ON THE MODELLING OF ASYMMETRIC HYSTERETIC LOOPS

TUDOR SIRETEANU
Institute of Solid Mechanics – Romanian Academy
Str. Constantin Mille, nr. 15, 010141 Bucharest
ROMANIA

Abstract: Hysteresis is a highly nonlinear phenomenon occurring in many disciplines involving system that possess memory, including inelasticity, electricity, magnetism etc. A wide variety of shapes of hysteretic loops can be portrayed by Bouc-Wen differential model. In general, differences between material strengths under tensile and compressive loading may result in hysteretic patterns, which are asymmetric. Due to the symmetry of Bouc-Wen differential equation with respect to the applied input, the standard version of this model cannot describe asymmetric hysteretic loops. In this paper, the asymmetry of experimental hysteretic characteristics are modeled by matching the solutions of a modified Bouc-Wen differential equation. The proposed approach is illustrated by its application to portraying the asymmetric hysteretic behavior of a novel type of base isolation device designed for forging hummer foundations .
Key-Words: asymmetric hysteresis, Bouc-Wen model, fitting experimental data, base isolation device, forging hammer

1 Introduction

The Bouc-Wen model, widely used in structural and mechanical engineering, gives an analytical description of a smooth hysteretic behavior. It was introduced by Bouc [1] and extended by Wen [2], who demonstrated its versatility by producing a variety of hysteretic characteristics. The hysteretic behavior of materials, structural elements or vibration isolators is treated in a unified manner by a single nonlinear differential equation with no need to distinguish different phases of the applied loading pattern. Usually, the mechanical hysteretic loops are obtained experimentally by imposing cyclic relative motion between the mounting ends on the testing rig of a material sample, structural element or vibration isolator and by recording the evolution of the developed force versus the imposed displacement. In practice, the Bouc-Wen model is mostly used within the following inverse problem approach: given a set of experimental input–output data, how to adjust the Bouc-Wen model parameters so that the output of the model matches the experimental data. Once an identification method has been applied to tune the Bouc-Wen model parameters, the resulting model is considered as a “good” approximation of the true hysteresis when the error between the experimental data and the output of the model is small enough from practical point of range. Various methods were developed to identify the model parameters from the experimental data of periodic vibration tests. In its standard form, the Bouc-Wen model can portray experimental data, which are symmetric with respect to the applied cyclic input. Therefore, this model can fit only asymmetric loops obtained for asymmetric inputs and not those due to the asymmetry of physical or mechanical properties of the tested material, structural element or vibration isolator. Asymmetric models, derived from Bouc-Wen model by using switching functions to uncouple the hysteretic loops into loading and unloading components, were proposed and applied to study the random vibration of hysteretic systems [3],[4]. Analytical methods, based on matching the solutions of two standard Bouc-Wen differential equations with different parameter values for positive and negative values of the displacement input, were also employed to fit asymmetric hysteretic loops [5]. In this paper, a slightly modified Bouc-Wen model is proposed for fitting the asymmetric hysteretic force developed by a novel type of base isolation device designed for forging hammer foundations [6],[7].

2. Experimental data

The new support for base isolation of forging hammers was designed as to meet both the shock transmissibility mitigation and serviceability requirements. The device consists in several packages of disks and annular plates bound as a sandwich type assembly. The hysteretic characteristic is the superposition result of a geometrically nonlinear elastic force of hardening type and a lubricated friction damping force. Since the support works only in compression – decompression mode around the static load, the resulting hysteretic loops recorded for imposed deformations within the working range are inherently asymmetric. Figs. 1 and 2 show the experimental setup and the time histories of imposed harmonic displacement and of the force developed by the device around the static load.

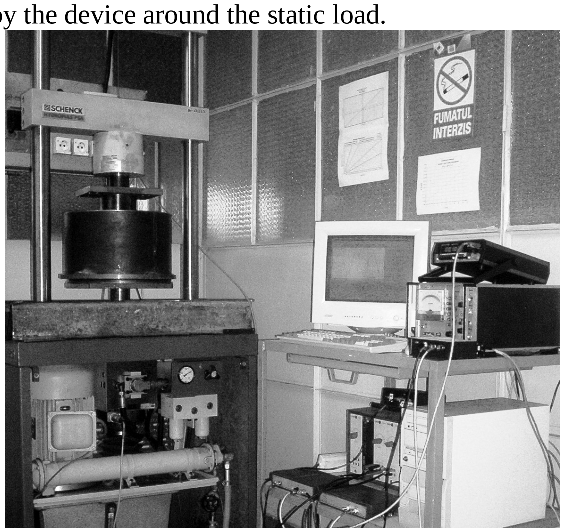

Fig.1 Experimental setup

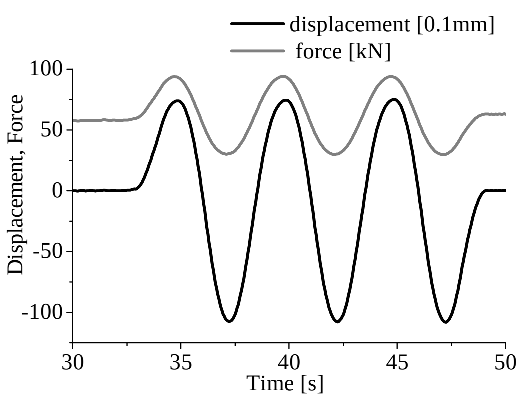

Fig.2. Time histories of imposed displacement and developed force

The hysteretic loop obtained by subtracting the static load ($F_s = 61\text{kN}$) is plotted in fig.3. As one can see, these curves are highly asymmetric with respect to imposed displacement, being of hardening type for compression (positive displacement) and of softening type for decompression (negative displacement). The asymmetry of displacement amplitude (smaller for compression stroke than for decompression stroke) is similar to that encountered in operating conditions.

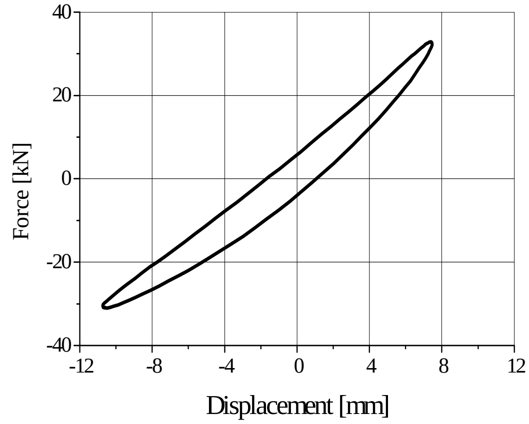

Fig.3. Hysteretic characteristic of the base isolation Device.

3. Analytical model

Suppose the experimental hysteretic characteristic is an asymmetric loop $-F_{\text{m2}} \le F(x) \le F_{\text{m1}}$, obtained for a periodic motion $-x_{\text{m2}} \le x(t) \le x_{\text{m1}}$, imposed between the mounting ends of the tested element.
By introducing the dimensionless magnitudes

$$\tau = t/T\,,\ \xi(\tau) = x(\tau T)/x_{\text{u}}\,, \xi'(\tau$$
$$z(\xi) = F(x_{\text{u}}\xi)/F_{\text{u}}\,,$$
$$\xi_{\text{m1}} = \max\left[\xi_+(\tau)\right], \xi_{\text{m2}} = \text{m}$$
$$z_{\text{m1}} = \max\left[z(\xi_+)\right],\ z_{\text{m2}} = \text{ı}$$
(1)

where T is the period of the imposed cyclic motion and x_{u}, F_{u} are displacement and force reference units such as $\xi_{\text{m1,2}} \le 1$, $z_{\text{m1,2}} \le 1$, the standard Bouc-Wen model, is described by the first order differential equation

$$\frac{dz}{A - |z|^n\left[\beta + \gamma \operatorname{sgn}(\xi' z)\right]} = d\xi \qquad (2)$$

where A, β, γ, n are loop parameters controlling the shape and magnitude of the hysteresis loop $z(\xi)$. The steady-state solution of equation (2), $z(\xi)$, is practically symmetric with respect to ξ.
The fitting procedure of asymmetric hysteresis loops consists in matching the solutions of two different equations, corresponding to positive $(\xi_+(\tau))$ and negative $(\xi_-(\tau))$ values of the imposed

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displacement. To illustrate the method, let us consider one of the normalized asymmetric hysteresis loops (see fig.3), derived from those from fig.4 by taking in (1) the reference units $x_{\mathrm{u}} = 15\mathrm{mm}, F_{\mathrm{u}} = 40\mathrm{kN}$.

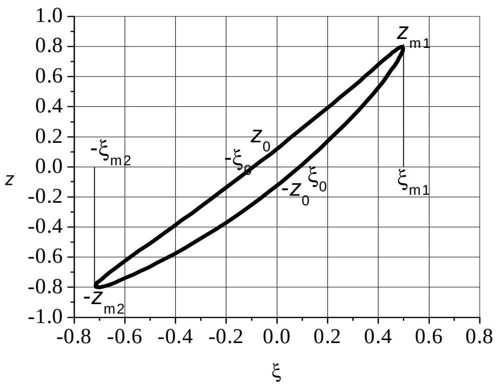

Fig.3. Normalized asymmetric hysteretic loop

The asymmetric hysteretic loop is given by

$$\begin{aligned} z(\xi) &= \frac{1}{2}\left\{z_1(\xi)[1+\mathrm{sgn}\xi] + z_2(\xi)[1-\mathrm{sgn}\xi]\right\}, \\ \xi(\tau) &= \frac{1}{2}\left\{\xi_+(\tau)[1+\mathrm{sgn}\xi] + \xi_-(\tau)[1-\mathrm{sgn}\xi]\right\} \end{aligned} \quad (3)$$

where $z_1(\xi)$ and $z_2(\xi)$, are the solutions of the Bouc-Wen equations

$$\begin{aligned} \frac{dz_1}{A_1 - |z_1|^{p_1}\left[\beta_1 + \gamma_1 \mathrm{sgn}(\xi' z_1)\right]} &= d\xi\ , \\ \frac{dz_2}{A_2 - |z_2|^{p_2}\left[\beta_2 + \gamma_2 \mathrm{sgn}(\xi' z_2)\right]} &= d\xi \end{aligned} \quad (4)$$

In the above equations, p_1 and p_2 are positive real numbers.

With the notations from fig.3, the continuity and smoothness conditions at the crossing points of hysteretic loop with force axis are [5]

$$\begin{aligned} I_{1i} &= \int_{\pm z_0}^{\pm z_{mi}} \frac{dz}{A_i - |z|^{p_i}(\beta_i + \gamma_i)} = \int_0^{\pm\xi_{mi}} d\xi = \pm\xi_{mi}\ , \\ I_{2i} &= \int_{\pm z_{mi}}^{0} \frac{dz}{A_i - |z|^{p_i}(\beta_i - \gamma_i)} = \int_{\pm\xi_{mi}}^{\pm\xi_0} d\xi = \pm\xi_0 \mp \xi_{mi}\ , \quad (5) \\ I_{3i} &= \int_0^{\mp z_0} \frac{dz}{A_i - |z|^{p_i}(\beta_i + \gamma_i)} = \int_{\pm\xi_0}^{0} d\xi = \mp\xi_0 \end{aligned}$$

$i = 1,2$

$$\left.\frac{dz_1}{d\xi}\right|_{\xi=0} = A_1 - z_0^{p_1}(\beta_1 + \gamma_1) = \quad (6)$$
$$= \left.\frac{dz_2}{d\xi}\right|_{\xi=0} = A_2 - z_0^{p_2}(\beta_2 + \gamma_2)$$

4. Numerical results

The experimental values used in (5) and (6) were

$$\xi_{m1} = 0.5,\ \xi_{m2} = 0.72,\ z_{m1} = z_{m2} = 0.8, \quad (7)$$
$$\xi_0 = 0.1,\ z_0 = 0.12$$

By using an iterative algorithm, with a cost function derived from fitting conditions (5) and (6), the following values of the asymmetric loop parameters were found

$$\begin{aligned} &A_1 = 1.4,\ \beta_1 = -1.3,\ \gamma_1 = 1.5,\ p_1 = 1.5, \\ &A_2 = 1.25,\ \beta_2 = 0,4,\ \gamma_2 = 1.5,\ p_2 = 1.5 \end{aligned} \quad (8)$$

The experimental and predicted normalized hysteretic loops are plotted in fig. 4.

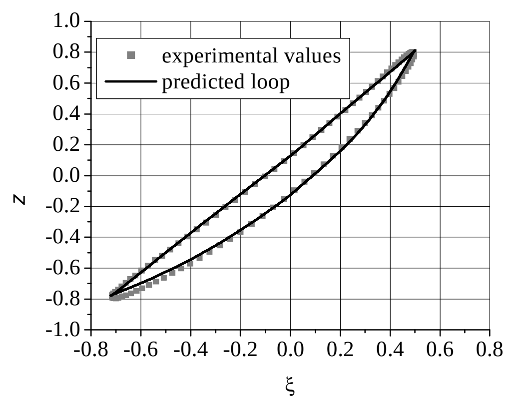

Fig.4 Experimental and predicted hysteretic loops

Introducing the numerical values (7) and (8) in the fitting relationships (5) and (6) yields

$$\begin{aligned} &I_{11} = 0.51 \cong \xi_{m1}; \\ &I_{21} = -0.39 \cong -\xi_{m1} + \xi_0 = 0.4, \\ &I_{31} = -0.09 \cong -\xi_0 = -0.1, \\ &I_{12} = 0.79 \cong \xi_0 = 0.72, \\ &I_{22} = 0.61 \cong \xi_{m2} - \xi_0 = 0.62, \\ &I_{32} = 0.097 \cong \xi_0 = 0.1. \\ &\left.\frac{dz_1}{d\xi}\right|_{\xi=0} = A_1 - z_0^{p_1}(\beta_1 + \gamma_1) = 1.39, \\ &\left.\frac{dz_2}{d\xi}\right|_{\xi=0} = A_2 - z_0^{p_2}(\beta_2 + \gamma_2) = 1.21 \end{aligned} \quad (8)$$

5. Conclusions

The proposed method, for fitting hysteresis loops by matching the solutions of two Bouc–Wen differential equations, can portray the asymmetric hysteretic behavior of materials, structural elements or vibration control devices.

It should be mentioned that the obtained model fits very well the experimental hysteresis loops only for the considered extreme values of the imposed displacement $\xi(\tau)$. The same procedure can be applied for other imposed cyclic motions. Then, the model parameters could be finally chosen by a trade-off based on practical considerations.

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