Mathematical Model of Temperature Distribution Field in Electrical Discharge Machining of Ceramics Composites

CONSTANTIN OPRAN, MIHAIELA ILIESCU
Manufacturing Department
“POLITEHNICA” University of Bucharest
Splaiul Independentei no. 313 Street, District no. 6, zip code 060042
ROMANIA

Abstract: - Composite ceramics materials have known an increasing development, because of their important characteristics and advantages. Among them, an important role is that of electro-conductive structural ceramics composites – CCsE-Al2O3/TiC. In order to obtain a part with specified shape and dimensions, made of this material type, most of the times a massive electrode electrical discharge machining - by shape copying is needed - PPE-E_mCo. This paper presents a theoretical study on temperature distribution field, for the previously mentioned machining procedure, carried on in order to get a mathematical model of temperature distribution. Once obtained, the theoretical assumption should be validated by experiments.

Key-Words: - ceramics composites, matrix, reinforcing element, electrical discharge, shape copying, mathematical model, temperature distribution

1 Introduction
Technical ceramics are characterized as a composite type engineering material, with more than one fragile phase, discretely crystalline, or amorphous, not soluble into water. It is obtained in a solidification and forming technological process, at high temperatures and pressures where the resulting material should be, at least, 30% of crystalline structure [1, 2].

Composite, thus, ceramic composite, represents a hole material system, made of two or more various materials, with specially created interfaces but, each of the involved materials maintaining its own properties. There are two main material’s phases: the matrix and the reinforcing element, the last one being purposely added, as to create a new material. Usually, ceramics composites are anisotropic, non-homogenous and heterogeneous, the interface of matrix and reinforcing element being easy to recognize because it represents the bond of the two main phases.

Metal-ceramics, that represent one of the ceramics composites types, are known as cermets, and represent an heterogeneous combination of one, or more, ceramic phases (oxides, carbides, etc.). The ductile metal phase acts as a metal – metallic matrix binder that incorporates the fragile ceramic phase.

Machining ceramic materials, involves special care, as this process results depend on important factors, such as: composite ceramics’ composition, material’s obtaining technology, fragile fracture’s behavior, ceramics’ thermal and electrical characteristics. One very important machining procedure of structural ceramics composite is represented by massive electrode electrical discharge and, more specifically, by shape copying electrical discharge.

Dealing with thermal characteristics of structural ceramics composites that do influence the massive electrode electrical discharge process, one can stress the importance of thermal shock. By thermal shock, there do appear thermal internal stresses that do have real displacement. So, when materials’ thermal shock resistance decreases the result is that, both, deformation resistance in thermal shock, and thermal fatigue resistance get lower.

Technical composites ceramics, such as Al2O3/TiC are widely used into various fields of advanced technologies (aerospace, navy, etc) and, thus, adequate machining procedures of the parts, made of this materials type have to be developed.

The ceramics composites, studied by this paper, are electro-conductive composite ceramics, made of two refractory materials, one electrical non-conductive (Al2O3) and the other electrical conductive (TiC). They are made of the ceramic phase - Al2O3 , whose size is 3 μm, the metal phase - TiC, whose size is 5 μm and the binder – ZrO2. As the percent of TiC is from 30% up to 45% and the maximum binder percent is 1% it results that the Al2O3 percent is the biggest (all results in 100%). Most of the studies presented by this paper were
carried out on Al$_2$O$_3$ + 30%TiC, that is conventionally named CC$_{cE}$-Al$_2$O$_3$/TiC.

2 Research Methodology

Fundamental principle of massive electrode electrical discharge machining by shape copying (PPE-E$_{m}$C$_o$) is that of material’s controlled erosion, as result of controlled electrical discharges between the part and the electrode, within a dielectric fluid environment.

Mechanism of material’s detachment, in PPE process, is that of thermal or, thermo-mechanical erosion as result of melting, vaporizing and expulsion of the detached material, together with the dielectric within the discharge zone.

The PPE process, as thermal one, is not affected by material’s mechanical properties but, it is affected by material’s properties such as thermal and electrical conductivity, density, melting temperature [3, 4].

Schematic representation of the PPE-E$_{m}$C$_o$ process is made in figure 1, while the microscope images of the sample part – before and, after the electrical discharge – are shown in figure 2 (a. and b.).

Obtaining the mathematical model of temperature distribution field, by considering the inter-dependences of the electrical discharge process factors (energy distribution within part, electrode, dielectric; plasma channel and pressure ball limits, etc.) is possible only if specific assumptions are made.

![Fig.1 Schematic model of PPE-E$_{m}$C$_o$ for CC$_{cE}$-Al$_2$O$_3$/TiC](image)

![Fig.2 Microscope images of Al$_2$O$_3$ + 30%TiC](image)

(a. initial sample Al$_2$O$_3$ + 30%TiC (x 1200))

(b. electrical discharged Al$_2$O$_3$ + 30%TiC (x 300)

$i_e = 3.13$ A; $t_i = 6$ $\mu$s; $t_0 = 190$ $\mu$s)

Fig.2 Microscope images of Al$_2$O$_3$ + 30%TiC
$(i_e$ – electric current intensity; $t_i$– impulse time; $t_0$ , pause time)
So, some of the hypotheses considered to be true are as follows:
- the part is considered to be continuous, heterogeneous and isotropic, as an elastic environment under thermal shock;
- the interactions of material, dielectric and their constituents are ignored;
- the resultant thermal flux toward the ceramic material is considered to be a semi-infinity solid;
- the intensity of thermal flux, Q(t), acts as a non-linear field, depending on time, t, and point’s position, x:

\[ Q(t) = Q(x,t) = Q(x,y,z,t) \]  

and its value is different from zero, when 
\[ t \in \left[ t_a + t_b + t_f \right] \], indexes a, b are associated to intermediate process moments and f is associated to final moment discharge;
- the thermal flux radius, r_t, acts like a non-linear field, depending on time;
- there is, also, a thermal influenced semi-spherical zone, within the material, whose radius is considered, r_{ti};
- the molten, or detached zone, resulting from electro-erosion thermal shock is determined by the position of temperature’s isotherms;
- the reference surface is always situated between two subsequent electrical discharges, so as the isotherms for material’s detaching refers to the new resulting surface of the previously discharge;
- because of the fact that the thermal extension coefficient, α, differs for the considered components, Al₂O₃ and TiC, while electrical discharging, there do appear an oscillatory spatial residual contractions field (compression, elongation);
- the oscillatory spatial residual contractions field determines the micro-fractures, micro-cracking and has significant role in their propagation, as well as in material’s detaching, by fragmentation, on electrical discharge thermal shock.

Heat distribution, within the considered granular Al₂O₃/TiC material is according to Cattaneo Iław [5]:

\[ \tau \frac{dQ(t)}{dt} + Q(x,t) = -k grad(\theta - \theta_0)(x,t) \]  

where:
- t - singular electrical discharge process time [μs];
- Q(x,t) - thermal flux intensity variation, as function of time and of three-dimensional vector, 
\[ x = (x_1, x_2, x_3) \]
- K – material’s thermal conductivity [W/mm°K]
- \( \theta_0 \) – material’s initial temperature, \( \theta_0 = 293 \ ^\circ \text{K} \);
- \( \theta \) – material’s temperature, at time, t [°K];
- \( \tau \) - relaxing time, when a stationary thermal flux is set into the material [μs].

### 3 Data and Results

The constitutive equations characterize the answer of material’s body to a certain solicitation and include relationships of stresses, strains, heat, temperature, internal energy. Because of the composite ceramics’ characteristics, the constitutive equations are considered to be established for a granular thermo-visco-elastic linear integral type, axial-symmetric, isothrop material, submitted to thermal field and with heat finite propagation speed, as a weave, within material.

According to the assumed hypotheses, the simplified constitutive equations of thermal field intensity is:

\[ Q_i(x,t) = -\frac{1}{\tau} \int_0^t k(s)(\theta - \theta_0)_i(x,s)e^{-\frac{t-s}{\tau}} ds \]  

where:
- s is time variable;
- \( k(s) \) - thermal conductivity function, time dependent;
- 0 index – initial state, at time \( t_0 \) (the beginning of singular electro-erosion discharge);
- \( i = 1,2,3 \)

Because of the assumptions made, that the discharge process is on while very shot period of times, the \( k(s) \) function is considered to be constant and, equal to its initial value, \( k_0 \) so, equation (2) turns to:

\[ Q_i(x,t) = -\frac{k_0}{\tau} \int_0^t (\theta - \theta_0)_i(x,s)e^{-\frac{t-s}{\tau}} ds \]

Other equations, necessary to the temperature distribution field study are the ones of impulse and energy equilibrium, expressed as:

\[ \rho_0 \ddot{\theta} - T_{\theta j} = 0 \]  

\[ \dot{\theta} + \frac{1}{\rho_0} Q_j = 0 \]

where::
- \( T_{\theta j} \) is the Cauchy stress tensor, expressing the stresses within material, because of erosive thermal shock, as function of position \( (x,y,z) \) and time, \( t \);
e – material internal energy, as function of position \((x,y,z)\) and time, \(t\);

\(\rho_0\) – material’s initial mass density, \([\text{g/cm}^3]\)

Knowing that material’s displacement is due, mainly, to electro-erosive thermal shock and, that, additional material deformation and cracking must take place, the thermal shock has to have the propagation wave in front of the longitudinal deformation wave.

Considering the real conditions of electrical discharge process, the thermal flux should be considered as:

\[
(\theta - \theta_0)(r,z,t) = \sum_{n=1}^{\infty} \frac{j_1}{2} \left( \frac{\alpha n r_p}{r_t} \right) J_0 \left( \frac{\alpha n r}{r_t} \right) \int_0^{\frac{s}{v_0}} \frac{1}{r^2} J_0 \left( \frac{v_0 \alpha n s}{r_t} \right) \left( 1 + 2 \alpha n t_1 \right) dt + \\
\sum_{n=1}^{\infty} \frac{j_1}{2} \left( \frac{\alpha n r_p}{r_t} \right) J_0 \left( \frac{\alpha n r}{r_t} \right) \int_0^{\frac{s}{v_0}} \frac{1}{r^2} J_0 \left( \frac{v_0 \alpha n s}{r_t} \right) \left( 1 + 2 \alpha n t_1 \right) dt
\]

When checking the obtained model, for real experimental conditions: \(k_0 = 20\ \text{W/mm}^2\k; v_0 = 0.22; r_1 = 10^{-5}\text{m}; \tau = 10^{-5}\text{s}; \alpha_0 = 8 \times 10^6\text{s/K}\), the result was:

\[
(\theta - \theta_0)(0,0,10^{-6}) < 3 \cdot 10^{-5}; \frac{5}{2} \left( 1 - e^{-10^6} \right) \approx 3 \cdot 10^{-5}\ \text{[°C]}
\]

4 Conclusion

Composites ceramics, Al\(_2\)O\(_3\)/TiCb, were studied and the machining procedure considered was massive electrode electrical discharge - by shape copying.

There was determined mathematical model of temperature field, as function of thermal flux radius, thermal influenced zone radius, radial direction of machined part and, time. Further research should be developed on specific stresses and displacement

Applying the model for real condition, showed good concordance to experimentally obtained data.

References:


