“Design of a fault tolerant control through Bond Graphs and algebraic-differential tools: application on a DC motor”

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Abstract - In this paper we propose a controller designed using algebraic techniques for a DC motor. The failures are estimated through a reduced order observer to reject their effect on the system. This paper represents the first phase in a Bond Graphs’ based approach to determine the diagnosability condition.

Key words: Diagnosis condition, observer, Bond graphs, differential algebra.

I. INTRODUCTION

Systems diagnosis has been studied for more than three decades, see for instance [1]. Also numerous are the journals and articles that present control systems’ applications [2, 3]. In [4] is given a review of the different fault diagnosis approach for deterministic nonlinear dynamic systems. Another appealing approach is the one based on differential geometric methods, shown in [2, 3, 5, 6, 7, 8, 9, 10, 11]. Alternatively some authors have proposed solutions to the fault detection and identification problem for a nonlinear system class in a differential and algebraic setting, see [12, 13, 14, 15, 16, 17]. For instance, in [12, 13] an approach has been considered to solve the diagnosis problem. It consists in translating the solvability of the problem in terms of the algebraic observability of the variable modeling the fault. In [16, 17, 18, 19, 20] the methodologies employed for the observer design only include full order observers without considering uncertainty estimation. In this work, the fault dynamics is considered as an uncertainty. In the proposed procedure, the construction of a full order observer is not necessary, instead, a reduced-order uncertainty observer is constructed using differential algebraic techniques applied to the fault estimation in the diagnosis problem. Using those estimations, a trajectory tracking controller was constructed through algebraic methods.

The main achievement of this work is to fuse algebraic differential techniques of diagnosis with Bond Graphs modeling. This will be used to verify the diagnosability condition, see [21], for which this paper represents the initial phase.

The need for applying Bond Graphs modeling methodology to obtain the differential transcendence degree, comes from the difficulty that presents verifying the diagnosis condition in some systems, let us remember that the theorem found in [21] gives a non constructive proof of said condition, which, sometimes makes more difficult to satisfy the theorem than to obtain the condition itself. The Bond Graph methodology allows a graphical construction of the model and to visually verify the relationships among outputs, known inputs and faults, thus the differential transcendence degree.

The system classes to which this approach can be applied include input dependant systems and its derivatives in polynomial form. In this work, we present an application to a DC motor, which is widely used in robotics where speed and positional control are of utmost importance. The proposed faults are a parasitic current in the field section and a nonlinear friction. A friction coefficient was not chosen because this would represent a motor under ideal conditions, instead, the LuGre’s mathematical friction model was utilized, see [23], to show the operation of the motor and of the approach under more realistic conditions.

The rest of this paper is organized as follows: in section 2 the differential algebraic definitions are given along with the faults diagnosability definitions. In section 3, the motor model is obtained, and the differential transcendence degree through Bond graphs of the output vector. In section 4 are shown the numerical simulations and the parameters used. In section 5 this paper is closed with concluding remarks and future work proposals.

II. BASIC DEFINITIONS

We start by introducing some basic algebraic differential definitions, these can be found in [12, 13, 14, 18, 21].

Definition 1. A differential field extension \( L/k \) is given by two differential fields \( k \) and \( L \), such that: (i) \( k \) is a subfield of \( L \), (ii) the derivation of \( k \) is the restriction to \( k \) of the derivation of \( L \).

Example. \( \mathbb{Q}, \mathbb{R} \) and \( \mathbb{C} \) are trivial differential field extensions where \( \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C} \).
Definition 2. Let $L/k$ be a differential field extension. A differential transcendental family, which is the greatest with respect to the inclusion, is called a differential transcendental base of $L/k$. The cardinality of the base is called the differential transcendental degree of $L/k$ and is denoted by

\[ \text{difftrd}(L/k) \]  

(1)

Example. Consider the following system:

\[
\begin{align*}
x_1 &= x_1 + x_2 + 2x_3 \\
x_2 &= x_2 + x_3 + u \\
x_3 &= x_1
\end{align*}
\]  

(2)

Where $u$ is an input variable which is by definition, differentially transcendental over $\mathbb{R}$. From equation (2), it is not hard to obtain the following relationships:

\[
\begin{align*}
0 &= \dot{x}_3 - x_1 \\
0 &= -\dot{x}_3 + x_3 + x_2 + 2x_3 \\
0 &= \ddot{x}_3 - 2\dot{x}_3 - \dot{x}_3 + x_3 \\
\end{align*}
\]  

(3) \quad (4) \quad (5)

Then, according to Definition 2 and from Equations (3) and (4), it can be concluded that $x_1$ and $x_2$ are both differentially algebraic over $\mathbb{R}(x_3)$, since both $x_1$ and $x_2$ satisfy an algebraic polynomial with coefficients in the differential field $\mathbb{R}(x_3)$. We can see that $x_3$ is differentially transcendental over $\mathbb{R}$, since $x_3$ satisfies an algebraic polynomial over $\mathbb{R}(u)$ (see (5)), and not over $\mathbb{R}$. Then it is concluded that the cardinality of the transcendental base of the extension $\mathbb{R}(x_1,x_2,x_3)/\mathbb{R}$ related to system (2) is equal to 1:

\[ \text{difftrd}(\mathbb{R}(x_1,x_2,x_3)/\mathbb{R}) = 1 \]

Definition 3. A dynamics is a finitely generated differential algebraic extension $G/k(u)$ of $k(u,\xi), \xi \in G$. Any element of $G$ satisfies an algebraic differential equation with coefficients being rational functions over $k$ in the elements of $u$ and a finite number of their time derivatives.

Example: Let consider the input-output system

\[
\begin{align*}
\dot{y} + a^2 \sin(y) &= u, \\
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -a^2 \sin(x_1) + u \\
y &= x_1
\end{align*}
\]  

(6)

This system is a dynamics of the form $\mathbb{R}(u,y)/\mathbb{R}(u)$ where $G = \mathbb{R}(u,y), \ y \in G \ y k = R$. Any solution of (6) satisfies the following differential algebraic equation:

\[
(y^{(3)} - \dot{u})^2 + (\dot{y}(y^{(2)} - u))^2 = (a^2 \dot{y})^2
\]

Definition 4. Let a subset $\{u,y\}$ of $G$ in a dynamics $G/k(u)$. An element in $G$ is said to be algebraically observable with respect to $\{u,y\}$ if it is algebraically over $k(u,y)$. Therefore, a state $x$ is said to be algebraically observable if, and only if, it is algebraically observable with respect to $\{u,y\}$. A dynamics $G/k(u)$, with output $y$ in $G$ is said to be algebraically observable if, and only if, the state has this property.

Example. System (6) with output $y \in \mathbb{R}(u,y)$ is algebraically observable, since $x_1$ and $x_2$ satisfy two differentially algebraic polynomials with coefficients in $\mathbb{R}(u,y)$, i.e.

\[
\begin{align*}
x_1 - y &= 0 \\
x_2 - \dot{y} &= 0
\end{align*}
\]

Statement of the problem

Let consider the class of nonlinear systems described by:

\[
\begin{align*}
\dot{x}(t) &= A(x, \bar{u}), \\
y(t) &= h(x, u),
\end{align*}
\]  

(7)

Where $x = (x_1, ..., x_n)^T \in \mathbb{R}^n$ is a state vector, $\bar{u} = (u, f) = (u_1, ..., u_{m-\mu}, f_1, ..., f_\mu) \in \mathbb{R}^{m-\mu} \times \mathbb{R}^\mu$ where $u$ is a known input vector and $f$ is an unknown fault vector, $y = (y_1, ..., y_p) \in \mathbb{R}^p$ is the output, $A$ and $h$ are assumed to be analytical vector functions.

Definition 5. (Algebraic observability). An element $f \in k(\bar{u})$ is said to be algebraically observable if $f$ satisfies a differential algebraic equation with coefficients over $k(u,y)$

Definition 6. (Diagnosability). A nonlinear system is said to be diagnosable if it is possible to estimate the fault $f$ from the system equations and the time histories of the data $u$ and $y$, i.e. it is diagnosable if $f$ is algebraically observable with respect to “$u$” and “$y$”.

In other words it is required that each fault component be able to be written as a solution of a polynomial equation $f_i$ and finitely many time derivatives of $u$ and $y$ with coefficients in $k$

\[
H_i(f, u, \dot{u}, ..., y, \dot{y}, ...) = 0.
\]

Theorem 1. If system (7) is observable then it is diagnosable, if and only if, $f$ is observable with respect to $u$, $y$, and $x$. [13]

Remark 1. This is an immediate consequence of the general transitivity property of the observability condition [13].
The diagnosability conditions of \( f \) with respect to \( u, y \), and \( x \) are generally expected to be simpler than those of \( f \) only in terms of \( u \) and \( y \). In particular, if the system is observable then it is possible to reduce the number of time derivatives of the data in the fault differential algebraic equation.

**Theorem 2.** System (7) is diagnosable if, and only if, \( dff\frac{trd}{\mu k(u,y)/k(u)} = \mu \) where \( \mu \) is the number of components of the fault \( f \). For proof of this theorem see [21]

### III. DC MOTOR

Now will be described the reduced order observer design which was applied to the DC motor model, see Fig. 1. The algebraic observer is of great importance since it can monitor the variables that the motor needs to work, particularly field and armor voltage and current, feeding information to the controller.

Let us define the DC motor model proposed by [22]

\[
\begin{align*}
\dot{x}_1 &= \frac{1}{Le} u_i - \frac{Re}{Le} x_1 + F_L \\
\dot{x}_2 &= \frac{1}{La} u_i - \frac{k}{La} x_2 - \frac{1}{La} x_1 x_3 \\
\dot{x}_3 &= \frac{k}{J} x_1 x_2 + F_L
\end{align*}
\]  

(8)

Where \( Le \) is the field inductance, \( Re \) is the field resistance, \( La \) is the armor inductance, \( Ra \) is the armor resistance. The state variables are \( x_1 = i_e, x_2 = i_a, y x_3 = \omega \). It is considered that all the states are available; this means that the output vector is given as:

\[ y_i = x_i \text{ for } i = 1, 2, 3. \]

In this model two faults are considered: \( F_1 \) is a parasitic current, to obtain \( F_L \) we applied the LuGre mathematical model, which has been previously validated through experimentation and represents a nonlinear friction (It must be noted that the motor load is included in the nonlinear friction), a more detailed description of this model is on [23, 24].

\[
F_L = \sigma_0 z + \sigma_1 z + \sigma_2 \dot{q} \\
\dot{z} = -\sigma_0 a(\dot{q}) z + \dot{q} \\
a(\dot{q}) = \frac{\dot{q}^2}{\sigma_0 + \sigma_1 e^{\frac{\sigma_2}{\dot{q}^2}}}
\]

(9)

On table 1, can be seen the parameters used for model (9).

On Figure 2, it is shown the Bond graphs DC motor model, its construction was based on the Bond graphs formalism introduced by [25].

According to **Theorem 2** it is enough to find an algebraic relation between fault and output; in the model proposed this relation is simple to see, in Fig. 3 and 4 the fault appears explicitly on the output node. Once confirmed that the system is diagnosable, the following step is to obtain a reduced order observer to estimate the faults. The convergence proof for this observer can be found in [21] so we will not show it here.

The general form of the reduced order observer is:

\[
f_e = \beta(f - f_e)
\]

(10)

Where \( f_e \) denotes the fault estimate \( f \) and \( \beta \in \mathbb{R}^+ \) determines the convergence ratio desired by the observer.

It must be noted that \( f \) is replaced on the observer because of its diagnosis condition, this means, an algebraic

![Fig. 1 DC motor diagram](image1)

![Fig. 2 Motor diagram using Bond graphs](image2)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_0 )</td>
<td>10^5</td>
<td>[N/m]</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>( \sqrt{10^5} )</td>
<td>[Ns/m]</td>
</tr>
<tr>
<td>( \sigma_2 )</td>
<td>0.4</td>
<td>[Ns/m]</td>
</tr>
</tbody>
</table>
differential equation whose coefficients are in the differential field $k(u, y)$. 

**Note.** Sometimes the output time derivatives (which are unknown), appear in the fault algebraic equation, and then an auxiliary variable is needed. The system dynamics (10) together with

$$
\dot{y} = \psi(x, u, y) \quad \text{with} \quad y_0 = \gamma(0) \quad y, \gamma \in \mathbb{C}^1
$$

The diagnosis condition for fault $F_1$ is show next:

$$
F_1 = \dot{x}_1 - \frac{1}{L_e} u_1 + \frac{R_e}{L_e} x_1
$$

And the $F_1$ observer is the given next:

$$
\dot{y}_1 = \beta_1 \left( - \frac{1}{L_e} u_1 + \frac{R_e}{L_e} x_1 - y_1 - \beta_1 x_1 \right)
$$

Applying the change of variable:

$$
y_1 = \dot{F}_1 - \beta_1 x_1
$$

It is evident that $\dot{F}_1$ recovers from

$$
\dot{F}_1 = y_1 + \beta_1 x_1
$$

In the same way for $F_L$, we must have the diagnosis condition.

$$
F_L = \dot{x}_3 - \frac{k}{j} x_1 x_2
$$

Which observer is:

$$
\dot{y}_2 = \beta_2 \left[ \frac{-k}{j} x_1 x_2 - \beta_2 x_3 - y_2 \right]
$$

$$
y_2 = \dot{F}_L - \beta_2 x_3
$$

Where the gains are $\beta_1$ and $\beta_2$ and they are 2500 $y$ 1000 respectively.

**Obtaining a fault tolerant control**

Now will be shown the algebraic controller construction that is used to compensate for the faults of a parasitic current $F_1$, and the nonlinear friction $F_L$. The algorithm used in this controller design can be seen on [20].

Let us define the error dynamics as:

$$
\dot{e}_i = -\lambda_i e_i
$$

If $\lambda_i > 0$ then the error dynamics is asymptotically stable. Then we can propose the following reference tracking control for $x_i$:

$$
e_i = y_{d_i} - x_i \quad \text{where} \quad y_{d_1} = cte.
$$

Then

$$
u_i = -L_e \left[ -\lambda_i (y_{d_1} - x_i) - \frac{R_e}{L_e} x_1 + F_1 \right]
$$

(12)

Substituting $u_i$ on $\dot{x}_i$ can be observed that:

$$
\dot{x}_1 = \frac{1}{L_e} \left( -L_e \left[ -\lambda_i (y_{d_1} - x_i) - \frac{R_e}{L_e} x_1 + F_1 \right] - \frac{R_e}{L_e} x_1 + F_1 \right)
$$

Similar terms can be reduced:

$$
\dot{x}_1 = \lambda_i (y_{d_1} - x_i)
$$

From where, it is clear to see that if $\lambda_i > 0$ then the system is stable and (12) allow us to have a good tracking of the reference signal.

Now let us suppose that it is desired that $x_3$ follows a time variant reference $y_{d_2}(t)$, then:

$$
\dot{e}_2 = -\lambda_2 e_2
$$

$$
e_2 = y_{d_2}(t) - x_3
$$

We repeat the procedure for the next controller proposing the following tracking reference signal controller $u_2$. 

\[ 
\]
Substituting $u_2$ we obtain that:

$$
\dot{x}_3 = \lambda_2 (y_{d2}(t) - x_3)
$$

(14)

With $\lambda_2 > 0$, $u_2$ is a good tracking reference control.

For the case shown here $\lambda_1=10$ and $\lambda_2=50$.

IV. NUMERICAL SIMULATIONS

The parameters used on the numerical simulations are shown on Table 2.

The simulations were done with MATLAB Simulink.

On Fig. 5 and 6 are shown the field and armor current. The objective of the controller is to force the field current to follow a constant reference signal even though the parasitic current fault was present, because the control of a DC motor is through the armor current. On Fig. 7 can be seen the LuGre’s friction against its estimated, and how the observer’s performance it’s quite good.

On Fig. 8 it is shown the reference signal’s simulation for velocity, for which a sine function was chosen, with 80 rad/s in amplitude, also in the figure are shown the motor velocities using the controller and with consideration to the fault effects (dotted line) and without consideration to the fault effects (segmented line). It is clear that the controller’s performance improves when utilizing the faults estimates.

Finally on Fig. 9 are shown the field and armor voltages that represent the system control vector.

Fig. 5. Field current

Fig. 6. Armor current

Fig. 7. LuGre’s Friction

Fig. 8. Velocities comparisons

Fig. 9. Field and Armor voltages comparison
V. CONCLUDING REMARKS

On this article, we have introduced a first approach to the relation between fault diagnosis using differential algebra and Bond graphs. Also, a tracking reference controller was designed that is able to use the fault estimates. As future work, it is planned to develop a systematical method to obtain the differential transcendence degree of $k(u,y)/k(u)$ for more complex systems.

REFERENCES