Synchronization of an uncertain Genesio chaotic system via adaptive CMAC

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Abstract: This study proposes an adaptive cerebellar model articulation controller (ACMAC) for synchronizing uncertain Genesio chaotic system. The proposed ACMAC system attempts to use a combination of backstepping method and cerebellar model articulation controller (CMAC) for synchronizing uncertain chaotic system. CMAC is a nonlinear adaptive network with simple computation, good generalization capability and fast learning property. In the proposed control system, a CMAC is used to mimic an ideal backstepping controller, and an $H^\infty$ robust controller is designed to attenuate the effect of the approximation error with desired attenuation level. Finally, numerical simulations for the Genesio system are presented to illustrate the effectiveness of the proposed control system.

Key-Words: Adaptive; CMAC; backstepping; chaotic; $H^\infty$; synchronization.

1 Introduction

Synchronization of chaotic system has been great interested since Pecora and Carroll introduced a method to synchronize two identical chaotic systems with different initial conditions [1]. The idea of synchronization is to use the output of the drive system to control the response system, so that the response system can follow the drive system asymptotically. Backstepping method is a recursive procedure by the choice of a Lyapunov function; it can guarantee global stability and good tracking performance for most of strict-feedback systems. In the past decade, many methods have been presented for the synchronization of chaotic systems such as backstepping control [2-4], adaptive control [5, 6] and adaptive backstepping control [7].

CMAC is a type of associative memory neural network inspired by the neurophysiologic theory of the cerebellum. CMAC has been first proposed by Albus in the literatures [8, 9]. The conventional CMAC uses local constant binary receptive-field basis functions. The disadvantage is that its output is constant within each quantized state and the derivative information is not preserved. On the other hand, for acquiring the derivative information of input and output variables, Chiang and Lin developed a CMAC with non-constant differentiable Gaussian receptive-field basis function, and provided the convergence analyses of this network [10]. The advantages of using CMAC over neural network in many practical applications have been presented in recent literatures [11-12].

The $H^\infty$ robust control can be applied to the robust tracking control design problems for a general class of uncertain nonlinear systems. The problems of designing robust control for uncertain nonlinear systems have been proposed for achieving specified tracking error attenuation [13]. In this study, an ACMAC system is proposed, which combines with the backstepping method and CMAC, to force the drive-response chaotic systems to be synchronized in different initial conditions and with system uncertainty and external disturbance. In the proposed control system, a CMAC is used to mimic an ideal backstepping controller (IBC), and an $H^\infty$ robust controller is designed to attenuate the effect of the approximation error with desired attenuation level. Simulation results are demonstrated to illustrate the effectiveness of the proposed control method.

2 Problem Formulation

The synchronization of chaotic system can be described in the following with a drive system and a response system.

Assume a $n$-dimensional drive system as follows:

\[
\begin{align*}
\dot{x}_1(t) &= f_1(x_1(t), x_2(t)) \\
\dot{x}_2(t) &= f_2(x_1(t), x_3(t), x_4(t)) \\
& \vdots \\
\dot{x}_n(t) &= f_n(x_1(t), x_2(t), \ldots x_n(t))
\end{align*}
\]

where $x_i(t)$ are the system states of drive system, and $f_i$ is a linear function and
Consider a class of response system as follows:
\[
j_1(t) = f_1(y_1(t), y_2(t), \ldots, y_n(t))
\]
\[
j_2(t) = f_2(y_1(t), y_2(t), \ldots, y_n(t))
\]
\[
\vdots
\]
\[
j_n(t) = f_n(y_1(t), y_2(t), \ldots, y_n(t))
\]
where \( y_i \) \((i = 1, 2, \ldots, n)\) are the system states of response system, \( u(t) \) is the control input and \( d(t) \) denotes the unknown external disturbance and \( \Delta f_i(y_1(t), y_2(t), \ldots, y_n(t)) \) denotes the unknown uncertain term.

Different initial conditions of two chaotic systems (1) and (2) are considered. The design purpose of synchronization is to design a suitable control system \( u(t) \), such that the states of response system can follow the states of drive system. Thus, two systems are synchronized, that is
\[
\lim_{t \to \infty} |x_i(t) - y_i(t)| \to 0, \quad i = 1, 2, \ldots, n
\]  

3 Ideal Backstepping Control of Chaotic System

This section focuses on designing an ideal backstepping control for Genesio chaotic system synchronization. The design procedures are described in the following. Considered the Genesio systems described by [15]
The drive system
\[
x_1(t) = x_2(t)
\]
\[
x_2(t) = -a x_1(t) - b x_2(t) - c x_1(t) + x_3^2(t) = f(x)
\]
where \( a, b, c \) are known parameters and \( f(x) = -a x_1(t) - b x_2(t) - c x_1(t) + x_3^2(t) \), in which \( x = [x_1, x_2, x_3]^T \).
The response system
\[
j_1(t) = y_2(t)
\]
\[
j_2(t) = y_3(t)
\]
\[
j_3(t) = -a y_1(t) - b y_2(t) - c y_3(t) + y_3^2(t) + d(t) + \Delta f(t)
\]
where \( d(t) \) denotes the unknown external disturbance, \( \Delta f(y_1, y_2, y_3) \) denotes the unknown uncertain term, \( y_i \) \((i = 1, 2, 3)\) are the system states of response system, \( u(t) \) is the control input and \( f(y) = -a y_1(t) - b y_2(t) - c y_3(t) + y_3^2(t) \), in which \( y = [y_1, y_2, y_3]^T \).

Step 1: Define the tracking error
\[
e_1(t) = x_1(t) - y_1(t)
\]
\[
e_2(t) = x_2(t) - y_2(t)
\]
\[
e_3(t) = x_3(t) - y_3(t)
\]
Then
\[
x_3(t) - y_3(t) = -a e_1(t) - b e_2(t) - c e_3(t) + (x_1(t) + y_1(t)) e_1(t) - d(t) - \Delta f(y) - u(t)
\]
Define the following stabilizing function
\[
a_i(t) = k_i e_i(t) + x_i(t)
\]
where \( k_i \) is a positive constant.
A Lyapunov function is chosen as
\[
V_1(t) = 0.5 c_i^2(t)
\]
Define
\[
e_1(t) = a_i(t) - y_1(t) = \dot{e}_1(t) + k_i e_1(t)
\]
The derivative of \( V_1(t) \) is
\[
\dot{V}_1(t) = e_1(t) \dot{e}_1(t) = -k_i e_1^2(t) + e_1(t) e_i(t)
\]
Then, if \( e_1(t) = 0 \), \( \dot{V}_1(t) = -k_i e_1^2(t) \leq 0 \) will be achieved.

Step 2: The derivative of \( e_2(t) \) is expressed as
\[
\dot{e}_2(t) = \dot{a}_2(t) - y_2(t) = x_1(t) - y_2(t) + k_i \dot{e}_2(t)
\]
The \( y_2(t) \) can be viewed as a virtual control in the above equation. Define another stabilizing function
\[
a_2(t) = e_2(t) + k_i e_2(t) + a_2(t)
\]
where \( k_i \) is a positive constant.
A Lyapunov function is chosen as
\[
V_2(t) = V_1(t) + 0.5 c_i^2(t)
\]
Define
\[
e_2(t) = a_2(t) - y_2(t)
\]
The derivative of \( V_2(t) \) is
\[
\dot{V}_2(t) = -k_i e_2^2(t) - k_i e_2^2(t) + e_2(t) e_i(t)
\]
Then, if \( e_2(t) = 0 \), \( V_2(t) = -k_i e_2^2(t) - k_i e_2^2(t) \) will be achieved.

Step 3: The derivative of \( e_3(t) \) is expressed as
\[
\dot{e}_3(t) = \dot{a}_3(t) - y_3(t)
\]
\[
= \dot{a}_3(t) - [f(y) + d(t) + \Delta f(y) + u(t)]
\]
The other Lyapunov function is chosen as
\[
V_3(t) = V_2(t) + 0.5 c_i^2(t)
\]
The derivative of \( V_3(t) \) is
\[
\dot{V}_3(t) = -k_i e_3^2(t) - k_i e_3^2(t) + e_3(t) [e_i(t) + \dot{a}_2(t) - f(y) - d(t) - \Delta f(y) - u(t)]
\]
Step 4: In case of system dynamics and the external disturbance being well known, an ideal
backstepping control can be obtained as
\[ u^{\ast}_{ibc} = \hat{a}_2(t) - f(y) - d(t) \]
\[ - \Delta f'(y) + e_1(t) + k_3 e_3(t) \]
where \( k_3 \) is a positive constant.
Substituting (20) into (19), the following equation can be obtained:
\[ \dot{V}_3(t) = -k_3 e_3^2(t) - k_1 e_1^2(t) - k_2 e_2^2(t) \]
\[ = -E^T K E \leq 0 \]  \hspace{1cm} (21)
where \( E = [e_1(t), e_2(t), e_3(t)]^T \) and \( K = \text{diag}(k_1, k_2, k_3) \). Since \( \dot{V}_3(E(t)) \leq 0 \), \( \dot{V}_3(E(0)) \) is negative semi-definite (i.e., \( \int_0^\infty \Omega(t) \, dt < \infty \)).

Now define the following term:
\[ \Omega(t) = E^T K E = -\dot{V}_3(E(t)) \]  \hspace{1cm} (22)

Because \( \dot{V}_3(E(0)) \) is bounded and \( \dot{V}_3(E(t)) \) is nonincreasing and bounded, the following result can be obtained
\[ \lim_{t \to \infty} \int_0^t \Omega(t) \, dt < \infty \]
Also \( \Omega(t) \) is bounded, so by Barbalat’s Lemma [14], it can be shown that \( \lim_{t \to \infty} \Omega(t) = 0 \). This will imply that \( e_1(t) \), \( e_2(t) \) and \( e_3(t) \) converge to zero as \( t \to \infty \). Therefore, the ideal backstepping control in (20) will asymptotically synchronize the system.

4 Design of ACMAC System
In practical applications, \( f(y) \) cannot be exactly obtained in general, the external disturbance \( d(t) \) and the uncertain term \( \Delta f'(y) \) are always unknown. Therefore, the ideal backstepping control in (20) can not be obtained. Thus, ACMAC system is proposed to control the Genesio chaotic system. The control law takes the following form:
\[ u = u_{\text{CMAC}} + u_s \]  \hspace{1cm} (23)
where \( u_{\text{CMAC}} \) is the cerebellar model articulation controller (CMAC) utilized to approximate the ideal backstepping controller and \( u_s \) is the \( H_\infty \) robust controller utilized to suppress the influence of approximation error between the ideal backstepping controller and CMAC.

4.1 Cerebellar model articulation controller

A cerebellar model articulation controller (CMAC) is proposed and shown in Figure 1. The signal propagation and the basic function in each space are introduced as follows.

1) Input space \( Q \): For a given \( q = [q_1, q_2, \ldots, q_T]^T \), each input state variable \( q_i \) must be quantized into discrete regions (called elements) according to given control space. The number of elements, \( n_q \), is termed as a resolution.

2) Association memory space \( A \): Several elements can be accumulated as a block, the number of blocks is \( n_e \). In this space, each block performs a receptive-field basis function. The Gaussian function is adopted here as the receptive-field basis function
\[ \phi_k = \exp\left( -\frac{(q_i - m_k)^2}{v_k} \right) \], for \( k = 1, 2, \ldots, n_e \)  \hspace{1cm} (24)
where \( \phi_k \) represents the receptive-field basis function for the \( k \)-th block of the \( i \)-th input \( q_i \) with the mean \( m_k \) and variance \( v_k \). The mean \( m_k \) and variance \( v_k \) will be on-line adjusted during operation.

3) Receptive-field space \( T \): The number of receptive-field, \( n_r \), is equal to \( n_e \) in this study. Each location of \( A \) corresponds to a receptive-field. The multi-dimensional receptive-field function is defined as
\[ b_k = \prod_{i=1}^n \phi_k(q_i) = \exp\left( \sum_{i=1}^n -\frac{(q_i - m_k)^2}{v_k} \right) \]  \hspace{1cm} (25)
where \( b_k \) is associated with the \( k \)-th receptive-field.

The multi-dimensional receptive-field function can be expressed in a vector form as
\[ \Gamma(q, m, v) = [b_1, \ldots, b_1, \ldots, b_{n_e}]^T \]  \hspace{1cm} (26)
where \( m = [m_1, \ldots, m_{n_e}, \ldots, m_{n_e}]^T \) and \( v = [v_1, \ldots, v_{n_e}, \ldots, v_{n_e}]^T \).

4) Weight memory space \( W \): Each location of \( T \) to a particular adjustable value in the weight memory space. Each location of \( A \) is associated with the \( n_q \) receptive-field.
space can be expressed as
\[ w = [w_1, \ldots, w_i, \ldots, w_n]^T \]  
(27)

where \( w_i \) denotes the connecting weight value of the output associated with the \( i \)th receptive-field. The weight \( w_i \) is automatically adjusted during on-line operation.

5) **Output space** \( Y \): The output of CMAC is the algebraic sum of the activated weights in the weight memory, and is expressed as
\[ y = w^T \Gamma(q, m, v) = \sum_{i=1}^{n} w_i b_i \]  
(28)

### 4.2 Synchronization of an Genesio unchaotic systems

The design of ACMAC system for the synchronization of the Genesio system is described as follows.

**Step 1:** Define the tracking errors \( e_i(t), e_i(t) \) and \( e_i(t) \) as (6), (10) and (15), and the stabilizing functions \( \alpha_i(t) \) and \( \alpha_i(t) \) as (8) and (13), respectively.

**Step 2:** Since adaptive CMAC is utilized to estimate the IBC in (20), a CMAC \( u_{\text{CMAC}} \) can be written as follows:
\[ u_{\text{CMAC}}(q, m, v) = y = w^T \Gamma(q, m, v) \]  
(29)

Assume there exists an optimal \( u_{\text{CMAC}} \) to approach the IBC \( u_{\text{IBC}} \) such that
\[ u'_{\text{CMAC}} = u'_{\text{CMAC}}(q, w', m', v') + \varepsilon = w'^T \Gamma' + \varepsilon \]  
(30)

where \( \varepsilon \) is a minimum estimation error; \( w', m', v' \) and \( \Gamma' \) are optimal parameters of \( w, m, v \) and \( \Gamma \), respectively. However, the optimal \( u'_{\text{CMAC}} \) cannot be obtained, so the on-line estimation of \( u_{\text{CMAC}} \) is used to approach the IBC \( u_{\text{IBC}} \). From (29), the control law (23) can be rewritten as follows:
\[ u = u_{\text{CMAC}}(q, \tilde{w}, \tilde{m}, \tilde{v}) + u_s = \tilde{w}^T \hat{\Gamma} + u_s \]  
(31)

where \( \tilde{w}, \tilde{m}, \tilde{v} \) and \( \hat{\Gamma} \) are some estimates of the optimal parameters \( w, m, v \) and \( \Gamma \), respectively. Subtracting (31) from (30), an approximation error term \( \tilde{u} \) is defined as
\[ \tilde{u} = u'_{\text{CMAC}} - u = w'^T \Gamma' + \varepsilon - u_s \]  
(32)

where \( \tilde{u} = w' - \tilde{w} \) and \( \Gamma = \Gamma' - \hat{\Gamma} \). Moreover, the expansion of \( \Gamma \) in Taylor series can be obtained as
\[ \Gamma' = [b_0, \ldots, b_i, \ldots, b_n]^T \]  
(33)

where \( \tilde{b}_i = b_i - \hat{b}_i \); \( \hat{b}_i \) is the optimal parameter of \( b_i \); \( \hat{b}_i \) is an estimate of \( b_i \); \( O_e \in \Re^n \) is a vector of higher-order terms; \( \frac{\partial b_i}{\partial m} \) and \( \frac{\partial b_i}{\partial v} \) are defined as
\[ \left[ \frac{\partial b_i}{\partial m} \right] = [0, \ldots, 0, \frac{\partial b_i}{\partial m_1}, \ldots, \frac{\partial b_i}{\partial m_n}, 0, \ldots, 0] \]  
(34)

\[ \left[ \frac{\partial b_i}{\partial v} \right] = [0, \ldots, 0, \frac{\partial b_i}{\partial v_1}, \ldots, \frac{\partial b_i}{\partial v_n}, 0, \ldots, 0] \]  
(35)

Rewriting (33), it can be obtained that
\[ \Gamma' = \hat{\Gamma} + C^Tm + H^Tv + O_e \]  
(36)

Substituting (33) and (36) into (32), yields
\[ \tilde{u} = \tilde{w}' \hat{\Gamma} + \tilde{w}' (C^T \hat{m} + H^T \hat{v}) + \xi - u_s \]  
(37)

where \( \xi = w' C^T \hat{m} + \tilde{w}' H^T \hat{v} + \tilde{w}' O_e \) is the approximation error term.

**Step 3:** In order to develop the robust controller, equation (17) can be expressed via (20) and (32) as
\[ \dot{e}_2(t) = \tilde{w}' \hat{\Gamma} + \tilde{w}' (C^T \hat{m} + H^T \hat{v}) \]  
(38)

Define a Lyapunov function as
\[ V_4(t) = V_4(t) + \frac{1}{2 \eta_1} \tilde{w}' \tilde{w} + \frac{1}{2 \eta_2} \tilde{m}' \hat{m} + \frac{1}{2 \eta_3} \tilde{v}' \tilde{v} \]  
(39)

where \( \eta_1, \eta_2, \eta_3 \) are positive constants. Taking the derivative of the Lyapunov function (39) and then using (38), yields
\[ \dot{V}_4(t) = -E_1 KE + \tilde{w}' [e_2(t) (\hat{\Gamma} C + \frac{1}{\eta_1}) \tilde{w}] + e_2(t) [\xi - u_s] \]  
(40)

**Step 4:** The ACMAC system is designed as (23). The adaptive laws of CMAC are chosen as
\[ \dot{w} = \eta_1 e_1(t) \hat{\Gamma}_w \]  
(41)

\[ \dot{m} = \eta_2 e_1(t) C \tilde{w} \]  
(42)

\[ \dot{v} = \eta_3 e_1(t) H \tilde{w} \]  
(43)

and the robust \( H_\infty \) controller is chosen as
\[ u_s = \frac{0.5(\delta^3 + 1)}{\delta^2} e_1(t) \]  
(44)

where \( \delta \) is a positive constant. Form (41)-(43), equation (40) can be rewritten as
\[ \dot{V}_4(t) = -E_1 KE - 0.5 e_2^2(t) - 0.5 \frac{e_2(t)}{\delta} - \delta \xi^2 \]  
(45)

Assume \( \xi \in L_2[0, T], \forall T \in [0, \infty) \). Integrating the above equation from \( t = 0 \) to \( t = T \), yields


\[ V_s(T) - V_s(0) \leq -0.5 \int_0^T e_i(t) dt + 0.5 \delta^2 \int_0^T \xi^2(t) dt \]  \hspace{1cm} (46)

Since \( V(T) \geq 0 \), the above inequality implies the following inequality

\[ 0.5 \int_0^T e_i(t) dt \leq V_s(0) + 0.5 \delta^2 \int_0^T \xi^2(t) dt \]  \hspace{1cm} (47)

Using (39), the above inequality is equivalent to the following

\[
\frac{1}{\eta_1} \int_0^T e_i(t) dt \leq E(0)E(0) + \frac{1}{\eta_1} \int_0^T \xi^2(t) dt
\]

\[ \int_0^T \eta_2(m(0) - \hat{m}(0)) + \frac{1}{\eta_2} \int_0^T \xi^2(t) dt \]  \hspace{1cm} (48)

If the system starts with initial conditions \( E(0) = 0 \), \( \hat{m}(0) = 0 \), \( m(0) = 0 \) and \( \xi(0) = 0 \), the \( H^\infty \) tracking performance in (48) can be rewritten as

\[
\sup_{\xi(\cdot) \in L_2(0,T)} \left\| e_i \right\|_2 \leq \delta
\]  \hspace{1cm} (49)

where \( \left\| e_i \right\|_2 = \int_0^T \xi^2(t) dt \) and \( \left\| \xi \right\|_2 = \int_0^T \xi^2(t) dt \) . The attenuation constant \( \delta \) can be specified by the designer to achieve desired attenuation ratio between \( \left\| e_i \right\|_2 \) and \( \left\| \xi \right\|_2 \) [13]. Then, the desired robust tracking performance in (48) can be achieved for a prescribed attenuation level \( \delta \).

5 Numerical simulation

In this section, numerical simulations of Genios system are presented to demonstrate the effectiveness of the proposed ACMAC system. The ACMAC used in these examples is characterized by \( \rho = 4 \), \( n_b = 5 \), \( n_a = 8 \) and \( n_y = 2 \times 4 \).

The configuration of the ACMAC system for Genios chaotic system is depicted in Figure 2. The initial values of the parameters for the receptive-field basis functions are chosen as \( m_{1i} = -1.05 \), \( m_{2i} = -0.75 \), \( m_{3i} = -0.45 \), \( m_{4i} = -0.15 \), \( m_{5i} = 0.15 \), \( m_{6i} = 0.45 \), \( m_{7i} = 0.75 \), \( m_{8i} = 1.05 \) and \( \sigma_{ai} = 0.6 \) for all \( i \) and \( k \). It is supposed that the uncertain term \( \Delta f(y) = -0.1 y_1 \) and disturbance \( d(t) = 0.4 \cos(\pi t) \) are unknown. The system’s parameters are \( a = 6 \), \( b = 2.92 \), \( c = 1.2 \). The ACMAC system parameters are chosen as \( \eta_1 = 0.75 \), \( \eta_2 = \eta_3 = 0.0017 \), \( k_1 = 2 \) and \( k_2 = 2 \). The initial conditions of the drive system \( x_i(0) = 2 \), \( x_2(0) = -3 \), \( x_3(0) = -1 \) and the response system \( y_i(0) = -5 \), \( y_2(0) = 5 \), \( y_3(0) = -5 \) are employed. The simulation results of the ACMAC system with \( \delta = 1 \) and \( \delta = 1 \) are shown in Figure 3 and Figure 4, respectively. It can be seen that the proposed ACMAC scheme can achieve favorable synchronization performance; moreover the better tracking performance can be achieved as specified attenuation level \( \delta \) being chosen smaller.

![Figure 2. Block diagram of ACMAC system for Genesio chaotic system.](image-url)

![Figure 3. Simulation results of CMAC with $\delta = 1$ (---drives system trajectory, ——response system trajectory).](image-url)
siblings. All the adaptation laws of the robust controller demonstrate the effectiveness of the proposed control scheme for this chaotic system even under unknown uncertainty and disturbance. Thus, the proposed control method is suitable for the synchronization of uncertain nonlinear SISO systems.


References: