# The Role of Irrational Numbers in Physics

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*Abstract:* In this paper, additional evidence is supplied, following previous contributions in WSEAS Conferences, concerning the appearance of irrational numbers, such as the phi number, the square roots of 2 and 3, in the Universe and in Atomic Physics in particular. The appearance of those numbers might not be simply a matter of a simple numerological coincidence. Examples are given concerning the masses of leptons and the fine structure constant, α. Irrational numbers are applied also in the form of Galaxies and in the orbits of planets and satellites.

*Keywords*: muons, tau particles, Koide's relation, fine structure constant, square root of three, golden angle

## 1 Introduction

In Aghios Nikolaos , July 2007, we have presented a paper on the role of the irrational numbers, such as the phi number and the square roots of 2 and 3, in the structure of biological systems[1]. In Cambridge, February 2008, we provided evidence that a certain triangle, in which all the above mentioned irrational numbers appear, seems to be in the heart of biological branching structures, as it is the bronchial tree [2]. The purpose of this paper is to highlight the traces of a cosmic plan based on the irrational numbers, also in the field of Microcosmos, i.e. in the relationships between the masses of leptons and the associated fine structure constant, α, known also, as the Sommerfeld's constant, which is rich in physical meaning. Feynman [3], has wondered whether the numerical value of this constant has to do with π or the base of the natural logarithms, e. We suggest that the fine structure constant contain both π and φ (the number of golden structure).

### 2. Relationships of the leptons masses

It has been remarked that the muon and the electron, or equivalently the chiral and the electroweak breaking scales, are separated by the fine structure constant. The electroweak interaction, considered to ignite the nuclear reactions in the stars, also in our Sun, thus making life easy for us the habitants of planet Earth is depicted in the famous Feynman diagram below (Fig. 1). An example of a Penguin diagram is given in Fig. 2.

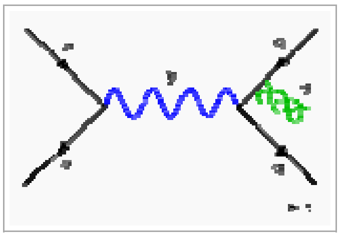

Figure 1. The annihilation of an electron and a positron gives the photon which results in a couple of a quark –antiquark with the release of a gluon[4].

It is known that the masses of leptons follow the Koide mass formula

$$(m_e+m_\mu+m_\tau)=\tfrac{2}{3}(\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau})^2$$

Consider that the mass of the electron approaches zero. Then the mass ratio between tau and muon becomes

$$[(1+\sqrt{3})/(1-\sqrt{3})]^2$$

i.e. an expression of the irrational number square root of three.

It is also known that the Koide formula has applied to the quarks. So, if we consider the masses of the elementary particles in a logarithmic plot , the "magic" numbers of α and √3 appear[5].

It is also known that, the strong and electromagnetic interactions conserve quark flavour, whereas the weak interaction may change it. In many weak decays, the changes are within a generation, e.g. in beta decay the W couples a u to a d quark; in the decay

$$D^+ \rightarrow K^0 \pi^+$$

it couples a c to an s. However, this is not always the case, e.g. in the decay

$$K^- \rightarrow \pi^0 e^- \bar{\nu}_e$$

the W couples an s to a u quark, and it was observed that such strangeness-changing decays were *slightly* weaker than strangeness-conserving weak decays.

Cabibbo explained this by proposing that the eigenstates of the weak interaction are different from those of the strong interaction. The strong interaction eigenstates are the u, d, s, c, b and t quarks, with well-defined isospin, strangeness etc. The eigenstates of the weak interaction, which does not conserve *I*, *S* etc., are said to be those of "weak isospin" *T*. For simplicity, let us first consider the first 2 generations alone. The weak eigenstates are the leptons and orthogonal linear combinations of the familiar quarks[6]. Let us consider only the u and s quarks. The relationship of the strong and weak eigenstates d, s, dc and sc are given by

$d_c = \cos\theta_c * d + \sin\theta_c * s$

$s_c = -\sin\theta_c * d + \cos\theta_c * s$

where $\theta_c$ is the Cabibbo angle k coupling constant with reaction type. As remarked by Rivero and Gsponer [5] the Cabibbo angle can be derived both from leptons and quarks. There the √2 appears in the mixing matrices.

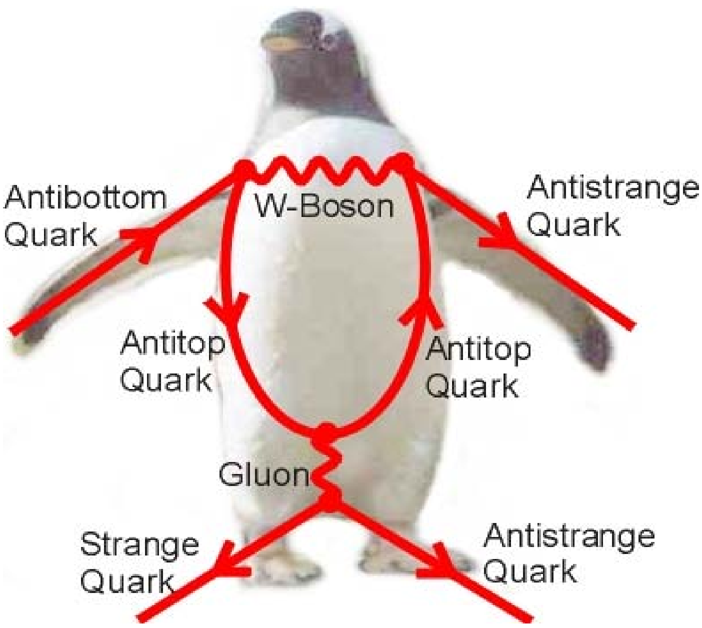


Figure 2. In the weak force three bosons (above the W boson) $W^+$ , $W^-$ are the mediators and Z (neutrally charged boson), instead of the photon. The Penguin diagram shows the quark

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## 3. The fine structure constant

The fine-structure constant or Sommerfeld fine-structure constant, usually denoted $\alpha$, is the fundamental physical constant characterizing the strength of the electromagnetic interaction. It is a dimensionless quantity, and thus its numerical value is idependent of the system of units used.

The best value currently, predicted by Quantum Electrodynamics, QED, is

$1/\alpha$= 137.035 999 068(96).

The physical meaning of $\alpha$ could be thought in various ways. One of the interpretations is the square of the ratio between the electron charge and the Planck charge. Also the ratio between the electron velocity in the Bohr atom and the speed of light. Take arbitrarily a length s and consider two energies, a)the energy needed to bring two electrons from infinity to a distance of S b)the energy of a single photon of wavelength equal to S but scaled by $2\pi$. Then the ratio of these two energies is the fine structure constant. Let us keep in our mind this physical interpretation of $\alpha$ for the following discussion about its numerical value[3,4].

According to Feynman:

*It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: is it related to π or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!* —Richard P Feynman, QED: The strange theory of light and matter, Prinveton University Press 1985, p. 129.

## 4. The golden angle

The golden angle is defined as in the following Fig.3 according to Elemeneary Theoretical Geometry textbooks

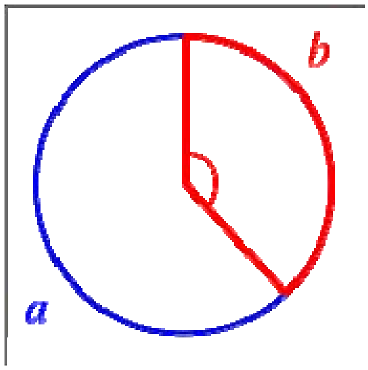


Figure 3. The angle b is $2\pi/\varphi$ or $360^o/\varphi$ or 137.

The importance of this angle in various forms encountered in Nature, such as in Phyllotaxis, in the helicoidal forms of certain flowers has been highlited by many authors, Not only in biological structures but also in Macrocosmos, in the geometry of the spiral galaxies, as Mario Livio[7] says in his wonderful book The Golden Section[7], Nature applies a Universal Plan.

## 5. Irrational Numbers and the Orbits of Planets and Satellites

According to Hippokrates Dakoglou the golden geometry provides a better approximation for the distances of the planets from our Sun , than the empirical Titius-Bode formula [1,8].

In this presentation we provide some data for the orbits of jovial satellites (Fig. 4) and also the satellites of Uranus (Fig.5 and Tables 1 and 2 respectively). From these data we can easily view the ratios of the distances of any pair of neighbouring satellites. The Europa to Io ratio as well as the Ganymede to Europa is close to the number of the golden section, φ. The same happens with the ratio of the uranian satellites Titania to Umbriel. The Kallisto to the Ganymede ratio approximates the √3 and the ratios of the Ariel/Miranda and Umbriel/Ariel approximate √2. We are tempting to conclude that for smaller distances, 100 000 to 300 000 km, the √2 prevails , for intermediate 400 000 to 1000 000, the φ number and for higher than 1 000 000 km the square root of 3.

***Table 1***

| *Satellites of Jupiter* | *Distance from the planet(km)* |
|---|---|
| Io | 421 600 |
| Europa | 670 900 |
| Ganymede | 1 070 000 |
| Kallisto | 1 883 000 |

***Table 2***

| *Satellites of Uranus* | |
|---|---|
| Miranda | 129 780 |
| Ariel | 191 240 |
| Umbriel | 265 970 |
| Titania | 435 840 |

## 6. Concluding Remarks

The Irrational numbers play an important role , not only in the biological structures as we have seen sofar but also generally in Physics, both in micro- and Macro-cosmos. We suggest that the fine-structure constant can be viewed numerologically as a number contasining φ and π. The golden angle plays a role both in the Macro- and Micro-cosmos. The same applies to the irrational numbers √2 and √3.

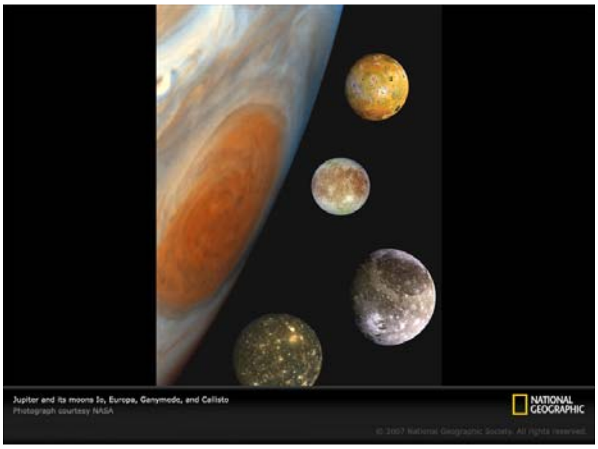

Figure 4. The Jovial Satellites

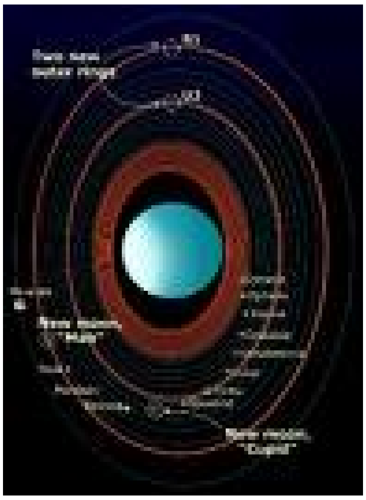

Figure 5. Uranus and its Satellites

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