner, W. Weihs, K. Hartmann
Centre for Sensor Systems
University of Siegen
Paul-Bonatz-Str. 6-8, 57076 Siegen
Germany

Abstract: The non-linearity of phase and amplitude of time-of-flight cameras' measurements with a single, usually coaxial illumination unit has been discussed excessively in literature. This effect is calibrated using a number of different approaches (B-Splines, polynomial approximation or with infinite Fourier series). In this paper, this concept is extended to the calibration of multiple distributed illumination units used simultaneously.

Key-Words: time-of-flight; phase non-linearity; multiple distributed illumination units

1 Introduction
Nowadays, time-of-flight cameras delivering a 3D-representation of the scene in real-time are widely used in computer vision for a number of purposes including for example surveillance purposes [2], autonomous navigation [7] or modeling of indoor environments [1]. In order to combine the advantages of depth imaging with the advantages of conventional 2D image processing, ZESS has developed a monocular 2D/3D camera called 'MultiCam'. It combines a PMD3k-S from PMDTec [6] with a conventional 2D CMOS sensor with high resolution. A detailed description of the (optical) set-up can be found in [4]. In addition to this new kind of 2D/3D combination and more important for this paper, the camera offers an adjustable modulation frequency, the possibility to emit the illumination modulation signal only during integration times, the availability of an external trigger input and of an auto-trigger function for temporally equidistant measurements and finally a highly configurable FPGA device for real-time processing, sensor synchronization and illumination control.

The phase/range measurement of these cameras shows systematic deviations to a linear phase/range relation [3,5,8]. These deviations emerge from the fact that neither the illumination nor the reference modulation signal is ideally sinusoidal. Usually, the deviations are measured, and then a B-Splines [5] or polynomial model [2] is fitted to the measurement. This model then serves to correct the measured phase. However, this model has not been extended yet to the operation with more than one illumination unit at the same time and frequency. The interest in this kind of calibration arises because simultaneously using more than one illumination unit a the same modulation frequency with overlapping illuminated areas, it is possible to selectively focus on certain parts of the scene, or to work with overlapping illumination areas when the individual units serve to illuminate a bigger spatial volume.

In this paper, a concept to calibrate multiple illumination units is explained, taking the example of a MultiCam with PMD-3kS TOF sensor. The concept should be independent of this specific sensor / camera. As to the structure of this paper, it starts to recite the operation principle with one illumination unit. In the next step, this principle is extended to multiple illumination units taking the example of two units for explanation. Last, an example for the increase in modulation contrast with more than one illumination unit is given.

2 Operation Principle with One Illumination Unit
Our MultiCam – like any common PMD camera (PMDTec, IFM etc.) – uses a near-infrared LED illumination unit which emits a modulated light signal \( R(t) \). As this signal is neither ideally sinusoidal nor rectangular, a Fourier series is used for the mathematical model [3] with the angular frequency \( \omega_1 \), the amplitudes \( \tilde{R}_{i1}, \tilde{R}_{2i} \), the constant component \( R_{01} \) and the (unknown) phase \( \phi \):

\[
R(t, \phi) = R_{01} + \sum_{i=1}^{\infty} \left( \tilde{R}_{i1} \sin(\omega_1 t + \phi) \right) + \tilde{R}_{2i} \cos(\omega_1 t + \phi) \quad (1)
\]

Its reflection is captured by a matrix of pixels with two push-pulled elements A and B in each pixel [3,6]. Here it is correlated with a reference modulation signal of similar form with the same angular frequency \( \omega_1 \), the constant component \( U_0 \), the
amplitudes \( \hat{U}_{1i}, \hat{U}_{2i} \) and the phase offset \( \psi \) to the illumination signal (+ for element A, - for B; typically \( \psi = 0^\circ, 90^\circ \)): 

\[
U_{A,B}(t, \psi) = U_0 + \sum_{i=1}^{\infty} (\hat{U}_{1i} \sin(i \omega_0 t + \psi) + \hat{U}_{2i} \cos(i \omega_0 t + \psi))
\]  

(2)

In each pixel, a correlation process is performed which can mathematically be described as a convolution of the two signals with the exposure time \( nT^2 \).

\[
N_{A,B}(\phi, \psi) = \int_0^{nT^2} U_{A,B}(t, \psi) R(t, \phi) dt
\]  

(3)

By neglecting higher-order harmonics and thus restricting to ideally sinusoidal signals, the well-known equations for the computation of the modulation amplitude \( M \) and the phase shift \( \phi \) are obtained (here at the example of the two-phase algorithm (\( \psi = [0^\circ, 90^\circ] \))):

\[
\phi = \tan^{-1} \left( \frac{\Delta I(\psi=90^\circ)}{\Delta I(\psi=0^\circ)} \right)
\]

(4)

\[
2M = \sqrt{\Delta I(\psi=90^\circ)^2 + \Delta I(\psi=0^\circ)^2}
\]

with \( \Delta I(\psi, \phi) = N_A(\psi, \phi) - N_B(\psi, \phi) \)

Without this simplification, however, solving the integral leads to (B analogously):

\[
N_A(\phi, \psi) = U_0 R_0 nT + \frac{nT^2}{2} \ldots
\]

(5)

\[
\left( H_1(\psi) \cos(\phi - \psi) + H_2(\psi) \sin(\phi - \psi) \right)
\]

with

\[
H_1(\psi) = \sum_{i=0}^{\infty} U_{1i} R_{1i} - U_{2i} R_{2i},
\]

\[
H_2(\psi) = \sum_{i=0}^{\infty} U_{2i} R_{1i} - U_{1i} R_{2i}
\]

(6)

These form factors depend on \( \psi \) because in this way systematic differences between the individual phase images are taken into consideration: systematic, albeit small variances in the control electronics of the PMD sensor directly affect the duty cycle. Such systematic variances could be shown experimentally. For this reason, two form factors are needed: \( H_{1,2} = f(\psi) \).

Building the difference between channels A and B leads to:

\[
\frac{\Delta I(\phi, \psi)}{nT} = H_1(\psi) \cos(\phi - \psi) + H_2(\psi) \sin(\phi - \psi)
\]  

(7)

The ratio of the Fourier coefficients in equation 6 is now described with form factors \( \alpha(\psi) : \)

1 For each phase image, \( \psi \) assumes a different value. For two phase images, we have \( \psi = 0^\circ, 90^\circ \).

2 In the digital control unit, the exposure time is an integer multiple of a the smallest time unit T, the periodic time of the modulation signal.

\[
H_2(\psi) = \alpha(\psi) \cdot H_1(\psi)
\]  

(8)

This factor characterizes the form of both signals. Vanishing form factors correspond to the case where at least one of the signals is ideally sinusoidal.

With the 2-phase-algorithm (\( \psi = [0, \pi/2] \)), \( \alpha_1 = \alpha(\psi=90^\circ); \alpha_2 = \alpha(\psi=0^\circ) \), one actually measures:

\[
\tan(\phi_{\text{meas}}) = \frac{\sin(\phi) + \alpha_1 \cos(\phi)}{\cos(\phi) + \alpha_2 \sin(\phi)}
\]  

(9)

Thus, the following model can be used to fit the deviations between ideal and measured phase:

\[
\Delta \phi = \phi - \arctan \left( \frac{\sin(\phi) + \alpha_1 \cos(\phi)}{\cos(\phi) + \alpha_2 \sin(\phi)} \right)
\]  

(10)

The parameters can be determined by a least-squares fit or by guiding a user graphically. Figure 1 shows a measurement of the deviations and a data fit using polynomial terms and the model-based data fit.

\[
\text{Figure 1: phase non-linearity, 1 ill., } \alpha_1=0.04; \alpha_2=0.03
\]

A polynomial fit with reasonable residues requires quite a high order (in the example order 11 for R=0.017). This can lead to problems with the numerical stability. However, considering only one illumination unit, the model-based data fit does not seem to be much better (R=0.018). Yet, the polynomial also fits to deviations that are not caused by the signal form, but e.g. by noise or systematic deviations due to intensity influences. The actual significance of the model-based fit does not get apparent until considering more than one illumination unit.

In general, the signal forms have significant influence on the phase. Yet, it must be noted that the phase is only influenced on by harmonics of the same order in both signals. Consequently, it would be sufficient to realize an ideally sinusoidal reference signal in order to obviate the impact of the harmonics.
3 Calibration of Two Illumination Units

Considering the simultaneous operation with two illumination units at the same modulation frequency and with overlapping or identical illuminated areas, equation 3 must be extended with a second illumination source:

\[ N_{A,B}(\phi, \psi) = \int_0^{\pi} \left( U_{A,B}(t, \psi) R^{L1}(t) + U_{A,B}(t, \psi) R^{L2}(t) \right) dt \]  

(11)

Analogously, we get for the difference between both channels:

\[ \frac{2|N_A(\phi, \psi) - U_0 R_0 nT|}{nT} = \ldots \]

\[ H_1^{L1}(\psi) \cos(\phi - \psi) + H_2^{L1}(\psi) \sin(\phi - \psi) + \ldots \]

\[ H_1^{L2}(\psi) \cos(\phi - \psi) + H_2^{L2}(\psi) \sin(\phi - \psi) \]

(12)

If one uses the 2-phase-algorithm equation, one actually measures:

\[ \tan(\phi_{max}) = \ldots \]

\[ \frac{\sin(\phi_1) + \alpha_1 \cos(\phi_1) + \gamma \sin(\phi_2) + \beta_1 \cos(\phi_2)}{\cos(\phi_1) + \alpha_1 \sin(\phi_1) + \gamma \cos(\phi_2) + \beta_2 \sin(\phi_2)} \]

(13)

with \( H_2^{L2} = \beta_1 \gamma H_1^{L2} \) and \( H_1^{L2} = \gamma H_1^{L1} \)

\( \beta_1, \gamma \) are the form factors of the second illumination, and \( \gamma \) is the ratio of both illuminations' intensities. For a simulated configuration of \( \alpha = \alpha_1 = \alpha_2 = 0.1 \), \( \beta = \beta_1 = \beta_2 = 0.1 \), \( \gamma = 1.0 \), the difference to the linearized phase is plotted in figure 2.

Modifying the intensity relationship to \( \gamma = 0 \), the effect looks like figure 3 shows. Only due to the change in the relation of both illuminations' intensities, there is a significant difference between the depicted deviations, which makes the usage of a non-model based correction like e.g. using polynomial terms quite difficult.

![Phase/Range Non-Linearity](image1)

**Figure 3: deviation to linearized phase, simulation 2**

A real measurement of this non-linearity effect for two illumination units is shown in figure 4. The colours indicate different differences between the first and second illumination's phase, i.e. red marked points indicate destructive interference whereas green points represent constructive interference. Destructive

![Phase/Range Non-Linearity](image2)

**Figure 4: deviation to linearized phase, measurement**

For this plot, a MultiCam has been positioned to look at a 90% greycard (“Zebra”, cf. [http://www.novoflex.com](http://www.novoflex.com)) at a distance of 0.5m, and the camera's modulation amplitude and phase measurement were acquired for a constant exposure time, while two illumination units identical in construction were shifted in phase in the interval \([0;2\pi)\) by means of a digital pulse generator.

3 Identical modulation frequencies are usually achieved by beat lines between camera and illumination units.

4 superscript “L1” for ill. 1, “L2” for ill. 2

5 For this plot, a MultiCam has been positioned to look at a 90% greycard (“Zebra”, cf. [http://www.novoflex.com](http://www.novoflex.com)) at a distance of 0.5m, and the camera's modulation amplitude and phase measurement were acquired for a constant exposure time, while two illumination units identical in construction were shifted in phase in the interval \([0;2\pi)\) by means of a digital pulse generator.
interference should not be considered for the evaluation of a model, as the primary interest in working with more than one illumination unit at the same time is the increase of the modulation contrast. It would be quite difficult to fit a polynomial model to this deviation, and the knowledge of the correction terms for the operation with one illumination unit does not help for this case, as one does not know how to combine the individual terms. This is where the model-based fit is helpful; as shown above, the parameters describing the deviation from the ideally sinusoidal form can be determined only the 1-illumination data. The same parameters can be used for the two-illumination-units data, as can be seen exemplarily for two selected fixed $\phi_B$ (figures 5 and 6 show the deviation and its model for two selected cross sections of figure 4 together with the resulting modulation amplitude measured).

Significant deviations between the model and the actual measurement occur only at destructive interference, i.e. when the difference between the individual illuminations’ phases in the observation point exceeds 180°. In this case, the resulting modulation amplitude and consequently the SNR drop low and the reliability of the PMD phase measurement is heavily attenuated by noise.

4 Application of Two Illumination Units

The main interest in simultaneously using two illumination units is to increase the SNR for a more reliable phase measurement or to retrieve reliable phase data in areas where illumination units overlap: according to [9], the standard deviation of the phase measurement is reciprocally proportional to the modulation amplitude (in this equation, $B$ is the background intensity):

$$\sigma_\phi = \frac{c}{4 \sqrt{2 \pi} f_M} \frac{\sqrt{B}}{M} \quad (14)$$

Although it is possible to achieve a suitable modulation amplitude by appropriate integration times, this can be problematic in dynamic environments: the frame rate decreases, the image smears and movement artefacts can occur [10]. To validate the increase of the contrast, the camera’s modulation amplitude and phase are acquired under the same conditions as described above (cf. footnote 5), but with an additional movement of the grey card in steps of 5mm by means of a linear axis, which also provides the real distance between camera and grey card as a reference. The plots in figure 7 compare the 1-illumination-unit data to the data acquired using both illumination units: the first subplot shows the computed range (for the 2-illumination case using the configuration with the maximum gain in modulation amplitude), the second subplot shows the gain in modulation amplitude, and the last subplot shows the phase’s standard deviation.
Evidently, the correct range data can be computed also with the 2-illumination configuration, and the increase of the modulation contrast results in a lower standard deviation corresponding to a more reliable phase measurement.

4 Conclusion
The non-linear relation between phase and range of time-of-flight cameras has been discussed with regard to the operation with multiple illumination units. If this effect is calibrated without a signal model but rather with a data fit model, the same kind of calibration for two illumination units used simultaneously gets very difficult. In this paper, a calibration concept has been discussed in the context of one illumination and the concept has been extended to the calibration of multiple distributed illumination units used simultaneously. The considerations should also hold for N illumination units without really complicating the model. In the last part of the paper, an example for the application of multiple, distributed illumination units showed the increase of the modulation contrast and thus the reliability of the phase/range measurement.

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