Optimal and MPC Control of the Quanser Flexible Link Experiment

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Abstract: - In this paper we developed a control scheme starting from Bond Graph representation of the Quanser Flexible link experiment. The objective of the experiment is to design a control system that positions the flexible arm as well as regulating vibration. The structure of the control scheme uses two types of control methods. In order to achieve a faster or robust response together with optimal control, we computed two model predictive controllers. LQR control and MPC control are based on the optimization of the cost function, except that MPC control handles constraints in the optimization.

Key-Words: - Flexible link, Bond Graph, Optimal control, MPC control

1 Introduction
System models may be constructed using Bond Graph method. In order to model and to simulate the behaviour of Quanser Flexible link experiment. System models will be constructed using a uniform notation for all types of physical systems which is bond graph method based on energy and information flow [1]. The method uses the effort-flow analogy to describe physical processes. A bond graph consists of subsystems linked together by lines representing power bonds. The objective in optimal design is to select the state variable feedback $K$ that minimizes the performance index $J$. The performance index $J$ can be interpreted as an energy function, so that making it small keeps small the total energy of the closed-loop system. The methods of model based predictive control have been widely presented and discussed in literature [2], [3], [4]. The predictive controller performs the set point adjustment for the underlying control loops in order to drive the process variables at desired set points or to maintain process variables within constraints.

2 Quanser flexible link experiment
The Flexible link module includes a SRV02 servomotor in the high gear ratio configuration. The SRV02 rotary plant module serves as the base component for the rotary family of experiments. Its modularity facilitates the change from one experimental setup to another and it consists of a DC motor in a solid aluminium frame equipped with a gearbox whose output drives external gears. The basic unit is equipped with a potentiometer to measure the output/load angular position.

Fig. 1. SRV02 plant – DC motor and gear box
The external gear can be reconfigured in two configurations:
- Low Gear Ratio - this is the recommended configuration to perform the position and speed control experiments with no other module attached to the output. The only loads that are recommended for this configuration are the bar and circular loads supplied with the system;
- High Gear Ratio - this is the recommended configuration for all other experiments that require an additional module such as the flexible beam, ball and beam, gyro, rotary inverted pendulum etc.

The servomotor is arranged for the high gear ratio as shown in Fig. 2

Fig. 2. High gear configuration
The Rotary Flexible Link module is designed as an attachment to the SRV02 plant. The model is designed to accentuate the effects of flexible links in robot control systems.

![Fig. 3. Quanser Flexible Link system](image)

The Flexible Link experiment is composed of a mechanical and electrical subsystem. The objective of the experiment is to design a control system that positions the flexible arm as well as regulating vibration.

The electrical subsystem involves modelling a DC servomotor that dynamically relates voltage to torque. The expression for the output torque in time domain is:

$$ T_L(t) = \eta K_m K_g R_a v_i(t) - \frac{K_m^2 K_g^2}{R_a} \dot{\theta}(t) $$

where: $R_a$ is the armature resistance, $K_m$ is the motor voltage constant, $K_g$ is motor torque constant, $v_i$ is the input voltage and $\theta$ is the angular position of the shaft.

The modelling of the mechanical model is used to describe the tip deflection and the base rotation for the experiment. The Flexible Link module consists of a flat flexible arm at the end of which is a hinged potentiometer. The flexible arm is mounted to the hinge. Measurement of the flexible arm deflection $\alpha(t)$ is obtained using a strain gage at the motor end of the link. For the measurement of the angular position of the shaft $\theta(t)$ an optical encoder attached to the shaft of the DC motor is used. Consider the Flexible Link schematic shown in Fig. 4.

![Fig. 4. Schematic representation of the Flexible link](image)

As it can be seen from the figure above the parameters of the module are as follows: $\theta$ servo gear angular displacement, $\omega$ servo gear angular velocity, $\alpha$ link angular deflection, $v$ link angular velocity, $\gamma$ total deflection.

The equations of motion involving a rotary flexible link, involves modelling the rotational base and the flexible link as rigid bodies. As a simplification to the partial differential equation describing the motion of a flexible link, a lumped single degree of freedom approximation is used. We first start the derivation of the dynamic model by computing link’s moment of inertia:

$$ J_{arm} = \frac{1}{3} m L^2 $$

where $m$ is the total mass of the flexible link, and $L$ is the total flexible link length.

For a single degree of freedom system, the natural frequency is related with torsional spring stiffness and rotational inertia in the following manner

$$ \omega_n = \sqrt{\frac{K_{stiff}}{J_{arm}}} $$

where $\omega_n$ is link’s damped natural frequency found experimentally and $K_{stiff}$ is the equivalent torsion spring constant.

Having computed the link’s moment of inertia, the torque due to the link acceleration is:

$$ T_{J_{arm}} = J_{arm} (\dot{\omega} + \ddot{\omega}) $$

The link torque due to torsional spring stiffness is assumed to be proportional to the link’s deflection

$$ T_{K_{stiff}} = K_{stiff} \alpha $$

$$ T_{J_{arm}} + T_{K_{stiff}} = 0 $$

or

$$ J_{arm} (\dot{\omega} + \ddot{\omega}) + K_{stiff} \alpha = 0 $$

The servomotor output torque is given by the following relation:

$$ J_{eq} \dot{\omega} + B_{eq} \omega + J_{arm} (\dot{\omega} + \ddot{\omega}) = T_L $$

where $J_{eq}$ is equivalent moment of inertia and $B_{eq}$ is the equivalent viscous friction referred to the secondary gear.
Using the expression of the servomotor output torque computed from the servomotor model and considering for the combined servomotor and the flexible link module the state variables 

\[
\begin{bmatrix}
\theta \\
\alpha \\
\omega \\
\nu
\end{bmatrix}
\]

we obtain the following state-space model:

\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\alpha} \\
\dot{\omega} \\
\dot{\nu}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & K_{\text{eff}} & 0 & 0 \\
J_{\text{eq}} & 0 & -\eta K_{\text{eq}}^2 K_{\text{eq}}^2 + B_{\text{eq}} R_{\text{eq}} \\
J_{\text{eq}}(J_{\text{eq}} + J_{\text{arm}}) & -\eta K_{\text{eq}}^2 K_{\text{eq}}^2 + B_{\text{eq}} R_{\text{eq}} & 0
\end{bmatrix}
\begin{bmatrix}
\theta \\
\alpha \\
\omega \\
\nu
\end{bmatrix}
\]

where \( J_{\text{eq}} \) is the armature inertia.

3 Bond Graph modelling of the system

System models may be constructed using Bond Graph method. In order to model and to simulate the behaviour of Quanser Flexible link experiment we will use 20-sim modelling and simulation environment (20-sim is a registered trademark of Controllab Products B.V. (2005) Enschede, Netherlands).

System models will be constructed using a uniform notation for all types of physical systems which is bond graph method based on energy and information flow. The method uses the effort-flow analogy to describe physical processes. A bond graph consists of subsystems linked together by lines representing power bonds. Each process is described by a pair of variables, effort (e) and flow (f), and their product is the power. In a dynamic system the effort and the flow variables, and hence the power fluctuate in time. Using the bond graph approach it is possible to develop models of electrical, mechanical, magnetic, hydraulic, pneumatic, thermal, and other systems using a small set of variables and these models can be express using a set of only nine elements, called elementary components. These elements are sufficient to describe any physical system regardless of the energy types processed by it. The elements used to model each system are: inertial elements (I), capacitive elements (C), resistive elements (R), effort sources (SE) and flow sources (SF), transformer elements (TF) and gyrator elements (GY), effort junctions (J0) and flow junctions (J1).

We started the modelling process by joining together the DC motor and the gear and modelling it as a system itself. We use a GY element to describe the electromechanical conversion in the motor relating the back emf from the electrical part to the angular velocity of the rotor from the mechanical part, respectively the armature current from the electrical part to the torque acting on the rotor. For this reason, the gyrators are called overcrossed transformers.

The electrical process in the armature is described in bond graph terms by the armature resistance \( R_{\text{a}} \) represented using a resistive element (R), and the armature inductance \( L_{\text{a}} \) represented using an inertial element (I). These two elements are joined through an effort junction (1 junction). The mechanical process is also described using an inertial element that models the equivalent moment of inertia \( J_{\text{eq}} \), and a resistive element that models the equivalent viscous friction \( B_{\text{eq}} \). These two elements are joined through an effort junction (1 junction) [5].

In order to model the component Flexible link it is necessary to introduce a capacitive element (C) to model the torsional spring stiffness and an inertial element (I) that models the link’s moment of inertia. A simplified model of the Flexible link experiment is presented in Fig.5.

![Fig. 5. Simplified Bond Graph model of the system](image)

4 Optimal control vs. MPC control

4.1 Linear Quadratic Regulator (LQR) State Feedback Design

A system can be expressed in state variable form as

\[
\dot{x} = Ax + Bu
\]
with \(x(t) \in \mathbb{R}^n\), \(u(t) \in \mathbb{R}^m\). The initial condition is \(x(0)\). We assume here that all the states are measurable and seek to find a state-variable feedback (SVFB) control \(u = -Kx + v\) that gives desirable closed-loop properties. The closed-loop system using this control becomes

\[
\dot{x} = (A - BK)x + Bv = A_x + Bv
\]  

(9)

With \(A\) the closed-loop plant matrix and \(v(t)\) the new command input.

If there is only one input so that \(m = 1\), then Ackermann's formula gives a SVFB \(K\) that places the poles of the closed-loop system as desired. However, it is very inconvenient to specify all the closed-loop poles, and we would also like a technique that works for any number of inputs.

Since many naturally occurring systems are optimal, it makes sense to design man-made controllers to be optimal as well. To design a SVFB that is optimal, we may define the performance index (PI)

\[
J = \frac{1}{2} \int_0^{\infty} x^T Q x + u^T R u dt
\]  

(10)

Substituting the SVFB control into this yields

\[
J = \frac{1}{2} \int_0^{\infty} (Q + K^T R K) x dt
\]  

(11)

We assume that input \(v(t)\) is equal to zero since our only concern here are the internal stability properties of the closed-loop system.

The objective in optimal design is to select the SVFB \(K\) that minimizes the performance index \(J\). The performance index \(J\) can be interpreted as an energy function, so that making it small keeps small the total energy of the closed-loop system. Note that both the state \(x(t)\) and the control input \(u(t)\) are weighted in \(J\), so that if \(J\) is small, then neither \(x(t)\) nor \(u(t)\) can be too large. To find the optimal feedback \(K\) we proceed as follows. Suppose there exists a constant matrix \(P\) such that

\[
\frac{d}{dt}(x^T P x) = -x^T (Q + K^T R K) x
\]  

(12)

where we assumed that the closed-loop system is stable so that \(x(t)\) goes to zero as time \(t\) goes to infinity which does not contain \(K\). This result is of extreme importance in modern control theory. The design procedure for finding the LQR feedback \(K\) is:

- Select design parameter matrices \(Q\) and \(R\)
- Solve the algebraic Riccati equation for \(P\)
- Find the SVFB using \(K = R^{-1}B^T P\)

### 4.2 Model based predictive control

The methods of model based predictive control have been widely presented and discussed in literature. A close literature review shows that in the academic works usually the model predictive controller is implemented as a single or two degree of freedom in the control architecture, whereas in the industrial control hierarchy model predictive controllers are supervisory applications, implemented on top of the regulatory control. The predictive controller performs the set point adjustment for the underlying control loops in order to drive the process variables at desired set points or to maintain process variables within constraints. Academic works often present architectures where the predictive controller is deployed at regulatory level as in Fig. 6.

![Regulatory MPC](image-url)

Fig. 6. Regulatory MPC

In general, a predictive control algorithm solves an on-line and optimal control problem subject to system dynamics and variable constraints. Consider the system model

\[
x(k + 1) = Ax(k) + Bu(k) + Gd_p(k) + Gd_m(k)
\]

\[
y(k) = Cx(k) + Du(k) + Hd_m(k)
\]  

(13)

where \(x(k) \in \mathbb{R}^n\) are the states, \(u(k) \in \mathbb{R}^m\) are the manipulated inputs and \(y(k) \in \mathbb{R}^p\) are the measured outputs. The vectors \(d_p(k)\) and \(d_m(k)\) are unmeasured disturbances to the state dynamics (process noise) and to the outputs (measurement noise), respectively. The controller predicts the future behavior of the actual system over a time interval defined by a lower and upper prediction horizon, denoted by \(N_u\) and \(N_p\), respectively. The optimal input to the plant is calculated by minimizing a cost function defined along the prediction horizon, usually specified as a sum of quadratic future errors between the reference
trajectory and predicted plant output, and the predicted control effort:

\[
J(k) = \sum_{i=N_u}^{N_k} \left( \hat{y}(k+i/k) - r(k+i) \right)^2 Q(i) + \sum_{i=0}^{N_u-1} \left\| \Delta u(k+i/k) \right\|^2 R(i) \\
\tag{14}
\]

Subject to constraints specified on the inputs, outputs and inputs increments:

\[
u_{min} \leq u(k) \leq u_{max} \\
y_{min} \leq y(k) \leq y_{max} \\
\Delta u_{min} \leq \Delta u(k) \leq \Delta u_{max}
\]

where: \(Q(i):\) positive definite error weighting matrix; \(R(i):\) positive semi-definite control weighting matrix; \(\hat{y}(k+i/k):\) vector of predicted output signals; \(r(k+i):\) vector of future set-point; \(\Delta u(k+i/k):\) vector of future control actions.

The cost function often used in MPC is similar to the cost in LQR. The main difference between MPC and LQR is that MPC handles constraints in the optimizations.

For LQR design, a full state feedback control law given by

\[
u(k) = -Kx(k)
\]
such that the full state feedback control law of satisfies the following criteria:

I. The closed loop state space system is asymptotically stable
II. The performance functional given by the (10).

\section{5 Experimental results}

We compute \(K\) using \(Q = \text{diag}(1,0.1,1,1)\) and \(R=1\). So \(K\) has the following form:

\[
K = [1 \quad -11.1797 \quad 0.6416 \quad -0.4336]
\]

Using these values we have simulated the systems’ behavior modeled by Bond Graph method. The control scheme was implemented in 20-sim modeling and simulation environment as is Fig. 7. The results for the system outputs (\(\theta\) and \(\alpha\)) are presented in Fig. 8.

The simulation result as well as the Simulink control scheme is presented in Fig. 9 and Fig. 10.

| Table 1 Parameters for the MPC controller design |
|-------------------------------|---------------------|
| MPC1                        | MPC2               |
| Control interval            | 0.1                | 0.1                  |
| Prediction horizon          | 10                 | 20                   |
| Control horizon             | 2                  | 3                    |
| Constraints                 | \(-10 < u < 10\)   | \(-10 < u < 10\)    |
|                            | \(-\pi / 2 < \theta < \pi / 2\) | \(-\pi / 2 < \theta < \pi / 2\) |
| Weight tuning               | 0.4                | 0.8                  |
6 Conclusions

The goal of this paper was to develop a control scheme for the particular Quanser Flexible link experiment using Bond Graph representation of the system and LQR and MPC controllers. From the results obtained by simulation it can be seen that optimal and faster control are obtained by modifying the parameters used for the design of MPC control. However, for a faster response one can see the presence of overshoots in terms of performance criteria. Future works will be focused on the implementation of this control structure on the real-time structure.

References:


*** Quanser Consulting Inc., Flexible Link Experiment ROTFLEX, 1998