On the Elastic Properties of Some Advanced Composite Laminates Subjected to Off-Axis Loading Systems

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Abstract: The paper presents the stiffness evaluation of three advanced composite laminates based on epoxy resin reinforced alternatively with HM carbon-, HS carbon- and kevlar49 fibers. The laminates have following plies sequences: [0/90/0/90], [0/45/-45/90], [45/-45/45/-45] and are subjected to off-axis loading systems. The elastic constants as well as the tensile-shear interaction have been determined. In order to obtain equal stiffness in all off-axis loading systems, a composite laminate have to present balanced angle plies.

Key-Words: stiffness, elastic properties, HM carbon, HS carbon, kevlar49, laminates, off-axis loading system.

1 Introduction

Stiffness evaluation of advanced composite laminates is for a great importance in designing composite structures especially suited for aerospace and automotive industries.

2 Problem Formulation

It is well known that composite laminates with aligned reinforcement are very stiff along the fibers, but also very weak transverse to the fibers direction. This fact is more obvious in the case of advanced composite laminates reinforced with anisotropic carbon- or aramid fibers. Getting equal stiffness of laminates is a demand.

3 Problem Solution

The solution to obtain equal stiffness of laminates subjected in all directions within a plane is by stacking and bonding together plies with different fibers orientations [1-3]. A composite laminate (fig. 1) formed by a number of unidirectional reinforced laminas subjected regarding to the loading scheme presented in fig. 2 is considered. The elasticity law for a unidirectional lamina \( K \) is:

\[
\begin{bmatrix}
\sigma_{xxK} \\
\sigma_{yyK} \\
\tau_{xyK}
\end{bmatrix}
= \begin{bmatrix}
r_{11K} & r_{12K} & r_{13K} \\
r_{21K} & r_{22K} & r_{23K} \\
r_{31K} & r_{32K} & r_{33K}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xxK} \\
\varepsilon_{yyK} \\
\gamma_{xyK}
\end{bmatrix},
\]

where \( r_{ijK} \) represent the transformed stiffness, \( \sigma_{xxK}, \sigma_{yyK} \) are the mean stresses of \( K \) lamina on x- respective y-axis and \( \tau_{xyK} \) represent the mean shear stress of \( K \) lamina against the x-y coordinate system. The balance equations of the laminate structure are:

\[
n_{xx} = \sum_{K=1}^{N} \sigma_{xxK} t_K = \sum_{K=1}^{N} n_{xxK},
\]

\[
n_{yy} = \sum_{K=1}^{N} \sigma_{yyK} t_K = \sum_{K=1}^{N} n_{yyK},
\]

\[
n_{xy} = \sum_{K=1}^{N} \tau_{xyK} t_K = \sum_{K=1}^{N} n_{xyK},
\]

where \( n_{xx}, n_{yy} \) are the normal forces on the unit length of the laminate on x- respective y-axis and \( n_{xy} \) represents the shear force, in plane, on the unit length of the laminate against the x-y coordinate system. \( \sigma_{xx}, \sigma_{yy} \) are the normal stresses on x- respective y-axis of the laminate, \( \tau_{xy} \) represent the shear stress of the laminate against the x-y coordinate system. \( t_K, t \) represent the thickness of the \( K \) lamina respective the laminate thickness, \( n_{xxK}, n_{yyK} \) are forces on the unit length of \( K \) lamina on x- respective y-axis directions and \( n_{xyK} \) is the shear force in plane, on the unit length of \( K \) lamina against the x-y coordinate system. Beside the balance equations, the geometric conditions must be also determined, to compute the stresses. For composite laminates these conditions imply that all laminas are bonded together and withstand, in a specific point, the same strains \( e_{xx}, e_{yy}, \gamma_{xy} \) as well as for the entire laminate:

\[
e_{xxK} = e_{xx}, \quad e_{yyK} = e_{yy}, \quad \gamma_{xyK} = \gamma_{xy}.
\]
According to equations (1)-(5), the elasticity law for entire laminate can be computed [4, 5]:

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\tau_{xy}
\end{bmatrix} = \begin{bmatrix}
\sum_{k=1}^{N} n_{1k} t_k \\
\sum_{k=1}^{N} r_{2k} t_k \\
\sum_{k=1}^{N} r_{2k} t_k
\end{bmatrix} \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy}
\end{bmatrix}, \tag{6}
\]

where the laminate stiffness \( C_{ij} \) are:

\[
\frac{E_y}{K} = \frac{N}{K} \left( r_{ij} K \cdot \frac{t}{t} \right). \tag{7}
\]

In other words, the laminate elasticity law becomes:

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\tau_{xy}
\end{bmatrix} = \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy}
\end{bmatrix}, \tag{8}
\]

Computing the laminate strains as a function of stresses, the expressions (8) are:

\[
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
\xi_{11} & \xi_{12} & \xi_{13} \\
\xi_{12} & \xi_{22} & \xi_{23} \\
\xi_{13} & \xi_{23} & \xi_{33}
\end{bmatrix} \begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\tau_{xy}
\end{bmatrix}, \tag{9}
\]

where \( \varepsilon_{ij} \) represents the laminate compliance tensor. This tensor can be computed as a function of elastic constants. Thus [6, 7]:

\[
E_y = \frac{1}{\xi_{11}}; \quad G_{xy} = \frac{1}{\xi_{33}}; \quad \nu_{xy} = -E_y \cdot \xi_{12}. \tag{10}
\]

It is obvious that the laminate will exhibit different elastic constants if the loading system is applied at a randomly angle, \( \Phi \), to the x-y coordinate system.

### 4 Three advanced composite laminates

The laminates taken into account at stiffness evaluation are based on epoxy resin reinforced alternatively with HM carbon-, HS carbon- and kevlar49 fibers. These laminates present the following plies sequence: [0/90/0/90], [0/45/-45/90] and [45/-45/45/-45].

Carbon fibers of type HM (high modulus) present a value of Young modulus larger than 300 GPa. High strength (HS) carbon fiber is a general purpose, cost effective carbon fiber, designed for industrial and recreational applications and is usually used for non structural components of aircrafts. Kevlar49 aramid fiber is characterized by low-density and high-tensile strength and modulus. These properties are the key to its successful use as reinforcement for plastic composites in aircraft, aerospace, marine, automotive, other industrial applications, and in sports equipment. It is available in continuous-filament yarns, chopped fiber, woven and
unidirectional fabrics, tissues or veils and tapes for reinforcement applications. Kevlar 49 aramid is used in high-performance composite applications where lightweight, high strength and stiffness, vibration damping and resistance to damage and fatigue are key properties. Reinforced composites can save up to 40% of the weight of glass-fiber composites at equivalent stiffness [8].

5 Results
General input data are: fibers volume fraction $\varphi = 0.5$ in all cases, laminates thickness $t = 1$ mm and off-axis loading systems varies between 0 and 90 degrees. The elastic constants $E_{xx}$, $G_{xy}$ and $\nu_{xy}$ are presented in figures 3 – 11.

Fig. 3. $E_{xx}$ Young modulus for [0/90/0/90] epoxy based composite laminate

Fig. 4. $G_{xy}$ shear modulus for [0/90/0/90] epoxy based composite laminate

Fig. 5. $\nu_{xy}$ Poisson ratio for [0/90/0/90] epoxy based composite laminate

Fig. 6. $E_{xx}$ Young modulus for [0/45/-45/90] epoxy based composite laminate

Fig. 7. $G_{xy}$ shear modulus for [0/45/-45/90] epoxy based composite laminate
6 Conclusion

Tensile-shear interactions lead to distortions and local micro-structural damage and failure, so in order to obtain equal stiffness in all off-axis loading systems, a composite laminate have to present balanced angle plies, e.g. [0/45/-45/90].

Under off-axis loading, normal stresses produce shear strains (and of course normal strains) and shear stresses produce normal strains (as well as shear strains). This tensile-shear interaction is also present in laminates but does not occur if the loading system is applied along the main axes of a single lamina or if a laminate is balanced.

References: