Compact Solution for a Robotomechanism of a Bimobile Orientation 
with Decoupled Moves

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Abstract - In this paper is presented a new kind of robotomechanism of a bimobile orientation, operated through wires, with decoupled moves, proposed by the authors. In comparison with other robotomechanisms for orientation, built out from other decoupling mechanisms with planetary units [1, 2, 3, 4, 5], the nominated variant is distinguished through an operating with wires and friction wheels, throughh compactness, and implicit, through reduced gauge. The solution proposed in contributes to the improving of the base of structural schemes of mechanisms for orientation activated through wires, and the kinematical relations obtained in the paper are useful in the organizing projection and command programming on the computer.

Key-Words: Robotomechanism activated through wires, Decoupling of the moves, Coupling grade, Differential mechanism with wires

1 Introduction
In this paper is presented a new kind of robotomechanism of a bimobile orientation, operated through wires, with decoupled moves, proposed by the author. In comparison with other robotomechanisms for orientation, built out from other decoupling mechanisms with planetary units [1, 2, 3, 4, 5], the nominated variant is distinguished through an operating with wires and friction wheels, through compactness, and implicit, through reduced gauge. The solution proposed in figure 1 contributes to the improving of the base of structural schemes of mechanisms for orientation activated through wires, and the kinematical relations obtained in the paper are useful in the organizing projection and command programming on the computer.

2 Notation
- $M$ the mobility degree of the mechanisms
- $M_i$, $M_{ii}$,... the mobility degree of the component mechanisms
- $L_C$ the number of the couplings between the mechanisms
- $\omega_a$ the angle speed of an element “a” reported to the base
- $i_{a,b}^{c}$ the transmission report from element “a” to element “b”, when the angle speed is $\omega_a=0$;
- $C$ the coupling degree of the moves;
- $M_a$ the moment of the element “a”;

3 The Structural Characterization of the Mechanism
The robotomechanism from figure 1 is built out of a kinematical open chain (0-1-2-3) with 2 rotation couples, with the axes a and b which are orthogonal and their relative moves $\alpha$ and $\beta$ characterize the orientation moves. For the training and decoupling of the moves $\alpha$ and $\beta$, too planetary units are being used, with pulley wheels and wires, pulled one in each other: I = 5-6-2(H)-7) and III = 13-14-2(H) and three monomobile units with wires II = 9-10-11-12=13, (IV = 4-5), (V = 8-9). The engine-operating is being made through the monomobile mechanisms IV and V.
Between the planetary bi-mobile units and the monomobile ones are five connections ($L_C = 5$), as follows: (5-11), (12-13), (2\textsuperscript{1}-2\textsuperscript{iii}), (5\textsuperscript{1}-5\textsuperscript{iv}), (9\textsuperscript{i}-9\textsuperscript{v}). Therefore, the roboto-mechanism from figure 1 is bimobile:

\begin{equation}
M = M_1 + M_{ii} + + M_V - L_C = \\
= 2 + 2 + 1 + 1 + 1 - 5 = 2 \end{equation}

and has 4 exterior connections : two entrances (4 and 8) and two outputs ($\alpha$ and $\beta$).
This robotomechanism assures the decoupling of the moves $\alpha$ and $\beta$, the wheel 4 trains only the move $\alpha$, and the wheel 8 trains only the move $\beta$. 
4 Transmission Functions for Speeds and Moments

Analytical, the characteristic that the robotomechanism has decoupled moves, comes out of the transmission functions of the speeds (fig. 1) [2, 3, 4]. For obtaining the analytical relations referring to the transmission functions of the speeds, there have been established relations between the input speeds and the output ones, using the method of overlapping the effects:

\[
\begin{align*}
\omega_a &= a_1 \cdot \omega_a + b_1 \cdot \omega_b = \omega_a^{(a)} + \omega_b^{(b)} \\
\omega_b &= a_2 \cdot \omega_a + b_2 \cdot \omega_b = \omega_a^{(a)} + \omega_b^{(b)}
\end{align*}
\]

Replacing successive \(\omega_a, \omega_b\) with zero, the coefficients \(a_1, a_2, b_1, b_2\) can be calculated.

In the case if \(\omega_a \neq 0\) and \(\omega_b = 0\) then:

\[
\begin{align*}
\omega_a^{(a)} &= a_1 \cdot \omega_a \\
\omega_b^{(a)} &= a_2 \cdot \omega_a
\end{align*}
\]

The determining of the constants \(a_1\) and \(a_2\):

- From the first equation of the system (3) and taking into consideration that \(\omega_a^{(a)} = \omega_a\), results:

\[
\begin{align*}
a_1 &= \frac{\omega_a^{(a)}}{\omega_a} = \frac{\omega_4}{\omega_a} = \frac{1}{4-\alpha} = \frac{1}{4-5} \cdot \frac{1}{5-11} = \\
&= \frac{R_5}{R_4} \cdot \frac{7}{11-5} = \frac{R_5}{R_4} \cdot 11/5 = \frac{R_5}{R_4} \cdot (1 - (-1)) = 2 \frac{R_5}{R_4}
\end{align*}
\]

\[\Rightarrow a_1 = 2 \frac{R_5}{R_4}\] (4)

- From the second equation of the system (3) and taking into consideration that \(\omega_b^{(a)} = \omega_b\), results:

\[
\begin{align*}
\omega_a &= \frac{\omega_a^{(a)}}{\omega_b} = \frac{\omega_4}{\omega_b} = \frac{1}{4-\alpha} = \frac{1}{4-5} \cdot \frac{1}{5-11} = \\
&= \frac{R_5}{R_4} \cdot \frac{7}{11-5} = \frac{R_5}{R_4} \cdot 11/5 = \frac{R_5}{R_4} \cdot (1 - (-1)) = 2 \frac{R_5}{R_4}
\end{align*}
\]

\[\Rightarrow a_1 = 2 \frac{R_5}{R_4}\]

4 Transmission Functions for Speeds

Considering that:

\[
\begin{align*}
i_{9-11} &= \frac{\omega_9}{\omega_11} \\
o_9 &= i_{9-11} \cdot \omega_h + i_{9-11} \cdot \omega_11 = \omega_12 (i_{9-12} + i_{9-11} \cdot \frac{\omega_9}{\omega_11})
\end{align*}
\]

And that:

\[
\begin{align*}
\omega_1 &= \omega_5 \\
\omega_12 &= \omega_5 \cdot \frac{1}{i_{12-13}} \\
\omega_11 &= \omega_5 \cdot \frac{1}{i_{12-13}} \cdot \frac{1}{i_{12}} = \omega_12 (2 + 2 \cdot 1 \cdot (-1)) = 0
\end{align*}
\]

\[\Rightarrow a_2 = 0\] (5)

Replacing the relations (4) and (5) in (3), we obtain:

\[
\begin{align*}
\omega_a^{(a)} &= 2 \frac{R_5}{R_4} \cdot \omega_a \\
\omega_b^{(a)} &= 0 \cdot \omega_a
\end{align*}
\]

In the case if \(\omega_a = 0\) and \(\omega_b \neq 0\):

From figure 1 we can observe that in this situation:

\[
\begin{align*}
\omega_2 &= \omega_7 = \omega_8 = \omega_5 = \omega_11 = \omega_4 = \omega_a = 0 \Rightarrow \\
\omega_a^{(a)} &= b_1 \cdot \omega_p \\
\omega_b^{(a)} &= b_2 \cdot \omega_p
\end{align*}
\]

The determining of the constants \(b_1\) and \(b_2\):

- From the first equation of the system (7) results:

\[
\begin{align*}
b_1 &= \frac{\omega_8}{\omega_p} = \frac{1}{i_{9-11-\beta}} = i_{9-11-\beta} \cdot i_{9-11} \cdot \omega_9 = \frac{1}{i_{9-11-\beta}} \cdot \omega_9 = \frac{1}{i_{9-11-\beta}} \cdot \omega_9
\end{align*}
\]

\[\Rightarrow \omega_a^{(a)} = \frac{2 \cdot \frac{R_5}{R_4} \cdot \frac{1}{i_{12-13}}}{R_5} \cdot \frac{1}{i_{12}} \cdot \frac{1}{i_{12}} = 0\] (6)

- From the second equation of the system (7) and taking into consideration that \(\omega_b^{(a)} = \omega_b\), results:

\[
\begin{align*}
b_2 &= \frac{\omega_8}{\omega_p} = \frac{1}{i_{9-11-\beta}} = i_{9-11-\beta} \cdot i_{9-11} \cdot \omega_9 = \frac{1}{i_{9-11-\beta}} \cdot \omega_9 = \frac{1}{i_{9-11-\beta}} \cdot \omega_9
\end{align*}
\]

\[\Rightarrow \omega_a^{(a)} = \frac{2 \cdot \frac{R_5}{R_4} \cdot \frac{1}{i_{12-13}}}{R_5} \cdot \frac{1}{i_{12}} \cdot \frac{1}{i_{12}} = 0\] (7)

Replacing the relations (8) and (9) in (7), we obtain:

\[
\begin{align*}
\omega_a^{(a)} &= 0 \cdot \omega_p \\
\omega_b^{(a)} &= b_2 \cdot \omega_p
\end{align*}
\]

Introducing the relations (6) and (10) in (2), we obtain:
\[
\begin{align*}
\omega_a &= \frac{2 R_2}{R_3} \cdot \omega_a \\
\omega_b &= \frac{2 R_2}{R_3} \cdot \frac{R_{14}}{R_3} \cdot \omega_b
\end{align*}
\] (11)

The relations (11) can be written in a matrix way, like:

\[
\begin{bmatrix}
\omega_a \\
\omega_b
\end{bmatrix}
= \begin{bmatrix}
\frac{2 R_2}{R_3} & 0 \\
0 & \frac{2 R_2}{R_3} \cdot \frac{R_{14}}{R_3}
\end{bmatrix}
\begin{bmatrix}
\omega_a \\
\omega_b
\end{bmatrix}
\] (12)

For the practical reason when \(R_4 = 2R_5\); and \(\frac{R_2}{R_3} = \frac{R_{13}}{R_{14}}\), we obtain:

\[
\begin{bmatrix}
\omega_a \\
\omega_b
\end{bmatrix}
= \begin{bmatrix}
1 & 0 \\
0 & 2
\end{bmatrix}
\begin{bmatrix}
\omega_a \\
\omega_b
\end{bmatrix}
\] (13)

In the premise of neglecting the rubbing and the inertia, out of the equation of energetic equilibrium follows:

\[
\begin{bmatrix}
M_a & M_b
\end{bmatrix}
\begin{bmatrix}
\omega_a \\
\omega_b
\end{bmatrix} + \begin{bmatrix}
M_a & M_b
\end{bmatrix}
\begin{bmatrix}
\omega_a \\
\omega_b
\end{bmatrix} = 0
\]

We obtain:

\[
\begin{bmatrix}
M_a & M_b
\end{bmatrix}
= \begin{bmatrix}
1 & 0 \\
0 & 2
\end{bmatrix}
\begin{bmatrix}
M_a & M_b
\end{bmatrix}
\] (14)

The relations (13) and (14) represent the functions of transmitting the speeds, respective the moments.

Considering the integration constants equal with zero, the functions of transmitting the angular coordinates have similar expressions with the speeds, obtaining so:

\[
\begin{align*}
\omega_a &\rightarrow \varphi_a \\
\omega_b &\rightarrow \varphi_b
\end{align*}
\]

\[
\begin{align*}
\varphi_a &= \varphi_a = \alpha \\
\varphi_b &= \varphi_b = 2\beta
\end{align*}
\] (15)

Writing down the connection relations between the versors of the coordination systems, we obtain the following expressions:

\[
\begin{bmatrix}
[i_3]_{x,y,z_1} \\
[i_3]_{x,y,z_2}
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
m_x & m_y & m_z
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
k_3 \\
k_3
\end{bmatrix}
= \begin{bmatrix}
n_x & n_y & n_z
\end{bmatrix}
\]

\[
\begin{bmatrix}
l_x^2 + l_y^2 + l_z^2 = 1 \\
n_x^2 + n_y^2 + n_z^2 = 1
\end{bmatrix}
\] (16)

which represent the matrixes of the versors of the trihedral \(Ox_3y_3z_3\), expressed in the trihedral \(Ox_1y_1z_1\).

The rotation angles \(\alpha\) and \(\beta\) between the axes of the system \(Ox_2y_2z_2\) when \(Oy_2=Oy_1\) and the rotation of the system \(Ox_3y_3z_3\) when \(Oz_3=Oz_2\):

\[
\begin{align*}
\alpha &= \varphi (Oz_2, Oz_1); \Rightarrow \varphi (Ox_2, Ox_1) \\
\beta &= \varphi (Ox_3, Ox_1); \Rightarrow \varphi (Oy_3, Oy_1)
\end{align*}
\]

Further we note:

- \(\varphi_a\), \(\varphi_b\) - the rotation angles of the pulley wheels 4 and 9, which form the angles \(\alpha\) and \(\beta\).
- \(T_{p-n} = T(x_p y_p z_p \leftrightarrow x_q y_q z_q)\) - notes the matrix which expresses the angular position of the trihedral \(Ox_3y_3z_3\) in the trihedral \(Ox_4y_4z_4\).
- \(C\) = cosine function
- \(S\) = sine function

In the direct kinematics, the angles \(\varphi_a\) and \(\varphi_b\) are considered as being known and the unknown are considered the direct cosine of the versors \(i_3, j_3, k_3\) expressed in the trihedral \(Ox_1y_1z_1\).

With this purpose we determine first of all the matrix \(T_{3-1} = T(x_1y_1z_1 \leftrightarrow x_3y_3z_3)\), depending on \(\alpha\) and \(\beta\).

\[
T_{3-1} = T_{2-1} T_{3-2}
\]

\[
\begin{bmatrix}
C\alpha & 0 & S\alpha \\
0 & 1 & 0 \\
-S\alpha & 0 & C\alpha
\end{bmatrix}
\begin{bmatrix}
C\beta & -S\beta & 0 \\
S\beta & C\beta & 0 \\
0 & 0 & 1
\end{bmatrix}
=
\begin{bmatrix}
C\alpha \cdot C\beta & -C\alpha \cdot S\beta & -S\alpha \\
S\beta & C\beta & 0 \\
S\alpha \cdot C\beta & S\alpha \cdot S\beta & C\alpha
\end{bmatrix}
\] (17)

Replacing (15) in (17) and taking into consideration that the direct cosines of the versors which are unknown are expressed in the matrix \(T_{3-1} = [i_3, j_3, k_3]_{x_1y_1z_1}\), we obtain the following relations:
\[ \begin{bmatrix} C\alpha \cdot C\beta - C\alpha \cdot S\beta & S\alpha \\ S\beta & C\beta & 0 \\ S\alpha \cdot C\beta & S\alpha \cdot S\beta & C\alpha \end{bmatrix} = \]
\[ \begin{bmatrix} C\phi_4 \cdot C \left( \frac{\phi_8}{2} \right) - C\phi_4 \cdot S \left( \frac{\phi_8}{2} \right) - S\phi_4 \\ S \left( \frac{\phi_8}{2} \right) \cdot C \left( \frac{\phi_8}{2} \right) \cdot 0 \\ S\phi_4 \cdot C \left( \frac{\phi_8}{2} \right) - S\phi_4 \cdot S \left( \frac{\phi_8}{2} \right) \cdot C\phi_4 \end{bmatrix} \]

\[ \text{(18)} \]

\[ \begin{bmatrix} C\phi_4 \cdot C \left( \frac{\phi_8}{2} \right) - C\phi_4 \cdot S \left( \frac{\phi_8}{2} \right) - S\phi_4 \\ S \left( \frac{\phi_8}{2} \right) \cdot C \left( \frac{\phi_8}{2} \right) \cdot 0 \\ S\phi_4 \cdot C \left( \frac{\phi_8}{2} \right) - S\phi_4 \cdot S \left( \frac{\phi_8}{2} \right) \cdot C\phi_4 \end{bmatrix} = \]

\[ \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix} \]

\[ l_x = C\phi_4 \cdot C \left( \frac{\phi_8}{2} \right) \quad m_x = -C\phi_4 \cdot S \left( \frac{\phi_8}{2} \right) \]

\[ l_y = S \left( \frac{\phi_8}{2} \right) \quad m_y = C \left( \frac{\phi_8}{2} \right) \]

\[ l_z = -S\phi_4 \cdot C \left( \frac{\phi_8}{2} \right) \quad m_z = S\phi_4 \cdot S \left( \frac{\phi_8}{2} \right) \]

\[ n_x = -S\phi_4 \]

\[ n_y = 0 \quad \text{(Oy \perp Oz)} \]

\[ n_z = C\phi_4 \]

From (19) it results that the values of the direct cosines of the versors \( l_1, l_2, l_3 \) cannot be chosen arbitrarily, but only in that way so they satisfy the following relations:

\[ \begin{align*}
C\phi_4 \cdot C \left( \frac{\phi_8}{2} \right) &= l_x \\
-C\phi_4 \cdot S \left( \frac{\phi_8}{2} \right) &= m_x \\
-S\phi_4 \cdot C \left( \frac{\phi_8}{2} \right) &= l_z \\
\Rightarrow \quad n_x \cdot m_y &= l_x \\
-n_z \cdot l_y &= m_x \\
-n_x \cdot m_y &= l_z \\
S\phi_4 \cdot S \left( \frac{\phi_8}{2} \right) &= m_z \\
S^2 \left( \frac{\phi_8}{2} \right) + C^2 \left( \frac{\phi_8}{2} \right) &= 1
\end{align*} \]

\[ \text{(22)} \]

6 Conclusion

- In comparison with other similar solutions, the roboto-mechanism of orientation, activated by pulley wheels and wires, with decoupled moves, proposed by the author in figure 1, assures an easy construction, a silent one, compact and with minimum gauge.
- The kinematical relations established in the paper are useful in the organological projection and in the command programming on the computer, of the analysed roboto-mechanism.
- The obtained results are useful to the inventors and the projectors from the industrial robots domain.

References:


