Reduced-set Vector Learning Based on Hybrid Kernels for Interval Type 2 Fuzzy Modeling

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Abstract: This paper presents a new interval type-2 fuzzy inference system to handle uncertainty using reduced-set vector learning mechanism based on hybrid kernels. Firstly, a novel concept, interval kernel, is proposed. It establishes a relationship between interval type-2 fuzzy membership and hybrid kernel. According to it, a particular interval type-2 fuzzy inference system is built, which abandons traditional type reduction procedure and utilizes directly defuzzification after inference. Subsequently, the model optimization is realized via a hybrid learning mechanism involving two sub-algorithms: bottom-up simplification algorithm and quadratic programming combined with back propagation algorithm. At last, simulation results show that the interval type-2 fuzzy model obtained possesses nice generalization and transparency.

Key-Words: interval type-2; fuzzy modeling; reduced-set; hybrid kernel

1 Introduction
By far, several authors had attempted to incorporate support vector machine (SVM) into fuzzy modeling so as to make the resulting fuzzy model possess nice generalization [1]-[3]. However, SVM produces too many support vectors which results in large number of fuzzy rules. Hence, some methods had been proposed to compensate this problem [4], [5].

Nevertheless, nearly all the approaches about the combination of FM (fuzzy modeling) and SVM are related to type-1 fuzzy logic. Mendel had claimed that type1 fuzzy logic is unable to handle uncertainties which actually lie in real world, whereas the type-2 fuzzy logic can model them and minimize their effects [6]-[8]. Therefore, this led to several fuzzy modeling algorithms based on type-2 fuzzy logic, especially the interval form owing to its decreasing computational intensity [7], [8]. Noticeably, no one has yet paid attention to the generalization of type-2 fuzzy systems when modeling. The work presented here will attempt to fill the blank. Loosely speaking, the type-2 fuzzy model, built on finite amount of given training data, generalizes best if the right trade-off is found between accuracy and the capability of fuzzy model set. The capability, here, could be understood as the number of fuzzy rules in rule base. Indeed, a good type-2 fuzzy rule base should have a small number of rules so as to make it more transparency and interpretability.

This paper will describe a new interval type-2 fuzzy modeling framework using reduced-set vector learning mechanism based on hybrid kernels, called RV-based IT2FIS. Firstly, a new concept of interval kernel is proposed. The idea is elicited from interval type-2 fuzzy logic. According to it, the ordinary Mercer kernel is generalized onto the interval. Consequently, a new kernel learning machine, interval support vector regression (SVR) model, is obtained. Noticeably, it is pointed out that the resulting model could be regarded as interval type-2 fuzzy inference system, and it also could be transformed into an ordinary SVR model with hybrid kernels for crisp input and output. Indeed, in order to get more transparent and general interval type-2 fuzzy rule base, a refined version is needed, i.e., RV-based IT2FIS. Hence, a hybrid learning algorithm is presented to achieve it. It involves two sub-algorithms: bottom-up simplification algorithm, which is exploited to extract reduced-set vectors for generating interval type-2 fuzzy IF-THEN rules instead of support vectors and quadratic programming combined with back propagation algorithm, which is employed to tune the parameters of RV-based IT2FIS. Finally, simulation results show that the proposed interval type-2 fuzzy model possesses nice generalization and transparency.

2 Relation between hybrid kernel and interval type-2 fuzzy membership
Smola et al. [9] indicated that the solution of the SVR approach is in the form of the following linear expansion of kernels:
\[ f_{\text{SVR}}(\mathbf{x}) = \sum_{i=1}^{M} \theta_i k(\mathbf{x}, \mathbf{x}_i) + b \]  

where \( \theta_i = \alpha_i - \alpha_i^* \) are subject to contains \( 0 \leq \alpha_i, \alpha_i^* \leq C \); \( c \) is the number of support vectors; \( C \) is the regularization parameter, \( k(\mathbf{x}, \mathbf{x}) \) is an admissible Mercer kernel. Chen et al. [3] introduced the reference function \( \mu(x) \) to construct the kernels.

\[ k(\mathbf{x}, \mathbf{x}) = \prod_{j=1}^{M} \mu(x_j, x_j, \Theta_j) \]  

Clearly, the reference function can be regarded as exact symmetrical type-1 fuzzy membership. It corresponds to the ordinary fuzzy set \( A_{\mu} \), and \( \Theta_j \) denotes the size of support of \( A_{\mu} \), \( x_j \) denotes the center of it. Hence, it is believed that the resulting product-type kernels are also “exact”. However, due to the known disadvantages of type 1 fuzzy membership, it seems doubtful whether SVR that utilizes these “exact” kernels is adequate to cope with uncertain or complicated real data. To this point, hybrid kernels had ever been proposed as followings [10], [11].

\[ k^i(\mathbf{x}, \mathbf{x}) = \sum_{j=1}^{M} a_j k(\mathbf{x}, \mathbf{x}_j) \]  

The hybrid kernel \( k^i(\cdot, \cdot) \) is encoded as nonnegative combination coefficients and admissible Mercer kernels. It makes the resulting kernels more flexible so as to accommodate various requirements. This seems very similar to the reason that interval type 2 fuzzy memberships are presented [6]. Therefore, it could be seen there might exist a road between interval type 2 fuzzy memberships and hybrid kernels.

Firstly, we generalize ordinary Mercer kernel onto interval. This leads to the concept of interval kernel, and is given by:

\[ k(x, x') = [k(x, x), \bar{k}(x, x)] \]  

where \( k(x, x) \) and \( \bar{k}(x, x) \) respectively represent the upper and lower kernel function of interval kernel, i.e., \( k(x, x) \leq \bar{k}(x, x) \) for all \( x, x \in \mathbb{R}^d \), and both are still ordinary Mercer kernel. While both are constructed by (2), and assuming that \( \mu(x_j, x_j, \Theta_j) \leq \bar{\mu}(x_j, x_j, \Theta_j) \), then (4) could be particularly formalized as:

\[ k^i(\mathbf{x}, \mathbf{x}) = \prod_{j=1}^{M} \mu(x_j, x_j, \Theta_j) \prod_{j=1}^{M} \bar{\mu}(x_j, x_j, \Theta_j) \]  

According to the theorem 3 in [12], (5) could reappear as:

\[ k^i(\mathbf{x}, \mathbf{x}) = \sup_{x_j} \prod_{j=1}^{M} \mu(x_j, x_j, \Theta_j) \prod_{j=1}^{M} \bar{\mu}(x_j, x_j, \Theta_j) / x / \]  

where \( \bullet \) denotes product \( t \)-norm; \( \mu(x_j, x_j, \Theta_j) \) and \( \bar{\mu}(x_j, x_j, \Theta_j) \) are upper membership function (UMF) and lower membership function (LMF) which bound the footprint of uncertainty (FOU) of an interval type-2 membership function \( \mu_{\alpha} \). In this way, it is clear that the interval kernel is equal to primary membership grades of an interval type-2 membership function. Here, we provide a special FOU for constructing interval kernel, i.e., symmetric FOU – Gaussian UMF and scaled Gaussian LMF [13].

Thus, we will obtain a new interval SVR model using the interval kernel instead of ordinary kernel for crisp input and output data:

\[ f_{\text{SVR}}(\mathbf{x}) = \sum_{i=1}^{M} \theta_i k^i(\mathbf{x}, \mathbf{x}_i) + [b, b]. \]  

\[ f_{\text{SVR}}(\mathbf{x}) = \left\{ \frac{1}{2} \left( \frac{x}{\sigma} \right)^2 \right\} \]  

Note that (9) employs mean computation which looks like the form of defuzzified output of an interval type-2 fuzzy model. Moreover, this model differs from support vector interval regression machine (SVIRM) [12], since it directly exploits interval kernel other than interval coefficients to construct regression model. It is apparent that the model presented is more simple and interpretable. By interval arithmetic, we formulate (9) in more visible expression:

\[ f_{\text{SVR}}(\mathbf{x}) = \sum_{i=1}^{M} \theta_i k^i(\mathbf{x}, \mathbf{x}_i) + b \]  

where \( k^i(\mathbf{x}, \mathbf{x}_i) = k(\mathbf{x}, \mathbf{x}_i)/2 + \bar{k}(\mathbf{x}, \mathbf{x}_i)/2 \) is a hybrid kernel with \( M = 2 \). Thus, the interval SVR model is transformed into a standard SVR model, in which hybrid kernels are utilized. Thus, the interval kernels seem to build a bridge between interval type-2 fuzzy memberships and hybrid kernels.

3 Modeling a new interval type-2 fuzzy inference system

In this section, we exploit reduced-set vectors learning based on hybrid kernel to build a new interval type-2 fuzzy inference system.

3.1 Model formulation

In [11], Chiang et al. regarded kernel function in (1) as the fuzzy basis function, since they claimed that the remove of denominator of the fuzzy basis
function (FBF) does not violate the spirit of fuzzy inference system. In the same way, we directly choose \textit{interval kernel} as interval type-2 FBF for FM. It means that traditional type-reduction procedure in interval type-2 fuzzy inference system is given up, and defuzzification procedure is successively implemented after inference. However, note that sometimes support vectors generates too many fuzzy rules when utilizing (1), as [11] suggests, for FM. Luckily, reduced-set method could be used to compensate this drawback [4]. Hence, a new interval type-2 fuzzy inference system using reduced-set vectors, named RV-based IT2FIS, is given:

$$f_{FM}(x) = \sum_{i} \theta' f_i(x) + \beta \sum_{i} k'(x, z_i) + \beta k(x, z_i)$$

(11)

$$f_{FM}(x) = \frac{f_{FM}(x) + \beta f_{FM}(x)}{2}$$

(12)

where \(k(x, z_i)\) are reduced-set vectors; and \(c'\) is the number of them. Therefore, \(\theta'\) construct a reduced set under the condition of \(c' < c\). Then, the model proposed extracts reduced-set vectors for generating fuzzy rules from the training data set. It makes the interval type-2 fuzzy inference system more transparent. As the same as (10), (12) could also be encoded with the form of hybrid kernels:

$$f_{FM}(x) = \sum_{i} \theta' f_i(k(x, z_i) + b, b)$$

(13)

Thus, the resulting model consists of a reduced set of linguistic rules in the following form:

IF \(x_i = \tilde{A}_{i1}\) and \(\ldots\) and \(x_d = \tilde{A}_{d1}\), THEN \(f_{FM}(x) = \theta'\). \(i = 1, \ldots, c'\). Where \(x_i (i = 1, \ldots, d)\) are input variables; \(\tilde{A}_{ij}\) is linguistic term characterized by its lower and upper primary membership function denoted respectively by \(\mu_{\tilde{A}_{ij}}\) and \(\tilde{A}_{ij}\). As the same as [3], bias \(b\) could be understood as an added rule which covers the whole input space, i.e., the grades of membership are constant “1”.

### 3.2 Learning algorithm

In this subsection, we present a hybrid learning mechanism which involves two sub-algorithms for estimating the parameters of (11), (12), in which (7) is employed.

Firstly, bottom-up simplification algorithm is utilized to reduce the number of support vector [14]. Thus, \(\theta'\) are estimated. Then, the second sub-algorithm is composed of a forward pass and a back pass which implement quadratic programming and back propagation algorithm in the following performance measure over \(\theta'\), \(s\) and \(\lambda\):

$$E = \frac{1}{2} l (y) + \beta \sum_{i} (f_{FM}(x_i) - y_i)^2$$

$$= \frac{1}{2} \sum_{i} \sum_{j} \theta' f_i k'(x, z_i) + \frac{2 \beta}{n} \sum_{i} k(x, z_i) k(x, z_i) + \frac{2 \beta}{n} \sum_{i} k(x, z_i) b - y_i)$$

(14)

where \(\{x_i, y_i\}_{i=1}^n\) are training data set; \(k'(x, z_i)\) and \(k(x, z_i)\) are constructed by (7); \(\beta\) is a regularization parameter. More specially, \(\theta'\) are adjusted by quadratic programming in the forward pass. In the backward pass, the error rates propagate backward, thus \(s\) and \(\lambda\) are updated by the gradient descent.

**Hybrid learning algorithm of RV-based IT2FIS:**

Input: Initial parameters of hybrid kernels, \(\epsilon\) error and regularization parameter \(C, \beta\); training data; learning rate.

Begin: epoch=0, while termination condition not satisfied do

Start

Use \(\epsilon\) – SVR with hybrid kernel to derive initial interval SVR model.

Employ the bottom-up simplification algorithm, thus reduced-set vectors are output as the centers of interval type-2 fuzzy memberships;

Adjust the \(\theta'\) by solving the quadratic programming;

Evaluate the error measure;

Update parameters \(s\) and \(\lambda\) with the back propagation method;

Set the updated parameters as new kernel parameters;

\(\text{epoch} = \text{epoch} + 1\);

End

End

### 4 Simulations

Let us consider the following function:

$$\sin(x) = \begin{cases} \frac{\sin(x)}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

(15)

61 data points are generated by (15) as training set, and the test set consists of 41 equally spaced data points. SVM-based fuzzy basis function inference system (SVM-based FBFIS) [1] is employed to simulate (15) compared to our proposed RV-based IT2FIS. Root average squared error (RASE) is used as measures for evaluating the approximation and generalization. Firstly, we consider the noiseless case. The results are shown in Figure.1 and Table.1. Obviously, our method produces small number of fuzzy rules, but maintains nice accuracy. On the
other hand, some noise points are added at the peaks and valleys of (15). Subsequently, SVM-based FBFIS is heavily affected by the noise, and the number reaches 21 (see Fig.2). Contrarily, RV-based IT2FIS could effectively decrease the number of fuzzy rules, at the same time possess acceptable performance.

![Fig. 1. Noiseless case. (a) Regression curve with 9 support vectors. (b) Regression curve with 7 reduced-set vectors](image1)

![Fig. 2. Noise case. (a) Regression curve with 21 support vectors. (b) Regression curve with 11 reduced-set vectors](image2)

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5 Conclusion
This paper tries to build a new interval type-2 fuzzy inference system, named RV-based IT2FIS, to cope with uncertain or complicated data. The reduced-set vector learning mechanism with hybrid kernel, which gives up traditional type reduction procedure, provides a new structural framework to extract reduced-set vector for the use of interval type-2 fuzzy rule generation. It effectively reduces the structure or the complexity of interval type-2 fuzzy model for interpretability and associated hybrid learning algorithm makes the model more accurate. It means that if we weight more flexibility about fuzzy membership function, i.e., type-2 form, then the rule base becomes more transparent. In result, the fuzzy inference system obtained preserves advantages of both the statistical learning and interpretable fuzzy model.

References:

