Syntactic pattern model classification with total fuzzy grammars

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Abstract: - The paper aims at representing structural information in image patterns by fuzzy structural rules. Several methods of pattern classifications, mainly based on numerical similarity measures are intensively studied and successfully applied in real world applications [17]. In these work we address structural pattern recognition through a syntactic method, based on formal language theory. Structural pattern recognition mainly concerns on how a pattern can be described and interpreted as a composition of simple sub-patterns (pattern primitives). Lee and Zadeh (1969) early introduced fuzzy grammars which deal with membership degrees associated to production rules. Since pattern classification assumes finding the right generative grammar for each pattern class, we introduce an extension operator and fuzzy facilities for nonterminal symbols of a grammar. Assessing and updating of membership information for each new (learned) pattern increases the recognition power.

Key-Words: - patterns, classification, object recognition, total fuzzy grammar, syntactic model, language, membership degree

1 Introduction

Several methods of pattern classifications, mainly based on numerical similarity measures are intensively studied and successfully put into practice [17]. But there also exist a completely different approach, known as structural pattern recognition. Its main concern is how a pattern can be described and interpreted as a composition of simple sub-patterns (pattern primitives).

In this paper we address the later one, namely structural pattern recognition through a syntactic method, based on formal language theory. For describing and analysing pattern structures in digital images, they are decomposed into simple elements called primitives.

Since we humans can sense and interpret the imprecise and vague visual characteristics of the world around us, we may conclude that our vision system is mainly fuzzy. In 1965 L. A. Zadeh introduced the fuzzy set theory which was a milestone for the area of visual and cognitive science. It provides a proper framework for dealing with vague and uncertain events.

As stated in [18] the imprecision in image patterns is derived from several factors including: ambiguity in gray levels of an image (describing whether a pixel is bright or dark), spatial ambiguity (imprecision in object boundaries or edges), imprecision in knowledge base (scene description, object recognition, region segmentation), and several combinations of these. The membership function is useful in modelling and quantifying several imprecise linguistic or ambiguous terms.

Fuzzy set and methods have been successfully been integrated into primitive extraction, production rules, collection of data from uncertain sources and parsing activities carried out during classification [10],[14],[17].

In section 2 we outline our approach that aims to apply recently developed methods [7],[5],[2],[1] as extension of well approved formal mechanisms.

Section 3 recalls some basic notation and concepts from fuzzy logics, although we assume familiarity with formal language theory.

Further we present the concept of fuzzy grammar combined with an extension operator. Extending the membership function to nonterminals establish a relationship between the syntactic analyze of a word from the language and his degree of membership to the language. The membership degree of a nonterminal is not a constant and depends on the derivation in which it is involved. How to choose the appropriate membership function for is out of the scope of this paper and will not be addressed here.

Conclusions and future work (Section 6) close the paper.
2 Approach

Classification in the perspective of formal language theory, means inferring a grammar and a language that captures all the objects we want to recognize. A grammar is associated to each class, and sentences are recognized as belonging to a given class if they can be generated/parsed in the grammar for that class. Usually all class grammars share the same primitives, so that all grammars can generate the sentence. Recognising piecewise an object then reduces to parsing a sentence to see if it can be generated by the grammar for that object class.

Syntactic pattern recognition applications [10],[13],[17] deal with shape recognition. The first task is to convert shape data into the string or tree that represents the shape formally. After that, all the well approved techniques of formal language theory, regarding parsing and generation of sequences, hold and can be used.

In the particular case of context free grammars (which are our concern) a tree type structural representation of class grammars and derivation chains eases up the automated membership degree mapping update and consequently the parsing process.

Our goal is to present a formal model that suites the classification process of object patterns. Implementation results will be issued in future works, as this research study is in an early stage of development.

3 Revisiting Basic Concepts

Some basic concepts and notations are revisited here before tackling the target concepts of the formal specification system.

Let $X$ be a collection of objects called universal set. Let $A$ be a regular (also called crisp) subset of $X$. For each $x$ in $X$ the characteristic function $f_A : X \to \{0,1\}$ determines whether $x$ belongs or does not belong to $A$. Any such function, whose values are either zero or one, defines a crisp subset of $X$.

A fuzzy set $A$ over a universe of discourse $X$ (a finite or infinite interval within which the fuzzy set can take a value) is a set of pairs $A = \{ \mu_A(x)/ x \mid x \in X, \mu_A(x) \in [0,1] \subseteq \mathbb{R} \}$ where $\mu_A(x)$ is called the membership degree of the element $x$ to the fuzzy set $A$. This degree ranges between the extremes 0 and 1 of the dominion of the real numbers.

Grades of membership for universes other than the real numbers are normally calculated by the firing of rules, estimated by an observer or some other method, rather than by membership functions.

Members of a numeric discrete fuzzy set always describe a numeric quantity. Such discrete fuzzy sets are called linguistic variables, with members linguistic terms. For instance, the set of all bright pixels in an image the intensity of a pixel is qualified as bright. In this case, pixel intensity is a linguistic variable, because it can take values like dark, bright and so on. Consequently, this means that we can deal further on with non precise values.

A linguistic term is the word, in natural language, that expresses or identifies a fuzzy set that may or may not be formally defined. Thus, the membership function $\mu_A$ of a fuzzy set $A$ expresses the degree in which $x$ verifies the category specified by $A$.

The theory of fuzzy sets generalizes the theory of classic sets, this means that the fuzzy sets allow operations of union, intersection, and complement. These and other operations can be found in Error! Reference source not found., Error! Reference source not found.

Let $A$ and $B$ be two fuzzy sets over $X$. Then $A$ is equal to $B$ if $\forall x \in X$, $\mu_A(x) = \mu_B(x)$, denoted by $A = B$.

Taking two fuzzy sets $A$ and $B$ over $X$, $A$ is said to be included in $B$ if $\forall x \in X$, $\mu_A(x) \leq \mu_B(x)$, denoted by $A \subseteq B$.

A Triangular Norm ($t$-norm) is a binary operation, $T : [0,1] \times [0,1] \to [0,1]$ that complies with the following properties:

a) Commutativity: $T(x,y) = T(y,x)$.

b) Associativity: $T(x,T(y,z)) = T(T(x,y),z)$.

c) Monotonicity: If $x \leq y$, and $w \leq z$ then $x T w \leq y T z$.

d) Boundary conditions: $T(x,0) = 0$ and $T(x,1) = x$.

A Triangular Conorm ($t$-conorm) is a binary operation, $\sqcup : [0,1] \times [0,1] \to [0,1]$ that complies with the following properties:

a) Commutativity: $\sqcup(x,y) = \sqcup(y,x)$.

b) Associativity: $\sqcup(x,\sqcup(y,z)) = \sqcup(\sqcup(x,y),z)$.

c) Monotonicity: If $x \leq y$, and $w \leq z$ then $x \sqcup w \leq y \sqcup z$.

d) Boundary conditions: $\sqcup(x,0) = x$ and $\sqcup(x,1) = 1$.

Since the $t$-norm of the Minimum and the $t$-conorm of the Maximum are most widely used of this type of functions, we consider in this paper the following form of the relations:

$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$, $x \in X$
4 Total fuzzy grammar and language

Fuzzy grammars Error! Reference source not found. ore linguistic rules are used to describe the syntax of languages or the structural relation of patterns. They can also be used to characterize a syntactic source which generates all the sentences in a language [9].

Let V be a countable, nonempty fuzzy subset of the universe of discourse X, named fuzzy alphabet. A fuzzy symbol is an element of the set V and a word over the alphabet V is a sequence of fuzzy symbols \( p = a_1a_2...a_n \), \( a_i \in V \), \( i = 1, n \).

The fuzzy alphabet or vocabulary \( V \) is defined through a membership function who maps to any \( x \in X \) the degree of membership \( \mu \) between \( [0, 1] \), \( \mu : X \rightarrow [0, 1] \).

In the particular case when \( \mu \) takes only values of 0 an 1, the fuzzy vocabulary \( V \) represents a classical subset of \( X \).

We introduce in [1] an extension operator defined as: \( E : [0, 1] \times [0, 1] \rightarrow [0, 1] \) that satisfy for \( \forall x, y, z \in [0, 1] \) the following properties:

a) Commutativity: \( E(x, y) = E(y, x) \).
b) Associativity: \( E(x, E(y, z)) = E(E(x, y), z) \).
c) Monotonicity: If \( x \leq y \), and \( w \leq z \) then \( xEy \leq yEz \).
d) Boundary conditions: \( E(x, 0) = 0 \) and \( E(x, 1) = x \).

If c) and d) are satisfied then \( E \) is a t-norm, or if the following properties hold
e) Monotonicity: If \( x \leq y \), and \( w \leq z \) then \( x \perp w \leq y \perp z \).
f) Boundary conditions: \( \perp (x, 0) = x \) and \( \perp (x, 1) = 1 \). then \( E \) is a t-conorm.

Consequently t-norms and t-conorms are extension operators.

For \( V \) a fuzzy alphabet over de universe of discourse \( X \), defined through the membership function \( \mu : X \rightarrow [0, 1] \), and the extension operator \( E : [0, 1] \times [0, 1] \rightarrow [0, 1] \). Then the membership function \( \mu : X \rightarrow [0, 1] \), for any \( L \subseteq X \), is defined as follows:

- \( \mu_L(a) = \mu(a) \), if \( a \in V \);
- \( \mu_L(pa) = E(\mu_L(p), \mu_L(a)) \), if \( p \in X^* \), \( a \in V \).

A fuzzy language over the fuzzy alphabet \( V \) is then fuzzy subset \( L \subseteq V^* \) defined through the membership function \( \mu_L : X^* \rightarrow [0, 1] \). In these case \( \mu_L(pq) = E(\mu_L(p), \mu_L(q)) \), \( p, q \in X^* \).

Since fuzzy languages are fuzzy sets all operations on fuzzy sets still hold.

A total fuzzy grammar is defined in [1] as:

\[ FG = (V_N, V_T, S, P, \mu_V, E) \]

where:

- \( V_N \) is the set of nonterminal symbols;
- \( V_T \) is the alphabet of terminal symbols, \( V_N \cap V_T = \emptyset \);
- \( S \in V_N \) is the starting symbol;
- \( P \) is a finite set of production rules of the form \( \alpha \rightarrow \beta \), where \( \alpha, \beta \in (V_N \cup V_T)^* \).
- \( \mu_V : X \rightarrow [0, 1] \) is the membership function that specifies the fuzzy alphabet \( V_T \);
- \( E : [0, 1] \times [0, 1] \rightarrow [0, 1] \) is the extension operator.

The language generated by the total fuzzy grammar \( FG = (V_N, V_T, S, P, \mu_V, E) \) is the fuzzy subset

\[ L(FG) = \{ p | p \in V_T^* \text{ and } S \Rightarrow p, \mu_V(p) > 0 \} \]

where \( \mu_V : X^* \rightarrow [0, 1] \) is the extension of \( \mu_V \).

5 Extending the membership function over nonterminals

In the following we show an extension of the membership function for nonterminals to, and establish a relationship between the syntactic analyze of a word from the language and his degree of membership to the language. The membership degree of a nonterminal is not a constant and depends on the derivation in which it is involved. Thus, for each nonterminal we will have a finite or infinite set of possible membership degrees in extend to the derivation implied in.

We denote \( V = V_T \cup V_N \) and extend the membership function \( \mu_V : X \rightarrow [0, 1] \) to the function \( \mu_V : X \rightarrow \mathcal{P}([0, 1]) \), where \( \mathcal{P}([0, 1]) \) is the set of partial subsets of the real interval \([0, 1]\), as follows:

a) If \( x \rightarrow r \in P \), than we have \( r = v_0v_1...v_n \), where \( v_0, v_1,..., v_n \in V_T \) and can define \( \mu_V(x) = \mu_V(v_0v_1...v_n) \).

b) If \( x \rightarrow r \in P \), than \( r, v_0xv_1...v_n \in V_T^* \), then \( x \rightarrow v_0v_1...v_n \), where \( v_0, v_1,..., v_n \in V_T \) and \( x, x_2,..., x_n \in V_N \). Thus, \( \mu_V(x) \) can be defined only if \( \mu_V(x_1), \mu_V(x_2),...\mu_V(x_n) \) are already defined.

Finally, the membership function \( \mu_V(x) \) extends recursively to the function \( \mu_V(x) \) as
follows:

a) If \( x \in V_f \) then \( \mu_\nu (x) = \mu_{V_f} (x) \).

b) If \( x \in V_N \) then
\[
\mu_\nu = \mu_\nu (x_1), \mu_\nu (x_2), \ldots, \mu_\nu (x_n) \quad \text{for al productions of from } x \rightarrow x_1, x_2, \ldots, x_n.
\]

In any type context free grammar with no useless symbols, for a nonterminal \( x \) exists at least one derivation of the form \( x \Rightarrow r \) where \( r \in V_f^- \). On the other hand, if there exists a derivation of the form \( x \Rightarrow r, r \in V_f^- \) then the membership degree \( \mu_\nu (x) \) can be calculated for the nonterminal \( x \).

The nonterminal membership degree calculus algorithm has as input a context free fuzzy grammar and produces as output the membership function \( \mu_{V_f} \). The method consists of constructing a series of sets \( \{K_n | n \in \mathbb{N}\} \), having the property \( K_0 \subseteq K_1 \subseteq \ldots \subseteq K_{n+1} \subseteq K_{n} \), \( K_{n} \) will contain all nonterminal symbols whose membership degrees have already been calculated.

We notice that a nonterminal may have more then one membership degree, this means each \( \alpha \)-production \( (\alpha \rightarrow \beta, \alpha \in V_f^+, \beta \in (V_f^+ \cup V_f^-) \) has another degree which is considered as membership degree of the production. In other words, every \( \alpha \)-production has his membership degree.

6. Conclusions and future work

To build classifiers [6],[7] capable to identify different objects using this approach we can outline the following general procedure. For instance, let’s consider as basic objects handwriting symbols. After digitalizing and eventually threshold operations, the objects have to bee thinned to obtain there skeletons. Then decompose them into sentences in over ore more languages over there grammar. Once particular objects are acquired, we will look for the production rules that generate correct representations of the objects in at least one of the chosen ore inferred grammars. One set of primitives may support many grammars, finding the primitives, the productions and the grammars can be considered as training procedure. Once the languages that cover the objects of interest are known, the process will be turned around to suite the classifier operation. A new object is firstly converted into a sentence using the existing primitives, and then parsed according to the available grammars. Matches represent class labels for the object.

Since the research goes on and our current result look quit promising, we have to admit that albeit the process is time and resource hungry. But in the spirit of Moor’s law, that issue shouldn’t be a major drawback.

Viewing computational linguistics as one main goal of linguistic interpretability by formal models, opens research interests in finding rule sets that suites more natural language text then fuzzy systems. Such approach assumes mapping sentences to semantic meanings in form of syntax driven semantic analyze to fuzzy level. After parsing a natural language text, the computed fuzzy set models the linguistic expression and it may be possible to reason about the degree of membership to a preliminary stored result.

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