An Algorithm for Drawing a Contour Line that borders a Plane Surface by increasing the number of knowledge points

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Abstract: – In this paper we present an algorithm for drawing a contour line that borders a surface from a horizontal plane when we know a finite number of points. Starting from these points that are given through their coordinates, this algorithm allows the increasing of the number of known points and therefore by binding all the points (old and new) it results a contour line which approximates better the real contour line.

Key–Words: – Knowledge points, contour line, surface, tangentoid triangle, center of gravity.

1 Introduction
The idea of an algorithm for drawing the contour line which borders a surface from a horizontal plane, starts from the remark below.

Remark 1 In a set of points from space, each point has a vicinity and every vicinity describes a geometric figure and each of these geometric figures has a center of gravity.

Using this remark, the new method of drawing of the contour lines that border various sections used in precise purposes, allows the drawing of these lines using the new proposed algorithm. Thus one eliminates the drawing approximations of the contour lines that are used to determine the areas of the sections or the surfaces which form the faces of a volume.

2 Problem Formulation
Let \( M_i \) (\( i=1,n \)) be a set of points from the contour of a surface, from a plane. The distance between these points is variable.

Now, we formulate the following problem:

It is required to increase the number of these points such that the irregular polygon that results binding the points \( M_i \) can be approximated to another irregular polygon with a large number of sides, which can be considered a regular or an irregular curve.

3 Algorithm description
The proposed algorithm has the following steps:

a) By using the coordinates, one defines the knowledge points from a plane which are on the contour that border the surface of the section;

b) The rotation of the plane in space with the slope angle of the plane, in order to be parallel with the plane \( xOy \);

c) Determining the binding order of the knowledge points in a contour and the determining of the advancing way of the contour, (Fig. 1), which will be clockwise.

Considering the contour types, we have the following cases:

1) When the contour line that borders a surface is closed (Fig. 1);

2) When the contour line of a surface is not closed.

Remark 2 In the second case, the number of the points for the first interval \([M_1, M_2]\) and the last interval \([M_{n-1}, M_n]\) can’t increase, because each of the points \( M_1 \) and \( M_n \) are influenced by just one point \( (M_2 \) for \( M_1 \) and \( M_{n-1} \) for \( M_n \)).
d) Writing the equations of the straight lines that are determined by two successive points, with the relation:

\[
\frac{x-x_i}{x_{i+1}-x_i} = \frac{y-y_i}{y_{i+1}-y_i};
\]

e) Determining the values of the angles formed with the segments which have a common knowledge point (using clockwise we will determine the angles to the right of this sense), with the relation:

\[
\alpha = \arctan \left( \frac{y_{i+2} - y_i}{x_{i+2} - x_i} \right) - \arctan \left( \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \right),
\]

where \(a_i, a_2\) are the slope of the two straight lines which form the angle:

\[
y = a_1x + b_1, \quad y = a_2x + b_2.
\]

Considering the values of these angles, we have the following situations:

- parts of the contour that contain only the angles < 180°, (2–3–4–5–6; 7–8–9–10–11; 11–12–13–14–15);
- parts of the contour which contain only the angles > 180°, (5–6–7–8; 10–11–12);
- parts of the contour which contain an angle > 180° and another angle < 180°, (6–7–8, 7–8–9; 10–11–12, 11–12–13);
- parts of the contour that contain an angle < 180° and another angle > 180°, (4–5–6, 5–6–7; 9–10–11, 10–11–12);
- parts of the contour with the angle = 180°, (1–2–3).

f) Using the angles formed by the segment [Mi Mi+1] with the segments [Mi–1 Mi ] and [Mi+2 Mi+1], we construct the tangentoid triangles.

Considering the values of these angles, we have the following situations:

- situation 1, (Fig. 2), one refers to the contour parts which contain only the angles < 180° (in Fig. 2 the angles with the vertexes in the points 3, 4, 5 are the angles < 180°). In this case, the tangentoid triangles are constructed on the segments [3–4] and [4–5].

A tangentoid triangle for a segment [Mi Mi+1], it is constructed as follows:

- the segments [Mi–1 Mi ] and [Mi+2 Mi+1] are extended;
- the bisectors of the angles which are resulted by the extension of the segments [Mi–1 Mi ] and [Mi+2 Mi+1] with the segment [Mi Mi+1], are traced; thus, it is obtain the tangentoid triangle;
- the coordinates of the intersection point Ti, between the two bisectors are calculated.

The equation of the bisector of an angle is:

\[
\frac{x-x_i}{x_{i+1}-x_i} = \frac{y-y_i}{y_{i+1}-y_i}.
\]

In order to determine the coordinates of the center of gravity G, of a tangentoid triangle Mi Mi+1 Ti one uses the relations:

\[
x_g = \frac{x_1 + x_2 + x_3}{3}, \quad y_g = \frac{y_1 + y_2 + y_3}{3}, \quad z_g = \frac{z_1 + z_2 + z_3}{3}.
\]

In the case of the tangentoid triangle (Fig. 2) of the segment line [3–4] we obtain the point 3’ and for the tangentoid triangle of the segment line [4–5] we obtain the point 4’. The contour line of the contour part 3–4–5 will contain these points.

- situation 2, one refers to the contour parts which contain only the angles > 180° (in Fig. 3 the angles with the vertexes in the points 6 and 7 are angles > 180°).

In this situation, the tangentoid triangle is constructed in the same way as in situation 1, but it is inverse positioned with respect to the segment (if the triangle is situated at the on of the segment, in the situation 1, then will be to the right of the segment, in the situation 2). In Fig. 3 the tangentoid triangle is constructed to the right of the segment 6–7, and the resulted center of gravity is the point 6′.
The contour line which contains the segment [6–7], also contains this point.

**Remark 3** In the situation 1 the center of gravity of the tangentoid triangle is a point that will be situated to the left of the advancing way of the contour. In the situation 2 the center of gravity of the tangentoid triangle is a point which will be situated to the left of the advancing way of the contour.

–situation 3, (Fig. 4), one refers to the contour parts which have the first angle $> 180^\circ$ (the angle with the vertex in the point 7) and the second angle $< 180^\circ$ (the angle with the vertex in the point 8). In this case, the both tangentoid triangles from the situations 1 and 2, will be constructed. By the intersection of the bisectors of the angles which are formed in the two points of the segment $[M_iM_{i+1}]$, a tetragon will be obtained. By drawing the second diagonal in this tetragon, four triangles are obtained, two to the left of the segment 1 and two to the right of the segment with respect to the sense of the binding of knowledge points situated on the contour line. Now, we will determine the center of gravity $G_{id}$ for the triangle, which has a vertex in the point $M_i$, and is situated to the right of the segment $[M_iM_{i+1}]$. Also, we will determine the center of gravity $G_{is}$ for the triangle which has a vertex in the point $M_{i+1}$, and is situated to the left of the segment $[M_iM_{i+1}]$. The first point will be situated on a convex contour, and the other point will be situated on a concave contour with respect to the segment $[M_iM_{i+1}]$. In the example from Fig. 4 it is determine the coordinates of the points $7^\prime$ and $7^\prime\prime$ which are on the contour line between the points 7 and 8.

**Remark 4** In this case, we obtain two points that belong to the contour line that is plotted between the two research points.

–situation 4, (Fig. 5), one refers to the contour parts that have an angle $< 180^\circ$ (in Fig. 3, the angle with the vertex in the point 10) and another angle $> 180^\circ$ (the angle with the vertex in the point 11). This situation is similarly with the situation 3, but the first determined point ($10^\prime$) will be situated on a convex curve (i.e. is situated to the left of the segment $[10–11]$), and the second determined point ($10^\prime\prime$) will be situated on a concave curve (i.e. will be situated to the right of the segment $[10–11]$).

The algorithm is iterative and it stops when there is obtained a proper density of points. In Fig. 7 we present iteration 3.

![Fig. 4](image_url)

![Fig. 5](image_url)

![Fig. 6](image_url)

![Fig. 7](image_url)
4 Example
Let $M_i(x_i, y_i, z_i)$ be the set of points that passes through the contour line and borders a surface with a regular contour (circle):

<table>
<thead>
<tr>
<th>$X_i$</th>
<th>$Y_i$</th>
<th>$Z_i$</th>
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<tr>
<td>15.0000</td>
<td>65.0000</td>
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The result of the binding of these points is a square (Fig. 8, curve no. 1), that has the area $A_1=2500$.

![Fig. 8](image)

In order to establish the form of the contour line which borders the given section we will use the algorithm for increasing the number of knowledge points from the set $M_i$. The set of points, which has resulted after the iteration no. 3, is presented graphically by curve no.2 from Fig. 8. The value of the area of the section bordered by the contour no. 2, Fig. 8 is $A_2=3559.6251$.

The set of given points $M_i$, are situated on a circle (Fig. 8, curve 3) which has the area $A_3=3876.2482$.

Between the values of the areas of the bordered sections of the two contours (contours 2 and 3) it exists the following difference:

$A_3 - A_2 = 3876.2482 - 3559.6251 = 316.6231$.

5 Conclusion
The contour line that results by applying the proposed model (the iteration no. 3) describes approximately a circle.

The difference between the form of the contour which results by applying the proposed algorithm and the circle, results from the fact that the newly obtained points are the centers of gravity of the tangentoid triangles that are not situated on the circle.

The algorithm for drawing the contours of certain irregular plane surfaces approximate highly precise their form by comparison with the form of the contours that result by applying the classical methods.

This algorithm can be successfully used for drawing the contours of hidden surfaces, when one knows a small number of points that pass through this contour line.

References: