Hybrid- and Pseudo-Distances in Pattern Recognition – Medical Applications

HORIA-NICOLAI L. TEODORESCU[1,2]
[1] Institute for Computer Science of the Romanian Academy
[2]Technical University of Iasi
B-dul Carol I nr. 8, 6600 Iasi
ROMANIA

Abstract: - We revisit several definitions and concepts related to distances; then, we propose the concept of proto-distance and of discrimination function, in view to better model sensorial discrimination and recognition processes. Applications to medicine, including pedometry, hearing, speech clues, and diagnosis in these fields are briefly discussed.

Key-Words: - Distance, pseudo-distance, bio-mimetic distance, classification, sounds, pedometry, foot pressure, gnathophonics.

1 Introduction

Distance functions play an important role in classification theory and in pattern recognition processes; moreover, they constitute the basis for several chapters in the metric theory in mathematics. The subject has been extensively studied in functional analysis in the frame of metric spaces theory and in physical, biological, information processing, and engineering applications. Many authors have proposed various distances to fit a class of applications. Good mathematical overviews are presented in [1]-[4]; examples of discussions of specific metrics in applications are [5]-[14]; several hybrid distances are covered in [5]-[8] (Wilson and Martinez). However, existing metrics may not always represent in a suitable form the manner humans perceive “distances” in the sense of differences between objects, phenomena and processes detected by senses. In fact, distances are mathematical abstractions that do not necessarily reflect the human discrimination processes.

In this paper we address several issues related to the modeling of the way humans discriminate between sensations and we introduce and analyze several new distances and functions that generalize distances. While the discussion in the paper is oriented toward fundamental issues, we also address applications related to classification, recognition and several medical applications, like diagnostic in pedometry, gnathophonics and gnathosonics; in the last two cases, we address classification and discrimination of sounds.

The paper organization is as follows: In the second Section we recall several definitions. The third section includes a brief discussion of sensing and discrimination processes. The next section introduces the concept of distinction function and includes several definitions of distances and distinction functions. The last sections address applications and present conclusions.

2 Basic definitions

In this section, we recall the definitions of several concepts related to distances and of several types of distances.

Definition. A ***quasi-metric*** is defined [3] (Istrățescu) as a function $p : X \times X \to \Re$ with the properties:

i) $p(x,x) = 0 \quad \forall x \in X$

ii) $p(x,y) \le p(x,z) + p(z,y) \quad \forall x,y,z \in X$

Notice that, according to the definition by Istrățescu in [3], the metric should not take only positive values, moreover it is not required to be commutative. Instead, for every metric p, the function defined by $q(x,y) = p(x,y)$ is also a (quasi-) metric, moreover is named conjugated (quasi-) metric. The conjugate quasi-metrics induce two topologies on X.

Semi-metrics

Definition. A ***semi-metric*** is a function $e : X \times X \to \Re$ satisfying the properties:

i) $e(x,y)=0 \quad \Rightarrow x=y$

ii) $e(x,y)=e(y,x) \quad \forall x,y$

iii) $e(x,y)\le e(x,z)+e(z,y) \quad \forall x,y,z\in X$

Notice that the quasi-metric has a weaker condition (i) and lacks the commutativity condition (ii) in the semi-metric definition. Some authors, like (Olga Costinescu, [2]), use the term "écart" for the semi-metric.

Definition. A set X is endowed with a ***metric*** if on X is defined a function $d: X\times X\to \mathbf{R}$ satisfying the conditions ([2], [4]):

i) $d(x,y)\ge 0 \quad \forall x,y\in X$ (1)

iia) $d(x,x)=0 \quad \forall x$ (2)

or, not equivalently

iib) $d(x,y)=0$ iff $x=y$ (3)

iii) $d(x,y)=d(y,x) \quad \forall x,y\in X$ (4)

iv) $d(x,z)\le d(x,y)+d(y,z) \quad \forall x,y,z$ (5)

The quasi-metric is a metric, according to [3] if it satisfies:

iv) $p(x,y)=0$ iff $x=y$.

However, most authors (excepting a few, like Istrăţescu), also require that the values taken by the function are positive, moreover that the function is symmetric i.e. if $d(x,y)=d(y,x) \quad \forall x,y$.

The condition (iv) defines a specific type of distance, the Archimedean distance. Only conditions (i)-(iii) are considered essential, in general.

Notice that, if (iv) is true, taking $z=y$, $d(x,y)\le d(x,y)+d(y,y)$ it results that $d(x,x)\ge 0$.

Any Banach space B with the norm denoted by || || is a metric space with

$$d(x,y)=\sqrt{\|\bar{x}-\bar{y}\|^2}\,. \quad (6)$$

Other typical distances are also related to the norm concept, Hilbert and Banach spaces concepts:

$$d_2(x,y)=|\bar{x}-\bar{y}| \quad (7)$$

$$d_3(x,y)=\max_i |x_i-y_i| \quad (8)$$

(the last one being sometimes named Tchebychev distance).

A typical example of Archimedean distance is the Euclidian distance in a Hilbert vector space:

$$d_E(\bar{x},\bar{y})=\sqrt{\sum_i (x_i-y_i)^2}=\sqrt{\langle \bar{x}-\bar{y}\rangle^2} \quad (9)$$

where $\bar{x}=(x_1,\dots,x_n)$, $\bar{y}=(y_1,\dots,y_n)$, n is the number of components of a vector (the dimension of the vector space). The absolute value of the difference distance, $d(x,y)=|x-y|$, and the Euclidean distance, $d(x,y)=\sqrt{\sum_{i=1}^{n}(x_i-y_i)^2}$, are among the most popular distances. More general than the Euclidean distance is the Minkowski distance, defined as

$$d_M(\bar{x},\bar{y})=\sqrt[q]{\sum_i (x_i-y_i)^q}\,. \quad (10)$$

The Euclidian distance is well exemplified on the space $\Re^n$, where $(\Re^n, \|\,\|, d^E)$ is a metric space.

Subsequently, we assume X is a normed vector space.

A quite natural condition for distances in vector spaces is the position (translation) invariant condition, which requires that a parallel displacement does not affect distances:

$$d(\bar{x}+\bar{a},y)=d(\bar{x},y) \quad (11)$$

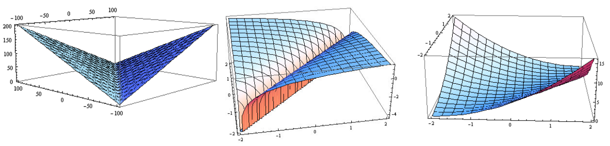
$$d(\bar{x},\bar{y}+\bar{b})=d(\bar{x},\bar{y}) \quad (12)$$

Also, the so-called homogeneity condition:

$$d(a\cdot\bar{x},a\cdot\bar{y})=|a|\cdot d(\bar{x},\bar{y})$$

While this set of conditions is well suited for Euclidean spaces and for vision in the plane, it is not justified in other cases, like hearing.

In Fig. 1, graphs of the elementary distances are exemplified.



(a) (b) (c)

Fig. 1. (a) The graph of the distance $d(x,y)=|x-y|$. (b) Graph of the Euclidean distance from the origin to a point in the plane, $d(x,y)=\sqrt{x^2+y^2}$. (c) Graph of the 'root of power 2" distance, $d(x,y)=\sqrt{|x^2-y^2|}$.

Instead of the conditions for distance functions, we suggest weaker conditions, namely, for $\delta: X\to\Re$,

i) $\delta(x,y)=\min_y \delta(x,y)\ne -\infty$ (13)

ii) $\delta(x,y)=\delta(y,x)$ (14)

iii) $\delta(x,y)<\delta(x,z)$ if $|x-y|<|x-z|$ (15)

We do not request that $\delta(x,y)=0$, but we request that $\delta(x,x)\ne-\infty$. A δ-distance will be named δ_0-distance if $\delta(x,y)=\min_y \delta(x,y)=0$.

Based on (iii) it follows that $\delta(x,y)=0\Rightarrow x=y$. The third condition, of strict monotony with respect to $|x-y|$, is a common-sense condition for proper

ISSN: 1790-5109

ISBN: 978-960-6766-85-5

discrimination. For example, a "distance" function with the graph as in Fig. 2 would not discriminate between A and B, moreover would classify C closer to D than to A.

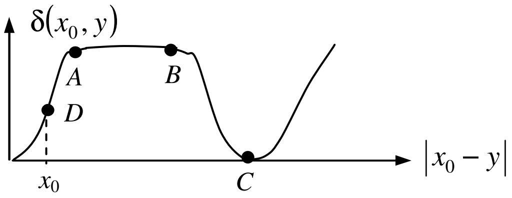


Fig. 2. Example of cases that can not be accepted from the common-sense point of view

A (very) strong condition is imposed in the definition of ultra-metrics:

$$d_u(x, y) \le \max\left(d(x, y), d(x, z)\right) \quad \forall x, y, z \tag{16}$$

3. Proto-distances

Consider a set X and a function $\Delta : X \times X \to \Re$ satisfying:

i) $\Delta(x, x) = \Delta(y, y) \quad \forall x, y \in X$ (diagonal $X \times X$ property);

ii) $\Delta(x, x) \ne \Delta(x, y) \quad \forall y \ne x$ (discrimination power property).

A proto-distance will be named symmetric if it satisfies:

iii) $\Delta(x, y) = \Delta(y, x) \quad \forall, y \in X$.

Condition (i) says that there is an element $o \in \Re$, unique, such that $\ker(\Delta - o) = \{(x, x) \in X \times X\}$, where $\Delta - o$ is the function obtained by subtracting from the function Δ the value o.

***Fact* 1**. Any distance is a proto-distance.

The verification of this fact is immediate.

***Fact* 2**. The function $\Delta^* = |\Delta - o| : X \times X \to \Re_+$ satisfies

i) $\Delta^*(x, x) = 0 \quad \forall x \in X$

ii) $\Delta^*(x, y) = 0 \Rightarrow x = y$

iii) $\Delta^*(x, y) = \Delta * (y, x)$

Notice that Δ^* is not a semi-metric, as it differs in two of the properties required for semi-metrics: $x \ne y \not\Rightarrow s_d(x, y)$ may equal 0; $s_d(x, z) \le s_d(x, y) + s_d(y, z)$ is not necessarily true (see, for example, the definition by Gaşpar, [1]).

Notes.

The definition of proto-distance from an object $x \in X$ to a set $A \subset X$ could mimic the definition of the distance from on object to a set:

$$\Delta(x, A) = \inf_{y \in A} \Delta(x, y).$$

However, with this definition

$$|\Delta(x, A)| \ne \Delta^*(x, A) + o \tag{17}$$

A more appropriate definition could be:

$$\Delta(x, A) = \inf_{y \in A} \left[|\Delta(x, y) - o| + |o| \right] \tag{18}$$

For the moment, we can not say if Δ^* induces a topology on X; the Archimedean condition $\left(d(x, z) \le d(x, y) + d(y, z)\right)$ plays an essential part in defining a topology through a metric, but this property is not satisfied by Δ^*.

3 Hybrid distances and two properties

Property. Any positive linear combination of (semi-) distances on the same set is a (semi-) distance.

Indeed, it is easy to verify that all the properties of (semi-)distances are satisfied by

$$d(x, y) = \sum_{i=1}^{r} a_i \cdot d_i(x, y) \tag{19}$$

if $d_i(x, y)$ are distances and $a_i \ge 0 \; \forall i = 1,.., r$. For example, the triangular (Archimedean) property reads:

For d_1, d_2 distances, let

$$(ad_1 + bd_2)(x, x) = ad_1(x, x) + bd_2(x, x) = 0 \; \forall x$$

$$(ad_1 + bd_2)(x, y) = 0 \Leftrightarrow ad_1(x, y) + bd_2(x, y) = 0$$

$$\Leftrightarrow d_1(, y) = 0 \;\; \& \;\; d_2(x, y) = 0$$

$$(ad_1 + bd_2)(x, z) \le (ad_1 + bd_2)(x, y) + (ad_1 + bd_2)(y, z) \Leftrightarrow$$

$$\Leftrightarrow ad_1(x, z) + bd_2(x, z) \le a\left[d_1(x, y) + d_1(y, z)\right] + $$

$$+ b\left[d_2(x, y) + d_2(y, z)\right]$$

$$d_1(x, z) \le d_1(x, y) + d_1(y, z) \; |\times a$$

$$d_2(x, z) \le d_2(x, y) + d_2(y, z) \; |\times b$$

The above property allows us to construct hybrid distances starting from simpler distances and using weighted averages of them. Moreover,

Property. If $d_i(x, y)$ are (semi-) distances on the same set, then $d(x, y) = \sqrt{\sum_{i=1}^{r} a_i^2 \cdot d_i^2(x, y)}$, is a (semi-) distance.

The above property can be extended in the form of q-root of a sum of q-powers of distances. Therefore, based on a set of elementary distances, we can build, using various strategies as in the above properties and comments, a large number of hybrid distances to suit specific applications. The process of weighting and the choice of the strategy can be made adaptive.

4 Bio-inspired Distances and Logarithmic Distances

Not every distance function mimics our senses. For example, the human two sounds separated by on octave, with frequencies $f_1, f_2 = 2f_1$ as being equally separated, irrespective of the basis of the octave: $\delta(f_1, 2f_1) = \delta(f_2, 2f_2) \quad \forall f_1, f_2$.

Taking logarithms, we obtain $\ln(2f) = \ln 2 + \ln f$, $\ln(2f) - \ln f = \ln 2$. Therefore, distances between logarithms should be considered to mimic the human ear. However, lower frequencies are more important in the discrimination of speech. A weighting through the inverse of the logarithm of the frequency is therefore justified:

$$\frac{\left|\ln f_2 - \ln f_1\right|}{\ln\left((f_1 + f_2)/2\right)}. \tag{20}$$

We start here with the absolute value distance. Taking the logarithm before applying the absolute value yields:

$$d_{L1}(x, y) = \left|\ln x - \ln y\right| = \left|\ln \frac{x}{y}\right| \tag{21}$$

Denoting $x = a\,y$ procedures

$$d_{L1}(x, y) = \ln a \tag{22}$$

Notice that d_{L1} is not a true distance in the classical sense, because, for a distance $d(x, y)$, $\ln d$ is no more a distance. However,

$$d_{Ld}(x, y) = \ln(1 + d(x, y)) \tag{23}$$

is a distance, in the sense of Definition 1, if $d(x, y)$ is a distance. Moreover, if $d(x, y) = |x - y|$, $x = a\,y$,

$$d_{Ld}(x, y) = \ln\left(1 + |a - 1| \cdot |x|\right) \tag{24}$$

and $d_{Ld}(x, y) \approx \ln|a - 1| + \ln|x|$ for $|a - 1| \cdot |x| >> 1$,

$$d_{Ld}(x, y) \xrightarrow{x \to y} 0 \tag{25}$$

$$d_{Ld}(x, y) \cong \frac{-1}{1 + |a - 1| \cdot |x|}|x| + 1 \text{ for } x \to y,\ a \to 1 \tag{26}$$

or

$$d_{Ld}(x, y) \cong \frac{-1}{1 + \left|\frac{x}{y} - 1\right|} + 1 \quad \text{for } \ x \to y \tag{27}$$

We may ask if and how the Archimedean property manifests for such a distance. The answer is that the property holds; in Fig. 3 is shown the graph corresponding to the property. More precisely, the graph of the Archimedean expression $f(y, z) = \log(1 + |y|) + \log(1 + |z - y|) - \log(1 + |z|)$, corresponding to the verification of the Archimedean inequality $d(0, z) \le d(0, y) + d(y, z)$ $(x = 0)$ is plot. The ranges of the variables are $0 \le y \le 100, 0 \le z \le 100$.

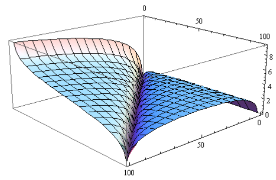


Fig. 3. Graph of the Archimedean property (see text)

Consider the distance

$$d_{L2}(x, y) = \frac{\ln|x - y|}{\ln\left|\frac{x + y}{2}\right|} = \frac{\ln|(a - 1)x|}{\ln|(a + 1)x|} = \frac{\ln|a - 1| + \ln|x|}{\ln|a + 1| + \ln|x| - \ln 2} \tag{28}$$

for

$$a \to 1\ (x \to y),\ d_{L2}(x, y) \to \frac{\ln|x|}{-\ln 2 + \ln|x|} \tag{29}$$

with the graph in Fig. 4. For the distance

$$d_{L1}(x, y) = \frac{\ln|x| - \ln|y|}{\ln|x| + \ln|y|} = \frac{\ln|a\,y| - \ln|y|}{\ln|a\,y| + \ln|yy|} = \frac{\ln|a| + \ln|y| - \ln|y|}{\ln|a| + \ln|y| + \ln|y|} = \frac{\ln|a|}{\ln|a| + 2\ln|y|} = f(a, y) \tag{30}$$

the graph is shown in Fig. 5.

Distances taking values on $[0, \infty)$ can be converted into distances on $[0,1)$ using a sigmoidal function, for instance $\sigma(x) = 2 \cdot \left(\frac{1}{1 + e^{-x}} - \frac{1}{2}\right)$.

We will denote by the index σ a distance transformed using a sigmoid, for example

$$d_{1\sigma}(x, y) = \sigma(d_1(x, y)) = \sigma(|x - y|) = \frac{2}{1 + e^{-|x - y|}} - 1 \tag{31}$$

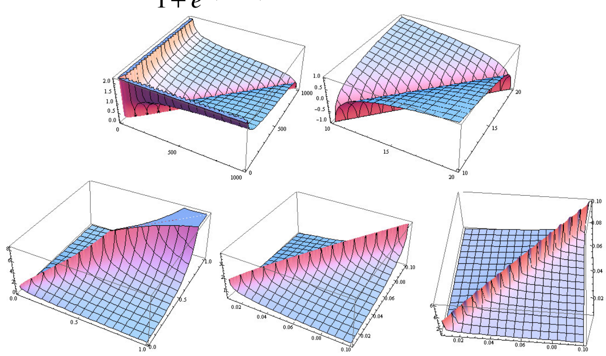


Fig. 4. Graph of the proto-distance $d_{L1}(x, y) = \ln|x - y| / \ln(|x + y| / 2)$, for various intervals of the variables: $x \in [1, 1000]$, $x \in [1, 1000]$, $y \in [1, 1000]$, $y \in [1, 1000]$, $x \in [0.01, 1]$, $y \in [0.01, 1]$, $x \in [0.01, 0.1]$, $y \in [0.01, 0.1]$.

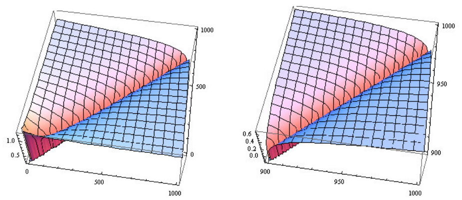


Fig. 5. The graph of $d_{L1}(x, y) = \ln|x - y| / \ln(|x + y|/2)$

Various hetero-distances can be conceived, tailored for specific applications; such a distance could be $d_{hetero-1}(x, y) = a \cdot d_{Camb}(x, y) + b \cdot d_3(x, y) + \cdots$.

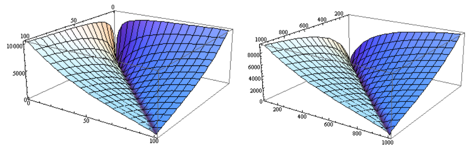


Fig. 6 . Graphs of a hetero-distance similar to the above, in the ranges $x \in [1,100]$, $y \in [1,100]$ and respectively $x \in [100,1000]$, $y \in [100,1000]$.

(Pseudo-) distances taking values on $(-\infty, +\infty)$ are converted to $(0,1)$ by the standard sigmoidal function, $\sigma_4(x) = 1/\left(1 + e^{-x}\right)$; for example

$$d_{L1\sigma}(x, y) = \sigma_s\left(\ln|x - y|\right) = \frac{1}{1 + e^{-\ln|x-y|}} = \\ = \frac{1}{1 + \frac{1}{|x - y|}} = \frac{|x - y|}{1 + |x - y|} \tag{32}$$

A logarithmic type distance using normalization to the minimum of the two values is:

$$d_2(x, y) = \frac{\ln|x - y|}{\ln(\min(x, y))} \tag{33}$$

The variation of this distance with the values of the variables is plot in Fig. , for several ranges.

The so-called Camberra distance (definition based on [5]) is a normalized distance, with the form:

$$d(x, y) = \frac{|x - y|}{|x + y|} \tag{34}$$

The plot for Camberra distance is shown in Fig. 7, for various ranges of the inputs.

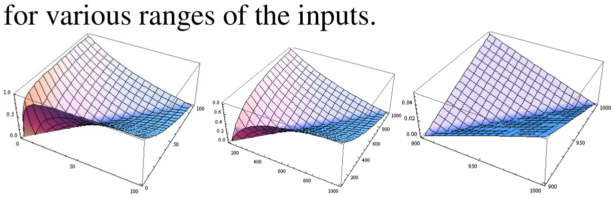


Fig. 7. Camberra distance behavior for a) $x \in [1,100]$, $y \in [1,100]$; b) $x \in [100,1000]$, $y \in [100,1000]$; and c) $x \in [900,1000]$, $y \in [900,1000]$

Another hybrid pseudo-distance, based on logarithms, is:

$$d_4(x, y) = \sqrt{\ln(1 + |x - y|) \cdot \ln(1 + |x + y|) + \left(\ln(1 + |x - y|)\right)^4}$$

$x \geq 0, y \geq 0$.

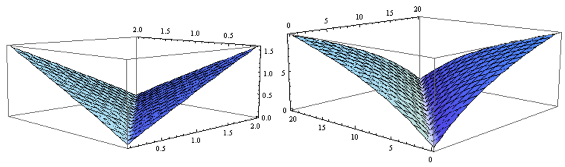


Fig. 8. Pseudo-distance d_4, for ranges a) $x \in [1,2], y \in [1,2]$ and b) $x \in [1,20], y \in [1,20]$

$$d_3(x, y) = \\ = \sqrt{\ln(1 + |x - y|) + \left(\ln(1 + |x - y|)\right)^2 + \left(\ln(1 + |x - y|)\right)^3}$$

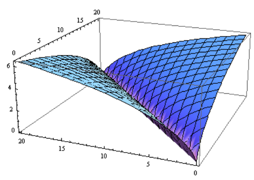


Fig. 9. Hybrid distance d_3, $d_3(x, y) = \\ = \sqrt{\ln(1 + |x - y|) + \left(\ln(1 + |x - y|)\right)^2 + \left(\ln(1 + |x - y|)\right)^3}$, for the range $x \in [1,20], y \in [1,20]$

4 Applications to Diagnosis in Medicine: Pedometry, Gnatho-phonics and Gnatho-sonics

Gnathic means “of or pertaining to the jaw” (see [16]). Gnatho-phonics is an interdisciplinary discipline we defined as the analysis of the speech in relation to the pathology of the gnathic system, that is, in relation with the state of dentition, mandible and temporo-mandibular joints.

The basic distance we defined and used is a normalized one, namely:

$$d(a,b) = \sum_{k=0}^{4} d_k = \sum_{k=0}^{4} \frac{\ln|F_{ka} - F_{kb}|}{\ln\left(\frac{F_{ka} + F_{kb}}{2}\right)} \tag{16}$$

where F_{ka} represents the formant number #k (k = 0 corresponds to the pitch); the second index (“a”) represents the sound. For each formant (k), a distance d_k is computed between the two sounds, and the total distance between the sounds is the sum of these elementary distances. When one of the sounds has no pitch, the elementary distance for that formant is assigned to 1. We found that the typical elementary distances d_k have values in the range 0.7 … 0.9. Values under 0.5 indicate that the possibility of discrimination between those sounds is poor at that frequency. Therefore, such values point to a poor spelling, that is, constitute a pathological sign. Similar uses have been made of the discussed hybrid and pseudo-distances in applications related to pedometry. I have used in this purpose the results obtained in the Grant CALORCO. The classification results have been encouraging and will be described in detail in another paper.

6 Discussion and conclusions

We have revisited the meaning of “distance” mathematical concept and the meaning of “discrimination”, as related to sensorial discrimination. We have noted that the distance concept is not always appropriate; therefor we introduced the proto-distance definition and briefly analyzed the properties of proto-distances. Next, we have defined several hybrid proto-distances and distances, in relation to their discrimination power. Applications in medicine have been briefly discussed.

Acknowledgments. The grants “CEEX – CALORCO” and “Program VIASAN - Proiect: Sistem automat de diagnostic paraclinic in sindromul disfunctional al sistemului stomatognat” have partly (about 30%) supported this research and fully supported the participation in the conference.

References

[1]. D. Gaşpar, Analiză funcţională, Ed. Facla, Timişoara, 1981, p. 20

[2]. Olga Costinescu, Elemente de topologie generală, Ed. Tehnică, Bucureşti, 1969, pp. 126-127

[3]. V. Istrăţescu, Introducere în teoria spaţiilor metrice probabiliste cu aplicaţii, Ed Tehnică, Bucureşti, 1974, pp. 13-14

[4]. L. Lusternik, V. Sobolev, Pécis d’analyse fonctionelle. Editions MIR, Moscow, 1989, pp. 27-28

[5]. D. Randall Wilson, Tony R. Martinez, Improved Heterogeneous Distance Functions. J. of Artificial Intelligence Research, 6 (1997) 1-34. (Also at http://axon.cs.byu.edu/~randy/ jair/wilson1.html#Contents)

[6]. D. Randall Wilson, Tony R. Martinez, Reduction Techniques for Instance-Based Learning Algorithms. Machine Learning, vol. 38, pp. 257-286, 2000.

[7]. D. Randall Wilson, Tony R. Martinez, Heterogeneous Radial Basis Function Networks. Proceedings of the International Conference on Neural Networks (ICNN’96), vol. 2, pp. 1263-1267, June 1996.

[8]. D. Randall Wilson, Tony R. Martinez, An Integrated Instance-Based Learning Algorithm. Computational Intelligence (Blackwell Publishers), Volume 16, Number 1, pp. 1-28, 2000

[9]. Andrea Frome, Yoram Singer, Jitendra Malik, Image Retrieval and Classification Using Local Distance Functions. http://www.cs.berkeley.edu/~afrome/papers/paper_localmetrics.pdf

[10]. Andrea Frome, Yoram Singer, Fei Sha, Jitendra Malik. "Learning Globally-Consistent Local Distance Functions for Shape-Based Image Retrieval and Classification". ICCV 2007.

[11]. Andrea Frome, Yoram Singer, Jitendra Malik. "Image Retrieval and Recognition Using Local Distance Functions". Proceedings of Neural Information Processing Systems (NIPS) 2006. pdf

[12]. Andrea Frome, Jitendra Malik. "Object Recognition using Locality Sensitive Hashing of Shape Contexts". Nearest-Neighbor Methods in Learning and Vision. Ed. Gregory Shakhnarovich, Trevor Darrell, and Piotr Indyk. MIT Press, 2006. pp. 221-247.

[13]. Andrea Frome, Daniel Huber, Ravi Kolluri, Thomas Bülow, and Jitendra Malik. Recognizing Objects in Range Data Using Regional Point Descriptors. Proceedings of European Conference on Computer Vision (ECCV), Prague, Czech Republic, 2004.

[14]. Jared Lundell, Dan Ventura, A Data-dependent Distance Measure for Transductive Instance-based Learning. Proc. IEEE Int. Conf. on Systems, Man and Cybernetics, pp. 2825–2830, Oct. 2007.

[15]. Peter Ladefoged, A Course in Phonetics. http://www.phonetics.ucla.edu/vowels/contents.html

[16]. Hutchinson Encyclopaedia. Helicon Publishing LTD 2008; see also www.dentaldictionary.net/words/27690-gnathion.html. Also, The American Heritage® Medical Dictionary © 2007, 2004 by Houghton Mifflin Company. http://medical-dictionary.thefreedictionary.com/gnathic

[17]. H.N. Teodorescu, Occlusal Sound Analysis Revisited. *Proc. 3rd Int. Conf. Advances in Medical, Signal and Information Processing*, *MEDSIP* 2006, IET, Glasgow, UK