A New Approach in Obtaining Transfer Function for a Large-Scale Linear Network

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Abstract: – A new approach in obtaining transfer function for a large-scale linear network is presented in this paper. The approach consists of the application of symbolic network analysis, Z-equivalent representation and a new proposed technique called Network-Impedance Relationship Equation (NIRE). In this approach, only the network’s impedances are required in obtaining its transfer function. By doing this, voltage nodes, current loops or simultaneous equations can be avoided which will become more complicated to be solved when the network becomes larger. An algorithm using software called MATLAB has been developed to verify the efficiency of using this new approach and technique. It can be adapted in a computer-based application and also can be introduced as an alternative method in obtaining transfer function for educational purpose.

Key-Words: – Transfer function, impedance, circuit, large-scale, linear, symbolic circuit analysis.

1 Introduction

Transfer function is a mathematical representation, in terms of spatial or temporal frequency, of the relation between the input and output of a linear, time-invariant (LTI) system. It is commonly used in the analysis of single-input single-output electronic network system where it is mainly used in signal processing, communication theory, and control theory. In its simplest form for continuous-time input signal \( x(t) \) and output \( y(t) \), the transfer function is the linear mapping of the Laplace transform of the input, \( X(s) \), to the output \( Y(s) \):

\[
H(s) = \frac{Y(s)}{X(s)}
\]

Since the input and output can be either voltage or current at any place in the circuit, there are four possible transfer functions [2]:

- Voltage Gain (VG): \( \frac{V_o(s)}{V_i(s)} \)
- Current Gain (IG): \( \frac{I_o(s)}{I_i(s)} \)
- Transfer Impedance (TZ): \( \frac{V_o(s)}{I_i(s)} \)
- Transfer Admittance (TY): \( \frac{I_o(s)}{V_i(s)} \)

The most common method used in obtaining the transfer function is nodal and mesh analysis. There are also some alternative method such as Modified Nodal Analysis [9], and Extra Element Theorem which has been extended with two and more extra element [6][7][8]. Even though these method seems to reduce problems regarding to nodes or loops increment, but the simultaneous equations involved still can’t be avoided. This new approach and technique will eliminate the nodes, loops and simultaneous equations and replace them with new relationship equations.
2 The Methodology
The main developments in symbolic network analysis have been in the realm of the frequency domain [1]. Symbolic analysis is a method to calculate the behavior or characteristic of a network represented by symbols. The method is complementary to numerical analysis where the variables and the circuit elements are represented by numbers [5]. The goal of such an analysis is the generation of a fully or partially symbolic transfer function that can be expressed as:

\[ H(s, X) = \frac{N(s, X)}{D(s, X)} \]

where \( N(s, X) \) and \( D(s, X) \) are numerator and denominator polynomials in s-domain and the symbolic network variables \( X \).

In the new approach, the network components are transformed into its symbolic Z-equivalent rather than to transform it into s-domain representation. There are two required parameters that has to be expressed in their numerator and denominator form, which is total input impedance, \( Z_T \) and output impedance, \( Z_0 \).

\[ Z_T = \frac{N_T}{D_T} \quad \text{and} \quad Z_0 = \frac{N_0}{D_0} \]

In order to implement this approach, all symbolic equations and terms must be in the form of ‘sum of product of the impedances’, where product of the impedances is called ‘element’ afterwards. For example, a symbolic equation \( Z_1Z_2 + Z_2Z_3 + Z_1^2 \) is said to be in the form of ‘sum of product of the impedances’ and the elements in this equation are \( Z_1Z_2 \), \( Z_2Z_3 \) and \( Z_1^2 \).

Since the transfer function can be divided into four types, each of the type has its own unique NIRE technique steps to be applied to it. Due to this, the desired transfer function’s type needs to be defined first before applying the NIRE technique. Those transfer function types are expressed by their numerator and denominator form:

\[ \begin{align*}
VG &= \frac{N_{VG}}{D_{VG}} \quad & IG &= \frac{N_{IG}}{D_{IG}} \\
TZ &= \frac{N_{TZ}}{D_{TZ}} \quad & TY &= \frac{N_{TY}}{D_{TY}}
\end{align*} \]

After applying the NIRE technique, it results in a symbolic Z-equivalent transfer function which can now be transformed into its s-domain representation and to be inserted with values (if any). The methodology can be summarized as in the flowchart in Figure 1.

Figure 1

2.1 The NIRE Technique
NIRE is an acronym for Network-Impedance Relationship Equation. NIRE has 5 steps which reflect the number of relationship equation it has to determine. These steps are taken regardless the size of the network.

The first step in this technique is the determination of the transfer function’s
denominator part. This first relationship equation is simply taken from the numerator or denominator part of $Z_T$.

\[ D_{YG} = N_T \quad D_{IG} = D_T \]
\[ D_{TZ} = D_T \quad D_{TY} = N_T \]

The second step and the steps afterwards are for finding the transfer function’s numerator part. The second relationship equation is called product equation where the remaining parts of numerator or denominator from the first relationship equation are multiplied with the output impedance.

\[ P_{YG} = D_T \times Z_0 \quad P_{IG} = N_T \times \frac{1}{Z_0} \]
\[ P_{TZ} = N_T \times \frac{1}{Z_0} \quad P_{TY} = D_T \times Z_0 \]

For the third step, every single element from the second relationship equation will be compared with elements from the first relationship equation that has been obtained. After that, the matched elements will be summed up. In mathematical expression, the product equation, $P$ and the denominator equation, $D$ can be written as:

\[ P = \sum_{i=1}^{j} p_i \quad \text{and} \quad D = \sum_{i=1}^{k} d_i \]

where $p_i$ are product’s elements and $d_i$ are denominator’s elements. Both of these elements can be grouped into sets, namely $E_P$ and $E_D$.

\[ E_P = \{p_1, p_2, \ldots, p_j\} \]
\[ \text{and} \quad E_D = \{d_1, d_2, \ldots, d_k\} \]

The intersection of set $E_P$ and set $E_D$ results in another set that contains the matched elements of those two sets, namely $M$.

\[ M = E_P \cap E_D = \{m_1, m_2, \ldots, m_n\} \]

A function that sums up all the elements in $M$ is introduced and named as ‘Sum of Matched Elements (SoME)’. This function makes up the third relationship equation and can be defined as:

\[ f_{SoME}(P, D) = \sum_{i=1}^{n} m_i \]

For all the four possible transfer functions, each third relationship equation can be written as:

\[ N_{VG} = f_{SoME}(P_{VG}, D_{VG}) \]
\[ N_{IG} = f_{SoME}(P_{IG}, D_{IG}) \]
\[ N_{TZ} = f_{SoME}(P_{TZ}, D_{TZ}) \]
\[ N_{TY} = f_{SoME}(P_{TY}, D_{TY}) \]

The fourth step is an optional depending on the type of transfer function required. If the type of the transfer function is a transfer impedance or transfer admittance, this step needs to be taken, but if it is a voltage gain or current gain type, this step can be skipped. When the step is taken, the result from the third relationship equation needs to be multiplied again with the output impedance. This fourth relationship equation can be written as:

\[ N_{TZ} = N_{TZ} \times Z_0 \quad N_{TY} = N_{TY} \times \frac{1}{Z_0} \]

Lastly, for the fifth step, the denominator result obtained from the first relationship equation is combined with the numerator result obtained from the fourth relationship equation to form the complete transfer function. All the steps taken in NIRE technique is summarized as shown in Table 1.
Table 1: The NIRE technique steps

<table>
<thead>
<tr>
<th>Steps</th>
<th>Description</th>
<th>Relationship Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Denominator equation</td>
<td>[D_{\text{VG}} = N_T]</td>
</tr>
<tr>
<td>2</td>
<td>Product equation</td>
<td>[P_{\text{IG}} = N_T \times \frac{1}{Z_0}]</td>
</tr>
<tr>
<td>3</td>
<td>Sum of matched elements</td>
<td>[N_{\text{IG}} = f_{\text{MSA}}(P_{\text{IG}}, D_{\text{VG}})]</td>
</tr>
<tr>
<td>4</td>
<td>Multiply with output impedance</td>
<td>[N_{\text{T1}} = N_{\text{IG}} \times Z_0]</td>
</tr>
<tr>
<td>5</td>
<td>Combine numerator and denominator</td>
<td>[V_G = \frac{N_{\text{T1}}}{D_{\text{VG}}}] [I_G = \frac{N_{\text{IG}}}{D_{\text{IG}}}] [I_L = \frac{N_{\text{T2}}}{D_{\text{T2}}}] [I_Y = \frac{N_{\text{T1}}}{D_{\text{T1}}}]</td>
</tr>
</tbody>
</table>

3 Results & Discussion
The results for any transfer functions either by using the new NIRE technique or the normal conventional methods are exactly the same. The difference is in the number of equation need to be solved in order to obtain the desired transfer function. The following example will show the number of equation needs to be solved when applying both methods.

Figure 2 shows some resistive ladder networks with size of network, \(n\) is increased in orderly. In this example the desired transfer function is the voltage gain. The number of equations involved for each size of network was computed and transformed in graphical illustration as shown in Figure 3.

The results clearly show that the number of equation increases as \(n\) increases when applying the nodal/mesh analysis. Meanwhile, the number of equation remains the same at five with the new technique. Those five equations are \(Z_T\) equation, denominator equation, product equation, SoME equation and combination equation.
4 Conclusion

By applying this new NIRE technique, the determination of transfer function is less complicated compared to nodal or mesh analysis method due to the avoidance of nodes, loops or simultaneous equations. The number of equation needs to be solved has been fixed and this is a major advantage when the network size is large.

The only drawback in this technique is that the number of elements in $Z_T$ and the symbolic transfer function will increase when the network becomes larger, which can be a tedious work when they are hand-calculated or manually obtained. However, the determination of $Z_T$ is done only one time for a particular network, and the increased of elements can be overcome by applying the technique in a computer-based algorithm programming such as MATLAB, for example.

This new approach and technique can also be a good method of circuit analysis and is very efficient when used as a computer-based application.

Reference:


