Fully Digital Fractional Frequency Synthesizer

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Abstract: The paper deals with architecture of a new pure digital frequency synthesizer based on pulse generators, counters and a register. The technique described here is much simpler then other method, e.g. S? fractional-N frequency synthesizers or direct digital synthesis. Presented synthesizer is the suitable for the design of VLSI architectures or for programmable Large Scale Integration circuits.

Key-Words: Fractional frequency synthesizer, direct digital synthesis, phase locked loop, counter, register.

1 Introduction
The frequency synthesizer can be described as an active electronic device that accepts a reference frequency and then generates one or more new frequencies as defined by a control word. Modern electronic and telecommunication systems demand frequency synthesizer of high resolution, wide bandwidth and fast switching speed. Conventional frequency synthesis techniques existing today may be classified as the following three types:

a) Phase-locked loop (PLL) based, or "indirect"
b) Mixer / filter / divide, or "direct analog"
c) Direct digital synthesis (DDS)

Each of these methodologies has advantages and disadvantages. Direct analog synthesis uses the functional elements of multiplication, division and other mathematical manipulation to produce the desired frequency, but this method is very expensive. DDS uses logic and memory to digitally construct the desired output signal. On the output, digital-to-analog (D/A) converter is used to convert the digital signal to analog domain. PLL-based frequency synthesis has been widely used in industry. However, one of major difficulties associated with PLL-based technique is that a PLL with wide frequency range cannot be achieved easily. Also, fast switching is difficult to achieve. Typically, the output frequency step size of this method is the reference frequency. With fractional-N synthesis technique [1], finer frequency control can be achieved; however, these systems typically have very narrow bandwidth.

In this paper a new simple architecture of digital frequency synthesizers with square wave output is presented. Described synthesizer is the most suitable for the design of VLSI architectures or for programmable Large Scale Integration. On the other hand, this synthesizer has a disadvantage in low output frequency, but this can be overcome by using this synthesizer together with phase locked loop.

The aim of frequency synthesis is to generate arbitrary frequency $f_x$ from a given standard frequency $f_s$, it means to solve the equation (1):

$$f_x = k_x \cdot f_s$$

where $k_x$ in the simplest case is a fraction formed by small, relatively prime integers. That is,

$$k_x = X_1 / Y_1$$

and the synthesizer is reduced merely to chain of one frequency divider and one multiplier. If $X_1$ and $Y_1$ in (2) are products of small prime numbers, the synthesizer may be realized by chain of frequency multipliers and dividers. However, there are difficulties with hardware solutions, mainly generation of spurious signals and frequent enhancement of the phase noise level.

![Fig. 1. Block diagram of the digital synthesizer](image)

2 Principles of New Synthesizer
In the Fig. 1, there is a block diagram of the digital frequency synthesizer [2]. It consist of Counter 1, which count up frequency $f_{C1}$ gated by input frequency $f_x$. Parallel output from Counter 1 is connected to Register input and Register output is connected to preset inputs of Counter 2 which counts down frequency $f_{C2}$. On the output of this Counter 2 there is frequency $f_y$. It is
expected, that \( f_{C1} > f_x \). Number \( C1 \) which is stored in Counter 1 during the period of the \( f_x \) is given by (3):

\[
C1 = \frac{f_{C1}}{f_x}
\]  

(3)

This number is written in the Register, where his value can be changed by the Control to \( C2 \):

\[
C2 = g(C1)
\]  

(4)

where \( g(.) \) denote some function of \( C1 \). Number \( C2 \) is given by (5):

\[
C2 = \frac{f_{C2}}{f_Y} = g(f_{C1}/f_x) = g(f_{C1}/f_X)
\]  

(5)

Output frequency \( f_y \) can be expressed from (5) by (6):

\[
f_y = f_{C2}/g(f_{C1}/f_X)
\]  

(6)

When, for example function \( g(.) = 1/k_i \) (which can be simply realized by shift binary number in the Register) output frequency \( f_y \) is given by (7):

\[
f_y = f_{C2} \cdot k_i \cdot f_x/f_{C1}
\]  

(7)

Equation (7) shows, that output frequency \( f_y \) is a products of frequency \( f_{C2} \), \( k_i \) and input frequency \( f_x \) divided by frequency \( f_{C1} \). All of these parameters can be individually set. The length of the counters and registers must be sufficient to prevent overrun. If the binary counter is expected, then minimal length \( L \) of the Counter 1 [bit] is given by (8):

\[
L = \text{Ceil}( \log_2 (f_{C1MAX}/f_{XMIN})) \text{ [bit]}
\]  

(8)

where \( f_{C1MAX} \) and \( f_{XMIN} \) are maximal clock and minimal input frequency and \( \text{Ceil} \) function converts numeric value to an integer by returning the smallest integer greater than or equal to its argument. In Figure 2, the synthesizer is shown as a building block.

Figure 3. Block diagram of the basic Phase-Locked Loop. PD - phase detector, LPF - low pass filter, VCO - voltage controlled oscillator, N - frequency divider by N (N is integer number).

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Figure 5. Possibility of the adaptive control using in the digital frequency synthesizer.

3 Digital Synthesizer and Phase Locked Loop

The phase locked loop (PLL) works as a feedback system shown in Figure 3 [3]. The task of PLL is to maintain coherence between input (reference) signal frequency, \( f_o \), and the respective output frequency, \( f_y \), via phase detector (PD) [4]. When PLL locks onto a reference signal the output frequency is given by (9):

\[
f_y = N \cdot f_i
\]  

(9)

where \( N \) is an integer divide number of divider. Normally, frequency dividers can only produce integer divide ratios (\( N \) is integer). Fractional division is accomplished by alternating the instantaneous divide number between \( N \) and \( N+1 \), but this causes phase modulation on the VCO [5]. Therefore a different complicated technique is used for correction of this error [6]. In Figure 4, the SYNT circuit is used in PLL [7]. Frequency on the SYNT input is \( f_y/N \) and frequency on the SYNT output is given by (10):
\[ f_i = f_{c2} \cdot k_1 \cdot f_o / (f_{c1} \cdot N) \]  
(10)

From (10) we can derive the frequency of voltage controlled oscillator which is shown in (11):

\[ f_o = f_{c1} \cdot N \cdot f_i / (f_{c2} \cdot k_1) \]  
(11)

In case that number \( C2 \) in register is given by (12) (binary number \( C1 \) is multiplied by \( m_1 \), e.g. register is shifted to left, instead of divided by \( k_1 \)), the frequency of voltage controlled oscillator is given by (13):

\[ C2 = m_1 \cdot C_1 \]  
(12)

\[ f_o = f_{c1} \cdot m_1 \cdot N \cdot f_i / f_{c2} \]  
(13)

From the (13) we can see, that output frequency \( f_o \) is a function of integer \( m_1 \), \( N \) and clock frequencies \( f_{c1}, f_{c2} \) [8, 9, 10, 11].

4 Error Correction in Synthesizer

The digital synthesizer shown in Figure 1, has a following disadvantage. When the numbers in counters are small (numbers are integer), the output frequency is not accurate. This error can be improved by adaptive control shown in Figure 5.

The Figure 5 is the almost the same as Figure 1, only adaptive control is added. Adaptive control block reads the contents of the Register. When the number is too small, the frequency of Generator 1 is multiplied and also frequency in Generator 2 is multiplied, so that the ratio of \( f_{c1}/f_{c2} = \) constant. On the other hand, if number in Register is too big, the frequencies of both generators are divided by the same number.

![Image](Figure 6. Example of the digital frequency synthesizer realized by using Lattice isp-LSI1016 IC's, oscillator and 2 one shot devices. In this experiment, Clk1 and Clk2 were connected together.)

5 Properties of New Synthesizer

The digital synthesizer which was described (Figure 1) has following disadvantages:

a) Accuracy depends on integer number in

Counter
b) Not suitable for high output frequency
c) Only square wave output

The main synthesizer advantages are:

a) Pure digital architecture
b) Wide range
c) No setting problems
d) Stable
e) Fast response
f) Easily realized by programmable logic array
g) Easily reprogrammable and reconfigurable
h) Microcontroller adaptive control can be simply added for quality improving.

![Image](Figure 7. Internal block diagram of the digital frequency synthesizer based on programmable logic Lattice isp-LSI1016)

6 Experimental results

The digital synthesizer was designed and built according to the above discussion. The synthesizer requires e.g. one Lattice ispLsi 1016 device (in-system programmable Large Scale Integration circuit), X-tal oscillator and two peripheral one shot devices (Figure 6). Internal block diagram of the 16-bit synthesizer is shown in Figure 7. The connection of two one shot devices is shown in Figure 8. For device testing, \( f_{c1} = f_{c2} = 31.111 \) MHz and \( k_1 = 1 \), so according to the relation (7), the ideal output frequency is:

\[ f_v = f_x \]

Photography of PC board of synthesizer is shown in Figure 9. Two, different delays one shot circuits were tested. For delay of 0.6 \( \mu \)s (load + clear) the input frequency \( f_i \), output frequency \( f_o \) were measured and \( C1 \) number in up-counter and \( C2 \) number in down-counter were computed and number difference dif = \( C1 - C2 \) was also computed. The results are shown in Table 1. From Table 1 it can be seen, that differences are constant and error in frequency can be easily corrected.
MHz to 3 MHz, the results are in Table 2. For \( f_1 = f_2 = 31.111 \text{ MHz} \), the maximal input frequency is approx. 3.5 MHz for good function. Minimal input frequency (to avoid an overflow of 16-bit counter) is 476 Hz.

<table>
<thead>
<tr>
<th>( fx ) [Hz]</th>
<th>( fy ) [Hz]</th>
<th>( C1 )</th>
<th>( C2 )</th>
<th>diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1502</td>
<td>1502</td>
<td>20713</td>
<td>20713</td>
<td>0</td>
</tr>
<tr>
<td>2010</td>
<td>2014</td>
<td>15478</td>
<td>15460</td>
<td>18</td>
</tr>
<tr>
<td>4008</td>
<td>4016</td>
<td>7762</td>
<td>7745</td>
<td>17</td>
</tr>
<tr>
<td>6004</td>
<td>6026</td>
<td>5181</td>
<td>5162</td>
<td>19</td>
</tr>
<tr>
<td>10008</td>
<td>10068</td>
<td>3108</td>
<td>3090</td>
<td>18</td>
</tr>
<tr>
<td>20004</td>
<td>20258</td>
<td>1555</td>
<td>1535</td>
<td>20</td>
</tr>
<tr>
<td>40000</td>
<td>40980</td>
<td>777</td>
<td>759</td>
<td>18</td>
</tr>
<tr>
<td>100000</td>
<td>106600</td>
<td>311</td>
<td>291</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 1. Input and output frequencies for 600 ns delay (load + clear), \( fx \) - input frequency, \( fy \) - output frequency, \( C1, C2 \) are the numbers in Counter1 and Counter 2, \( \text{diff} \) - counters difference \((C1 - C2)\)

<table>
<thead>
<tr>
<th>( fx ) [kHz]</th>
<th>( fy ) [kHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>800</td>
<td>800</td>
</tr>
<tr>
<td>1082.1</td>
<td>1083.0</td>
</tr>
<tr>
<td>2020</td>
<td>2032</td>
</tr>
<tr>
<td>3275</td>
<td>3276</td>
</tr>
</tbody>
</table>

Table 2. Input and output frequencies for 60 ns delay (load + clear).

The experimental digital synthesizer was also used in PLL feedback to produce a fractional PLL. The output frequency spectrum is shown in Figure 10.

At the present time, the digital synthesizer was also designed in VHDL language. This design was realized and tested on Altera FPGA development board with EP20K200E device. The clock signals \( f_{c1} \) and \( f_{c2} \) are driven by a 33.3 MHz free running oscillator. The whole design consumes only 6 % of available logic cells. Despite of a low demand of the logic cells the counter 1 is 24 bit long and the counter 2 is 32 bit long.
This width (32 bit), provides the input frequency range from 2 Hz up to 6.2 MHz (for 33.3 MHz clock). The 8 bit difference between the width of counters allows to divide or multiple output frequency up to 8th power of 2. Fine tuning of the output frequency is provided by the combinational logic for adding or subtracting 23 bit integer numbers to the content of the register.

![Figure 12](image1)

**Figure 12.** The synthesizer with frequency error correction example for input frequency multiplies by 11. a) pulses for ideal output frequency, b) output pulses of synthesizer with error correction, c) output pulses of synthesizer without error correction (the output frequency is higher than ideal).

![Figure 13](image2)

**Figure 13.** The synthesizer with frequency error correction example for input frequency multiplies by 11. a) pulses of input signal, b) output pulses of synthesizer with error correction, c) signal in digital accumulator for error correction.

7 Frequency error correction

The error in described frequency synthesizer (higher output frequency then ideal frequency value) is caused by removing “fractional part” after arithmetic operation on number \( C_1 \), because number \( C_2 \) can be only integer (according equation (4): \( C_2 = g(C_1) \)). This error can be corrected similarly, like in PLL fractional synthesizers. The sigma-delta modulation (accumulator and overflow output) is used in fractional PLL synthesizers. The digital accumulator and adder for frequency error correction is used for presented synthesizer. The synthesizer block diagram with correction is shown in Figure 11. The architecture described in Figure 11 was simulated in Matlab. Example of simulation results for input frequency multiplies by 11 and 5.7 are shown in Figure 12, 13 and 14. From this simulation can be seen, that simple digital error correction system can be added for better synthesizer performances.

![Figure 14](image3)

**Figure 14.** The synthesizer with frequency error correction example for input frequency multiplies by 5.7. a) pulses of input signal, b) output pulses of synthesizer with error correction, c) signal in digital accumulator for error correction.

8 Conclusion

The frequency synthesizers form, which is the basic of most radio system designs and their performance is often key to the overall operation. They are also an important building block in almost all digital and mixed signal integrated circuits as a clock multiplier. Apart from the usual integer-N PLL implementation of the clock multiplier, where a voltage controlled oscillator is locked to a clean reference clock [9, 10]. The architectures based on a Delay-Locked Loop (DLL) have been successfully
used recently as a clock multipliers. The main disadvantage of conventional DLL's, however, is their limited phase capture range.

A new design technique of the frequency synthesizer has been presented in this paper. The presented digital frequency synthesizer was patented in the Czech Republic. Schemes for direct and indirect synthesizers were shown and basic equations and block diagram were also described. The digital frequency synthesizer was realized as 16 bit device, by using Lattice ispLsi 1016 IC's (Counter max. frequency 80 MHz), and experimental results were introduced. It is important to note, that delay, caused LOAD and CLEAR can be easily corrected. The synthesizer can be best of all realized simply by using FPGAs or another types of programmable logic. The synthesizer is suitable for fractional frequency multiply, divide or for another frequency processing. Main advantage is that synthesizer has a fully digital structure and also, there are no stability problems. Also possibilities of wide range input frequency is important. The digital synthesizer can be used with phase-locked loop for simple production of the fractional PLL. The frequency error correction system was also designed and simulations of system with error correction were presented in this paper.

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References