## The Level Crossing Rate and Outage Probability of the SSC Combiner Output Signal in the Presence of Nakagami-*m* fading

MIHAJLO STEFANOVIĆ, DRAGANA KRSTIĆ, PETAR NIKOLIĆ, SRDJAN JOVKOVIC, DUSAN STEFANOVIĆ

Faculty of Electronic Engineering University of Niš Aleksandra Medvedeva 14, Niš SERBIA

*Abstract:* - Level crossing rate, outage probability and fade duration of the SSC combiner output signal are determined in this paper. The presence of the Nakagami-*m* fading at the input is observed. The results are shown graphically for different variance values and decision threshold values.

*Key-Words:* - Diversity reception, Level Crossing Rate, Fade Duration, Outage Probability, Nakagami-*m* fading, SSC Combining

### **1** Introduction

Many of the wireless communication systems use some form of diversity combining to reduce multupath fading appeared in the channel. Among the simpler diversity combining schemes, the two most popular are selection combining (SC) and switch and stay combining (SSC). SSC is an attempt at simplifying the complexity of the system but with loss in performance. In this case the receiver selects a particular antenna until its quality drops below a predetermined threshold. When this happens, the receiver switches to another antenna and stays with it for the next time slot, regardless of whether or not the channel quality of that antenna is above or below the predetermined threshold.

In the paper [1] Alouini and Simon develop, analyze and optimize a simple form of dual-branch switch and stay combining (SSC).

The consideration of SSC systems in the literature has been restricted to low-complexity mobile units where the number of diversity antennas is typically limited to two ([2], [3] and [4]). Furthermore, in all these publications, only predetection SSC has thus far been considered wherein the switching of the receiver between the two receiving antennas is based on a comparison of the instantaneous SNR of the connected antenna with a predetermined threshold. This results in a reduction in complexity relative to SC in that the simultaneous and continuous monitoring of both branches SNRs is no longer necessary.

In [5] the moment generating function (MGF) of the signal power at the output of dual-branch switch-and-stay selection diversity (SSC) combiners is derived. Blanco consider diversity receiver performance in Nakagami fading in [6].

In this paper level crossing rate, average outage probability and fade duration of the SSC combiner output signal in the presence of Nakagami-*m* fading will be determine. The results will be shown graphically for different variance values and decision threshold values.

### 2 System Model

The model of the SSC combiner with two inputs, considered in this paper, is shown in Fig. 1. The signals at the combiner input are  $r_1$  and  $r_2$ , and r is the combiner output signal. The predetection combining is observed.



# Fig. 1: Model of the SSC combiner with two inputs

The probability of the event that the combiner first examines the signal at the first input is  $P_1$ , and for the second input is  $P_2$ . If the combiner examines first the signal at the first input and if the value of the signal at the first input is above the treshold,  $r_T$ , SSC combiner forwards this signal to the circuit for the decision. If the value of the signal at the first input is below the treshold  $r_T$ , SSC combiner forwards the signal from the other input to the circuit for the decision.

If the SSC combiner first examines the signal from the second combiner input it works in the similar way.

The determination of the probability density of the combiner output signal is important for the receiver performances determination. The probability for the first input to be examined first is  $P_1$  and for the second input to be examined first is  $P_2$ .

The probability densities of the combiner input signals,  $r_1$  and  $r_2$ , in the presence of Nakagami-*m* fading, are:

$$p_{r_{1}}(r_{1}) = \frac{2m_{1}^{m_{1}}r_{1}^{2m_{1}-1}}{\Omega_{1}^{m_{1}}\Gamma(m_{1})}e^{-\frac{m_{1}r_{1}^{2}}{\Omega_{1}}}$$
$$r_{1} \ge 0$$
(1)

$$p_{r_{2}}(r_{2}) = \frac{2m_{2}^{m_{2}}r_{2}^{2m_{2}-1}}{\Omega_{2}^{m_{2}}\Gamma(m_{2})}e^{-\frac{m_{2}r_{2}^{2}}{\Omega_{2}}}$$
$$r_{2} \ge 0$$
(2)

The cumulative probability densities (CDFs) are given by:

$$F_{r_{1}}(r_{T}) = \int_{0}^{r_{T}} p_{r_{1}}(x) dx$$
(3)

$$F_{r_2}(r_T) = \int_{0}^{r_T} p_{r_2}(x) dx$$
 (4)

 $r_T$  is the treshold of the decision. In the presence of Nakagami-*m* fading CDFs are:

$$F_{r_{1}}(r_{T}) = \int_{0}^{r_{T}} \frac{2m_{1}^{m_{1}}x^{2m_{1}-1}}{\Omega_{1}^{m_{1}}\Gamma(m_{1})} e^{-\frac{m_{1}x^{2}}{\Omega_{1}}} dx = \gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2},m_{1})$$
(5)
$$F_{r_{2}}(r_{T}) = \int_{0}^{r_{T}} \frac{2m_{2}^{m_{2}}x^{2m_{2}-1}}{\Omega_{2}^{m_{1}}\Gamma(m_{2})} e^{-\frac{m_{2}x^{2}}{\Omega_{2}}} dx = \gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2},m_{2})$$
(6)

where  $\gamma(x,a)$  is incomplete gamma function defined by [7]:

$$\gamma(x,a) = \frac{1}{\Gamma(a)} \int_{0}^{x} t^{a-1} e^{-t} dt.$$

The joint probability densities of the combiner input signals,  $r_1$  and  $r_2$ , and their derivatives  $\dot{r}_1$  and  $\dot{r}_2$ , in the presence of Nakagami-*m* fading, are:

$$p_{r_{1}\dot{r}_{1}}(r_{1},\dot{r}_{1}) = \frac{2m_{1}^{m_{1}}r_{1}^{2m_{1}-1}}{\Omega_{1}^{m_{1}}\Gamma(m_{1})}e^{-\frac{m_{1}r_{1}^{2}}{\Omega_{1}}} \cdot \frac{1}{\sqrt{2\pi}} \frac{1}{\beta_{1}}e^{-\frac{\dot{r}_{1}^{2}}{2\beta_{1}^{2}}}}{r_{1} \ge 0}$$
(7)

$$p_{r_{2}\dot{r}_{2}}(r_{2},\dot{r}_{2}) = \frac{2m_{2}^{m_{2}}r_{2}^{2m_{2}-1}}{\Omega_{2}^{m_{2}}\Gamma(m_{2})}e^{-\frac{m_{2}r_{2}^{2}}{\Omega_{2}}} \cdot \frac{1}{\sqrt{2\pi}\beta_{2}}e^{-\frac{\dot{r}_{2}^{2}}{2\beta_{2}^{2}}}$$

$$r_2 \ge 0 \tag{8}$$

The probabilities  $P_1$  and  $P_2$  are:

$$P_{1} = \frac{F_{r_{2}}(r_{T})}{F_{r_{1}}(r_{T}) + F_{r_{2}}(r_{T})} =$$

$$= \frac{\gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2}, m_{2})}{\gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2}, m_{1}) + \gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2}, m_{2})} \qquad (9)$$

$$P_{2} = \frac{F_{r_{1}}(r_{T})}{F_{r_{1}}(r_{T}) + F_{r_{2}}(r_{T})} =$$

$$= \frac{\gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2}, m_{1}) + \gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2}, m_{2})}{\gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2}, m_{1}) + \gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2}, m_{2})} \qquad (10)$$

The expression for the joint probability density function of the SSC combiner output signal and its derivative will be determined first for the case:  $r < r_T$ :

$$p_{r\dot{r}}(r\dot{r}) = P_1 \cdot F_{r_1}(r_T) \cdot p_{r_2\dot{r}_2}(r\dot{r}) + P_2 \cdot F_{r_2}(r_T) \cdot p_{r_1\dot{r}_1}(r\dot{r})$$
(11)

and then for  $r \ge r_T$ :

$$p_{r\dot{r}}(r\dot{r}) = P_1 \cdot p_{r_1\dot{r}_1}(r\dot{r}) + P_1 \cdot F_{r_1}(r_T) \cdot p_{r_2\dot{r}_2}(r\dot{r}) + P_2 \cdot P_{r_2\dot{r}_2}(r\dot{r}) + P_2 \cdot F_{r_2}(r_T) \cdot p_{r_1\dot{r}_1}(r\dot{r}) \quad (12)$$

We have now for  $r < r_T$ :

$$p_{r\dot{r}}(r\dot{r}) = \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) \cdot \frac{2m_2^{m_2}r_t^{2m_2-1}}{\Omega_2^{m_2}\Gamma(m_2)} e^{-\frac{m_2r^2}{\Omega_2}} \cdot \frac{1}{\sqrt{2\pi}\beta_2} e^{-\frac{\dot{r}^2}{2\beta_2^2}} +$$

$$+\frac{\gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2},m_{1})}{\gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2},m_{1})+\gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2},m_{2})}\gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2},m_{2})\cdot$$
$$\cdot\frac{2m_{1}^{m_{1}}r^{2m_{1}-1}}{\Omega_{1}^{m_{1}}\Gamma(m_{1})}e^{-\frac{m_{1}r^{2}}{\Omega_{1}}}\cdot\frac{1}{\sqrt{2\pi}\beta_{1}}e^{-\frac{r^{2}}{2\beta_{1}^{2}}}$$
(13)

and for  $r \ge r_T$ :

$$p_{r\dot{r}}(r\dot{r}) = \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \frac{2m_1^{m_1}r^{2m_1-1}}{\Omega_1^{m_1}\Gamma(m_1)}e^{-\frac{m_1r^2}{\Omega_1}} \cdot \frac{1}{\sqrt{2\pi}\beta_1}e^{-\frac{r_1^2}{2\beta_1^2}} + \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) \cdot \frac{2m_2^{m_2}r^{2m_2-1}}{\Omega_2^{m_2}\Gamma(m_2)}e^{-\frac{m_2r^2}{\Omega_2}} \cdot \frac{1}{\sqrt{2\pi}\beta_2}e^{-\frac{r^2}{2\beta_2^2}} + \frac{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \frac{2m_2^{m_2}r^{2m_2-1}}{\Omega_2^{m_2}\Gamma(m_2)}e^{-\frac{m_2r^2}{\Omega_2}} \cdot \frac{1}{\sqrt{2\pi}\beta_2}e^{-\frac{r^2}{2\beta_2^2}} + \frac{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \frac{2m_2^{m_2}r^{2m_2-1}}{\Omega_2^{m_2}\Gamma(m_2)}e^{-\frac{m_2r^2}{\Omega_2}} \cdot \frac{1}{\sqrt{2\pi}\beta_2}e^{-\frac{r^2}{2\beta_2^2}} + \frac{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2) \cdot \frac{2m_1^{m_1}r_1^{m_2}}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \frac{2m_1^{m_1}r_1^{m_2}}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2) \cdot \frac{2m_1^{m_1}r_1^{m_2}}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_1^2, m_2)} \cdot \frac{2m_1^{m_2}r_1^2}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_1^2, m_2)} \cdot \frac{2m_1^{m_1}r_1^2}{\gamma(\frac{m_1}{\Omega_1}r_1^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_1^2, m_2)} \cdot \frac{2m_1^{m_2}r_1^2}{\gamma(\frac{m_2}{\Omega_1}r_1^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_1^2, m_2)} \cdot \frac{2m_1^{m_1}r_1^2}{\gamma(\frac{m_1}{\Omega_1}r_1^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_1^2, m_2)} \cdot \frac{2m_1^2}{\gamma(\frac{m_$$

For the channels with identical parameters it is, for  $r < r_T$ :

$$p_{r\dot{r}}(r\dot{r}) = \gamma(\frac{m}{\Omega}r_{t}^{2}, m)\frac{2m^{m}r^{2m-1}}{\Omega^{m}\Gamma(m)}e^{-\frac{mr^{2}}{\Omega}} \cdot \frac{1}{\sqrt{2\pi}\beta}e^{-\frac{\dot{r}^{2}}{2\beta^{2}}}$$
(15)  
and for  $r \ge r_{T}$ :

$$p_{r\dot{r}}(r\dot{r}) = \left(1 + \gamma(\frac{m}{\Omega}r_t^2, m)\right) \frac{2m^m r^{2m-1}}{\Omega^m \Gamma(m)} e^{-\frac{mr^2}{\Omega}} \cdot \frac{1}{\sqrt{2\pi}\beta} e^{-\frac{\dot{r}^2}{2\beta^2}}$$
(16)

The level crossing rate is:

$$N(r_{th}) = \int_{0}^{\infty} \dot{r} \, p_{r\dot{r}}(r_{th}, \dot{r}) \, d\dot{r}$$
(17)

for  $r_{th} < r_T$ :

$$N(r_{th}) = \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) \cdot \frac{2m_2^{m_2}r_{th}^{2m_2-1}}{\Omega_2^{m_2}\Gamma(m_2)}e^{-\frac{m_2r_{th}^2}{\Omega_2}}\frac{\beta_2}{\sqrt{2\pi}} + \frac{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2) \cdot \frac{2m_1^{m_1}r_{th}^{2m_1-1}}{\Omega_1^{m_1}\Gamma(m_1)}e^{-\frac{m_1r_{th}^2}{\Omega_1}}\frac{\beta_1}{\sqrt{2\pi}}$$
(18)

and for  $r_{th} \ge r_T$ :

$$N(r_{th}) = \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \frac{\gamma(\frac{m_1}{\Omega_1}r_t^{2m_1-1}, m_1) + \gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \frac{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) \frac{2m_2^{m_2}r_t^{2m_2-1}}{\Omega_2^{m_2}\Gamma(m_2)}}{\rho(\frac{m_1}{\Omega_1}r_t^2, m_1) \frac{2m_2^{m_2}r_t^{2m_2-1}}{\Omega_2^{m_2}\Gamma(m_2)}}{\rho(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \frac{2m_2^{m_2}r_{th}^{2m_2-1}}{\Omega_2^{m_2}\Gamma(m_2)} \cdot \frac{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \frac{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2) \frac{2m_1^{m_1}r_t^{2m_1-1}}{\Omega_1^{m_1}\Gamma(m_1)}}{\rho(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2) \frac{2m_1^{m_1}r_t^{2m_1-1}}{\Omega_1^{m_1}\Gamma(m_1)}}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2) \frac{2m_1^{m_1}r_t^{2m_1-1}}{\Omega_1^{m_1}\Gamma(m_1)}}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2) \frac{2m_1^{m_1}r_t^{2m_1-1}}{\Omega_1^{m_1}\Gamma(m_1)}}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2) \frac{2m_1^{m_1}r_t^{2m_1-1}}{\Omega_1^{m_1}\Gamma(m_1)}}{\gamma(\frac{m_1}{\Omega_2}r_t^2, m_2)} \cdot \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2) \frac{2m_1^{m_1}r_t^{2m_1-1}}{\Omega_2}}{\gamma(\frac{m_1}{\Omega_2}r_t^2, m_2)} \cdot \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2) \frac{2m_1^{m_1}r_t^{2m_1-1}}{\Omega_1^{m_1}\Gamma(m_1)}} + \frac{\gamma(\frac{m_1}{\Omega_2}r_t^2, m_2) \frac{m_1^{m_2}r_t^{2m_2}}{\gamma(\frac{m_1}{\Omega_2}r_t^2})}$$

For the channels with identical parameters it is valid for  $r_{th} < r_T$ :

$$N(r_{th}) = \gamma(\frac{m}{\Omega}r_{t}^{2}, m)\frac{2m^{m}r_{th}^{2m-1}}{\Omega^{m_{1}}\Gamma(m)}e^{-\frac{mr_{th}^{2}}{\Omega}}\frac{\beta}{\sqrt{2\pi}} (20)$$
  

$$r_{th} \ge r_{T} :$$
  

$$N(r_{th}) = \left(1 + \gamma(\frac{m}{\Omega}r_{t}^{2}, m)\right)\frac{2m^{m}r_{th}^{2m-1}}{\Omega^{m_{1}}\Gamma(m)}e^{-\frac{mr_{th}^{2}}{\Omega}}\frac{\beta}{\sqrt{2\pi}} (21)$$

The outage probability  $P_{out}(r_{th})$  is defined as:

$$P_{out}(r_{th}) = \int_{0}^{r_{th}} p_r(r) dr$$
(22)

For  $r < r_T$  probability density function is:

$$p_{r}(r) = P_{1} \cdot F_{r_{1}}(r_{T}) \cdot p_{r_{2}}(r) + P_{2} \cdot F_{r_{2}}(r_{T}) \cdot p_{r_{1}}(r)$$
(23)

for  $r \ge r_T$ 

$$p_{r}(r) = P_{1} \cdot p_{r_{1}}(r) + P_{1} \cdot F_{r_{1}}(r_{T}) \cdot p_{r_{2}}(r) + P_{2} \cdot p_{r_{2}}(r) + P_{2} \cdot F_{r_{2}}(r_{T}) \cdot p_{r_{1}}(r)$$
(24)

In the presence of Nakagami-*m* fading and for  $r < r_T$  probability density function is:

$$p_{r}(r) = \frac{\gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2}, m_{2})}{\gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2}, m_{1}) + \gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2}, m_{2})} \cdot \gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2}, m_{1}) \frac{2m_{2}^{m_{2}}r^{2m_{2}-1}}{\Omega_{2}^{m_{2}}\Gamma(m_{2})}e^{-\frac{m_{2}r^{2}}{\Omega_{2}}} + \frac{\gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2}, m_{1})}{\gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2}, m_{1}) + \gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2}, m_{2})} \cdot \gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2}, m_{2})\frac{2m_{1}^{m_{1}}r_{th}^{2m_{1}-1}}{\Omega_{1}^{m_{1}}\Gamma(m_{1})}e^{-\frac{m_{1}r^{2}}{\Omega_{1}}}$$
(25)

for  $r \ge r_T$ :

$$p_{r}(r) = \frac{\gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2}, m_{2})}{\gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2}, m_{1}) + \gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2}, m_{2})} \frac{2m_{1}^{m_{1}}r^{2m_{1}-1}}{\Omega_{1}^{m_{1}}\Gamma(m_{1})}e^{-\frac{m_{t}r^{2}}{\Omega_{1}}} + \frac{\gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2}, m_{2})}{\gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2}, m_{1}) + \gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2}, m_{2})} \cdot$$

$$\cdot \gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2},m_{1})\frac{2m_{2}^{m_{2}}r^{2m_{2}-1}}{\Omega_{2}^{m_{2}}\Gamma(m_{2})}e^{-\frac{m_{2}r^{2}}{\Omega_{2}}} + \\ + \frac{\gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2},m_{1})}{\gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2},m_{1}) + \gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2},m_{2})}\frac{2m_{2}^{m_{2}}r^{2m_{2}-1}}{\Omega_{2}^{m_{2}}\Gamma(m_{2})}e^{-\frac{m_{2}r^{2}}{\Omega_{2}}} + \\ + \frac{\gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2},m_{1}) + \gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2},m_{2})}{\gamma(\frac{m_{1}}{\Omega_{1}}r_{t}^{2},m_{1}) + \gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2},m_{2})} \cdot \\ \cdot \gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2},m_{2})\frac{2m_{1}^{m_{1}}r^{2m_{1}-1}}{\Omega_{1}^{m_{1}}\Gamma(m_{1})}e^{-\frac{m_{1}r^{2}}{\Omega_{1}}}$$
(26)

The outage probabilities  $P_{out}(r_{th})$  are defined as, for  $r_{th} < r_T$ :

$$P_{out}(r_{th}) = \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) \cdot \gamma(\frac{m_2}{\Omega_2}r_{th}^2, m_2) + \frac{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2) \gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2) \gamma(\frac{m_1}{\Omega_1}r_t^2, m_1)$$
(27)

For  $r_{th} \ge r_T$ :

$$P_{out}(r_{th}) = \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)} \cdot \left(\gamma(\frac{m_1}{\Omega_1}r_{th}^2, m_1) - \gamma(\frac{m_1}{\Omega_1}r_t^2, m_1)\right) + \frac{\gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}\gamma(\frac{m_1}{\Omega_1}r_t^2, m_1) \cdot \gamma(\frac{m_2}{\Omega_1}r_t^2, m_1) + \gamma(\frac{m_2}{\Omega_2}r_t^2, m_2)}$$

$$\cdot \gamma(\frac{m_2}{\Omega_2}r_{th}^{2}, m_2) + \frac{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1)}{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)} \cdot \frac{\gamma(\frac{m_2}{\Omega_2}r_{th}^{2}, m_2) - \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1) - \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)} \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2) \cdot \frac{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)} \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2) \cdot \frac{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)} \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2) \cdot \frac{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)} \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2) \cdot \frac{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)} \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2) \cdot \frac{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)} \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2) \cdot \frac{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)}{\gamma(\frac{m_1}{\Omega_1}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)} \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2) \cdot \frac{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)}{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_2)} \cdot \frac{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)}{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)} \cdot \frac{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_2)}{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)} \cdot \frac{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_2)}{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_2)} \cdot \frac{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_1) + \gamma(\frac{m_2}{\Omega_2}r_{t}^{2}, m_2)}{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_2)} \cdot \frac{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_1) + \gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_2)}{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_2)} \cdot \frac{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_1) + \gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_2)}{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_2)} \cdot \frac{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_1)}{\gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_1)} + \gamma(\frac{m_1}{\Omega_2}r_{t}^{2}, m_2)}$$

$$\cdot \gamma(\frac{m_1}{\Omega_1}r_{th}^2, m_1) \tag{28}$$

For the channels with identical parameters we have:  $r_{th} < r_T$ 

$$P_{out}(r_{th}) = \gamma(\frac{m}{\Omega}r_t^2, m)\gamma(\frac{m}{\Omega}r_{th}^2, m)$$
(29)

$$r_{th} \ge r_{T}$$

$$P_{out}(r_{th}) = \gamma(\frac{m}{\Omega}r_{t}^{2}, m)\gamma(\frac{m}{\Omega}r_{th}^{2}, m) + \left(\gamma(\frac{m_{2}}{\Omega_{2}}r_{th}^{2}, m_{2}) - \gamma(\frac{m_{2}}{\Omega_{2}}r_{t}^{2}, m_{2})\right)$$
(30)

Finnaly, fade duration is obtain from the expression:

$$T(r_{th}) = \frac{P_{out}(r_{th})}{N(r_{th})}$$
(31)

### **3** Numerical Results



Fig.2. The joint probability density function (PDF),  $p_{r\dot{r}}(r\dot{r})$  for  $r_T = 1, m = 0.7, \Omega = 1, \beta = 0.1$ 

The joint probability density functions (PDFs) of the SSC combiner output signal are shown in Figs. 2 and 3. for different values of  $r_T$ , m,  $\Omega$  and  $\beta$ . Level crossing rate curves,  $N(r_{th})$ , for some parameters are given in Figs. 4 to 6. Fade durations  $T(r_{th})$  are shown in Figs. 7. to 9.



Fig.3. The joint probability density function (PDF),  $p_{rr}(rr)$  for  $r_T = 1, m = 0.7, \Omega = 4, \beta = 0.2$ 



Fig. 4. Level crossing rate  $N(r_{th})$  for  $r_T=1$ , m=0.7,  $\Omega=4$ ,  $\beta=0.2$ 



Fig. 5. Level crossing rate  $N(r_{th})$  for  $r_T=1$ , m=1,  $\Omega=1$ ,  $\beta=0.1$ 



Fig. 6. Level crossing rate  $N(r_{th})$  for  $r_T=0.5$ , m=0.7,  $\Omega=1$ ,  $\beta=0.1$ 



Fig.7. Fade duration  $T(r_{th})$  for  $r_T=1$ , m=0.7,  $\Omega=4$ ,  $\beta=0.2$ 



Fig. 8. Fade duration  $T(r_{th})$  for  $r_T=1$ , m=1,  $\Omega=1$ ,  $\beta=0.1$ 

#### 4 Conclusion

In this paper the level crossing rate, outage probability and fade duration of the SSC combiner output signal are determined in the presence of Nakagami-m fading. The results are shown

graphically for different variance values and decision threshold values.



Fig. 9. Fade duration  $T(r_{th})$  for  $r_T=0.5$ , m=1,  $\Omega=1$ ,  $\beta=0.1$ 

References:

- [1] M. S. Alouini, and M. K. Simon, "Postdetection Switched Combining- A simple Diversity Scheme With Improved BER Performance", *IEEE Trans. on Commun.*, vol. 51, No 9, Sept. 2003, pp.1591-1602.
- [2] A.A. Abu-Dayya and N. C. Beaulieu, "Analysis of switched diversity systems on generalized – fading channels", *IEEE Trans. Commun.*, vol. 42, 1994, pp. 2959-2966.
- [3] A. A. Abu-Dayya and N. C. Beaulieu, "Switched diversity on microcellular Ricean channels", *IEEE Trans. Veh. Technol.*, vol. 43, 1994, pp. 970-976.
- [4] Y. C. Ko, M. S. Alouini and M. K. Simon, "Analysis and optimization of switched diversity systems", *IEEE Trans. Veh. Technol.*, vol. 49, 2000, pp.1569-1574.
- [5] C. Tellambura, A. Annamalai and V. K. Bhargava, "Unified analysis of switched diversity systems in independent and correlated fading channels", *IEEE Trans. Commun.*, vol. 49, 1994, pp. 1955-1965.
- [6] M.A. Blanco, "Diversity receiver performance in Nakagami fading" in *Proc. 1983 IEEE Southeastern Conf. Orlando*, pp. 529-532.
- [7] Cody, J., "An Overview of Software Development for Special Functions", *Lecture Notes in Mathematics, 506, Numerical Analysis Dundee,* G. A. Watson (ed.), Springer Verlag, Berlin, 1976.
- [8] Marvin K. Simon, Mohamed-Slim Alouni, *Digital Communication over Fading Channels*, Second Edition, Wiley Interscience, New Jersey, 2005, pp. 586.