An Optimal Controller Design for SUV Active Roll Control System

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Abstract: - An Anti-Rollover control algorithm for reducing the risk of rollover which is based on the optimal tracking control theory has been proposed in this paper. Vehicle parameters of a 1997 Jeep Cherokee which have been published by Vehicle Research and Test Center (VRTC) have been used to construct the 3DOF model developed from the Lagrangian dynamics. Simulation show that the controller is capable of reducing the likelihood of rollover of vehicle during various standard test maneuvers Also we compare the result of simulation with other control strategies like as PID controller and optimal regulator controller.

Key-Words: Optimal Control – Vehicle Dynamic Control – rollover resistance

1 Introduction
Rollover is the second most dangerous type of crash occurring on highways. During the eight years, 1991 through 1998, Fatality Analysis Reporting System (FARS) data showed that averages of 9,237 people were fatally injured each year in light vehicle rollover crashes. Comparing to other types of vehicles, Sport Utility Vehicles (SUV) had the highest rollover rates. [1], [2] Rollover prevention can be achieved by employing rollover warning or anti-rollover systems. In this paper, an active stabilizer system which consists of two actuators has been proposed to generate torques between the front/rear axle and the vehicle body when the vehicle is turning. [3], [4]

2 Vehicle modeling
Vehicle models typically consist of two components, a Chassis model which describes the dynamics of the vehicle, and a tire model which describes the forces generated at the contact point between tire and the road.

2.1 Tire model
All road vehicles interact with the road surface via tires. More specifically, the tires are responsible for generating those forces which are required to alter the vehicle’s speed and course according to the driver’s inputs. The physical mechanisms which tires functions are complicated, and modeling is therefore difficult.

A variety of different models exists, both theoretical and empirical. In this section a widely-used approach to tire modeling will be presented. A simple linear approximation of Magic Formula can be used in this paper. [12], [5]

\[ F_y = C_{\alpha} \alpha + C_{\gamma} \gamma \]  

(1)

\( C_{\alpha} \) is the lateral slip stiffness, \( \alpha \) is slip angle, \( C_{\gamma} \) is camber stiffness, \( \gamma \) is camber angle.

2.2 Nonlinear modeling of vehicle
Modeling of vehicle dynamics is a varied and a widely researched topic. In the past, researchers used the laws of physics and vehicle dynamics to model small portions of the vehicle. Depending on the application, these models can vary from complex and highly nonlinear to simple and linear. Recently, there has been increased effort on vehicle stability control, especially roll and yaw. Complex seven, eight, nine and even 104 degree-of-freedom (DOF) nonlinear models have been used to model different vehicle ride and handling aspects, but these models are too complex for vehicle electronics. Simpler, yet accurate models are needed for vehicle control systems. In this paper the design is based on a linear 3DOF vehicle model developed from the Euler-Lagrange method, including a constant but nonzero pitch angle \( \theta_r \). Obvious choices of coordinates for the system are longitudinal and lateral velocities \( u \) and \( v \), yaw rate \( \psi \) and roll angle \( \phi \). This model provides a good

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description of vehicle motion that is valid in more than 90% of driving conditions.

Vehicle parameters of a 1997 Jeep Cherokee which have been published by Vehicle Research and Test Center (VRTC) have been used to construct this model. (Fig 1)

The Euler-Lagrange equations state that: (Eq 2)

\[ \frac{d}{dt} \frac{\partial T}{\partial q_i} - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i \]  

(2)

Where T is the kinetic energy of the system, U is the potential energy, \( q_i \) are generalized coordinates and \( Q_i \) are generalized forces corresponding to the generalized coordinates. [5]

The translational kinetic energy is therefore given by: (Eq 3)

\[ T_{\text{trans}} = \frac{1}{2} m ((u - h \dot{\psi} \phi)^2 + (v + h \dot{\phi})^2) \]  

(3)

The rotational kinetic energy of the system (with \( \theta_i \) accounted for but assumed small, and \( \phi \) also assumed small) is given by: (Eq 4)

\[ T_{\text{rot}} = \frac{1}{2} (I_{xx} \dot{\phi}^2 + I_{zz}(\phi \psi)^2 + I_{xz}(\psi^2 - \phi^2 \psi^2 + 2 \theta_i \dot{\phi} \psi) - 2 I_{zz} \dot{\phi} \psi) \]  

(4)

The potential energy of the system is stored in the suspension springs and the height of the centre of gravity. It is given by: (Eq 5)

\[ U = \frac{1}{2} C_{r} \phi^2 - mg(h - h \cos \phi) \]  

(5)

The equations of motion may now be obtained by evaluating the Lagrangian equations. [12]

The 3DOF model can be written on the form:

\[ m_{\text{tot}} [\ddot{v} + \psi u] + mh (\ddot{\phi} - \psi \phi) = F_{\gamma T} \]  

(6)

\[ I_{zz} \ddot{\psi} - mh (\dot{u} - \psi \dot{\phi}) + (I_{zz} \theta_i - I_{xz}) \ddot{\phi} = M_{T} \]  

(7)

\[ (I_{zz} + mh^2) \ddot{\phi} + mh (v - \psi u) + (I_{zz} \theta_i - I_{xz}) \ddot{\psi} 
- (I_{yy} + mh^2 - I_{xz}) \psi^2 \phi + (K_{\phi} - mg h) \phi - C_{\phi} \dot{\phi} = 0 \]  

(8)

The parameters of vehicle model are summarized in table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>m_{\text{tot}}</td>
<td>Total Vehicle mass, 1987.935 Kg</td>
</tr>
<tr>
<td>m_R</td>
<td>Rolling sprung mass, 1663 Kg</td>
</tr>
<tr>
<td>h</td>
<td>Height of CG above roll axis, 0.306 m</td>
</tr>
<tr>
<td>a</td>
<td>Distance from the vehicle CG to the front axle, 1.1473 m</td>
</tr>
<tr>
<td>b</td>
<td>Distance from the vehicle CG to the rear axle, 1.4307 m</td>
</tr>
<tr>
<td>C_\gamma</td>
<td>Camber thrust coefficient at the front axle, 2038.8 N/rad</td>
</tr>
<tr>
<td>I_{xx}</td>
<td>Moment of inertia about x-axis, 602.82 Kg-m²</td>
</tr>
<tr>
<td>I_{xz}</td>
<td>Moment of inertia about z-axis, 2703.7 Kg-m²</td>
</tr>
<tr>
<td>I_{zz}</td>
<td>Product of inertia for x and z axes, 89.9 Kg-m²</td>
</tr>
<tr>
<td>K_\phi</td>
<td>Total roll stiffness, 56957 N-m/rad</td>
</tr>
<tr>
<td>C_\phi</td>
<td>Total roll damping, 3495.7 N-m-sec/rad</td>
</tr>
<tr>
<td>\theta_i</td>
<td>Angle between roll axis and x-axis, 0.0873 rad</td>
</tr>
<tr>
<td>C_{\gamma x}</td>
<td>Front tire cornering stiffness, 59496 N/rad</td>
</tr>
<tr>
<td>C_{\gamma f}</td>
<td>Rear tire cornering stiffness, 109400 N/rad</td>
</tr>
<tr>
<td>\partial \delta_x / \partial \phi</td>
<td>Partial derivative of the roll induced steer at the front axle, 0.07</td>
</tr>
<tr>
<td>\partial \delta_f / \partial \phi</td>
<td>Partial derivative of the camber thrust at the front axle, 0.8</td>
</tr>
</tbody>
</table>

Fig 1: The two-track model, showing suspension modeled as a torsional spring and damper

Fig 2: compare lateral acceleration on field-test and simulation

Fig 3: compare body roll motion on field-test and simulation

Table 1 Nomenclature and specifications data of vehicle
2.3 Linearization of vehicle model

To use this model in our controller design, it’s also linearized. Linear models use a number of assumptions and approximations, these include: (Eq 9)

- Constant longitudinal velocity
- Small steering angles
- Linear tire forces
- Simple approximations of tire slip values

These approximations imply that linear models be useful for designing control systems intended for use under standard maneuver and normal driving. (Fig 4) [7], [11]

\[ m_{in} [\ddot{\psi}u] + mh(\ddot{\phi}) = F_{\tau} \]
\[ I_{zz} \ddot{\psi} + (I_{zz} \theta - I_{wy}) \ddot{\phi} = M_{\tau} \]
\[ mhu \psi + (I_{zz} \theta - I_{wy}) \dot{\psi} + (I_{xx} + mh^2) \dot{\phi} + (K_{\phi} - mgh) \phi - C_{\phi} \phi = 0 \] (9)

The tire forces acting on the vehicle are obtained:

\[ F_{\tau} = -C_{\alpha f} + C_{\alpha w} \beta - \frac{(a C_{\alpha f} - b C_{\alpha w})}{U} \psi \]
\[ + (C_{\alpha f} \frac{\partial \delta}{\partial \phi} + C_{\alpha w} \frac{\partial \gamma}{\partial \phi}) + C_{\alpha f} \delta \]
\[ M_{\tau} = -(a C_{\alpha f} - b C_{\alpha w}) \beta + \frac{(a^2 C_{\alpha f} + b^2 C_{\alpha w})}{U} \psi \]
\[ - (b C_{\alpha f} \frac{\partial \delta}{\partial \phi} - a C_{\alpha w} \frac{\partial \gamma}{\partial \phi}) \phi + a C_{\alpha f} \delta \] (10)

The governing equations for Roll, yaw and lateral motions of the vehicle model, in state space form are derived as: (Eq 11)

\[ \dot{X} = AX + B\delta_{\text{steer}} \iff x = [\beta \ r \ \dot{\phi} \ \phi]^T \]
\[ Y = CX + D\delta_{\text{steer}} \] (11)

For the vehicle model the side slip angle, yaw rate, roll rate and roll angle are considered as the four state variables and steering angle is control input.

In the further parts Roll moment is control input, which must be determined from control low and the vehicle steering angle is considered as the external disturbance.

3 Test maneuvers

In order to obtain a common measure, a number of standardized maneuvers have been developed. The National Highway Traffic Safety Administration (NHTSA) has developed various standard maneuvers, including the so-called Fishhook and J-Turn maneuvers, which are described here. [2], [5]

3.1 Fishhook

The Fishhook maneuver is an important test maneuver in the context of rollover. It attempts to maximize the roll angle under transient conditions and is performed as follows, with a start speed of 80 km/h:

- The steering wheel angle is increased at a rate of 720 deg/sec up to 6.5 \( \delta_{\text{stat}} \), where \( \delta_{\text{stat}} \) is the steering angle which is necessary to achieve 0.3g. This value is held for 250ms.
- The steering wheel is turned in the opposite direction at a rate of 720 deg/sec until it reaches -6.5 \( \delta_{\text{stat}} \).
- No brake or accelerator commands are given during the maneuver. (Fig 5)

![Fishhook maneuver](image)

3.2 J-Turn

The J-Turn is a simple step in the steering wheel angle driving the vehicle towards the physical limits. This maneuver can cause a roll over of vehicles with critical load.

- The steering wheel angle is increased at a rate of 1000 deg/sec until it reaches 8 \( \delta_{\text{stat}} \). (Fig 6)

![J-Turn maneuver](image)
4 Optimal Controller Design

We used an active stabilizer system which consists of two actuators to generate torques between the front/rear axle and the vehicle body, and control body roll motion of vehicle with an anti-rollover control algorithm, based on linear tracking optimal control.

The control law consists of four state variable feedback terms being those of the side slip angle, yaw angle, yaw rate and roll angle. (Eq 12)

\[ U_t^* = Fx_t + V_t \]  

(Eq 12)

In above control law, according to the current state of technology, direct measurement of yaw angle, yaw rate and roll angle are quit feasible. However due to the impracticality of direct measurement of side slip angle, estimation of this state variable is most desirable.

We defined performance index in the following form:

\[ J_r = \frac{1}{2}(U_t^T R_t U_t + (X_t - \bar{x}_t)^T Q_t (X_t - \bar{x}_t)) \]  

(13)

\( \bar{x} \) is the desired roll angle and \( R_t \) is weight factor.

Minimization of the performance index must be sought in order to obtain the optimum body roll motion to ensure, vehicle roll angle doesn’t exceed from defined boundaries and the vehicle has enough body roll motion to create feeling about vehicle maneuver in driver.

It is important to note that the control effort must satisfy some physical constrains due to actuation system and energy consumption in vehicle.

By proper selection of weighting factor in (Eq 13) and the appropriate design of the control law, in order to minimize the performance index we achieve a good body roll behavior and also satisfy the physical limit of dynamic system.

To determine the values of feedback control gain, which are based on the defined performance index and the vehicle dynamic model, a LQR problem has been formulated, which its analytical solution is obtained.

The Hamiltonian function, in the expanded form, is therefore given by: (Eq 14)

\[ H = \frac{1}{2}(X_t - X)^T Q_t (X_t - X_t) + \frac{1}{2}U_t^T R_t U_t + P_t^T A (X_t + BU_t) \]  

(Eq 14)

After written the state an co-state equation, generally we can form a system of nonlinear ordinary differential equations which can be converted into a nonlinear algebraic system of equations by assuming that the solution of the equations converge rapidly to a constant values.(Eq 15, 16)

\[ 0 = -K_t A - A^T K_t - Q_t + K_t B R_t^{-1} B^T K_t \]  

(15)

\[ 0 = -[A^T - K_t B R_t^{-1} B^T] \bar{x}_t + Q_t X_{\bar{x}} \]  

(16)

The system of equations given above can be solved analytically in order to determine the corresponding value of feedback gains. (Eq 17)

\[ U_t^* = -R_t^{-1} B^T K X_t - R_t^{-1} B^T S_t \rightarrow U_t^* = Fx_t + V_t \]  

(17)

The numerical simulations of the vehicle roll motion based on standard tests like as J-Turn and Fishhook maneuver, with and without the optimal roll motion controller were carried out. (Fig 7, 8)

Simulation results indicate that the controller is capable of reducing the likelihood of rollover of vehicle during standard test maneuvers. [8], [9]

5 Compare results with other control systems

In the last, we compared the result of simulations with other control strategies like as regulator optimal and PID, to ensure performance of our controller. (Fig 9, 10)

Simulations indicate that performance of optimal tracking controller due to simple structure and good behavior of roll motion and less moment for controlling of roll in maneuver is better than other control systems.

Fig 11 and Fig 12 indicate the moment that use for controlling of roll motion, and so we can use weaker actuator that require less energy than other controller like as PID controller and optimal regulator controller.
In Fishhook maneuver, Actuator that control with PID controller, should produce about 8600 (N.m), Actuator that control with optimal regulator controller, should produce about 7100 (N.m), but in optimal regulator controller we use only about 6200 (N.m) to controlling roll motion in Fishhook maneuver so more than 25% less energy consuming compared to PID controller.

In J-Turn maneuver, Actuator that control with PID controller, should produce about 10500 (N.m), but in optimal regulator controller we use only about 7800 (N.m) to controlling roll motion in J-Turn maneuver so more than 25% less energy consuming compared to PID controller. (Fig 12)

6 Conclusion
A 3DOF yaw-roll model was constructed by using the Cherokee parameters and tuned to match the test data. We used an anti-rollover control algorithm, based on linear tracking optimal control and also we used an active stabilizer system which is consists of two actuators to generate torques between the front/rear axle and the vehicle body to control body roll motion of vehicle.

We design an optimal tracking controller and defined a desired roll motion that controller try to obey it and get that roll angle.

Simulation results indicate that our optimal tracking controller is capable of reducing the likelihood of rollover of vehicle during standard test maneuvers with less usage of energy compare with other controller.

References