On the Wavelet OFDM Performance in Time Variant Channels: Choosing the Number of DWT Iterations

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Abstract: - The influence of the number of Discrete Wavelet Transform (DWT) iterations on a wavelet OFDM (WOFDM) system performance is studied in this paper. The WOFDM transmission is tested on a flat, time variant radio channel. Basically, the number of DWT iterations coincides with the number of transmission scales used. Our simulations show that by increasing the value of this parameter, the BER performance degrades. This result is explained by the fact that a small number of iterations keeps the duration of the transmitted symbols significantly shorter than the coherence time of the channel.

Key-Words: - WOFDM, DWT, iterations, time variability

1 Introduction
Wavelets represent a successful story of the last decade in signal processing applications. Thus, these signals, with some remarkable properties, are widely used in various domains as compression, denoising, segmentation or classification. By the other hand, in data communications, the same successful story can be assigned to multi-carrier modulation techniques. Practically, most of the data transmission systems nowadays use Orthogonal Frequency Division Multiplexing (OFDM) or some versions of it. We can mention here WiFi (IEEE 802.11), WiMAX (IEEE 802.16) or ADSL. It is especially the very good resilience of OFDM to the inter-symbol interference (ISI) that makes from this technique a reliable candidate for transmission in any dispersive channel. The WOFDM technique, sometimes referred to as wavelet modulation, is the point where the above concepts meet with each-other. Although they are widely used in signal processing, few wavelets applications are known in data transmission. The idea that gathers the two concepts is to use wavelet signals as carriers in a multi-carrier data transmission. Despite its undoubted advantages, OFDM presents some drawbacks too. Recent research has shown that, by associating the multi-carrier concept and the wavelet signals, some of the OFDM’s classical drawbacks can be counteracted. Thus, the sidelobes of the OFDM spectrum contain an important amount of energy, causing interference in the adjacent bands. This is not the case for WOFDM, which, due to the waveform of its carriers provides significant out-of-band rejection by comparison to OFDM [1-3]. Furthermore, the orthogonality of OFDM carriers relies on their precise positioning and spacing on the frequency axis [4]. It is the Doppler shift effect that occurs in the time variable channels, which particularly attacks this orthogonality. Unlike in OFDM, the orthogonality of the wavelet carriers relies on both their time position and their frequency localization (scale). This makes the WOFDM transmission less sensitive to Doppler, leading to noticeable BER improvements, under certain circumstances [5]. All the above considerations prove that an extensive investigation of WOFDM may be worthwhile. An important detail related to the two techniques is that their practical implementation relies on digital signal processing algorithms. Thus, an inverse transform implements the modulator and the direct transform will be the key point of the demodulator. In the case of WOFDM, the implementation is based on the famous Mallat’s filter bank algorithm, which computes the DWT [6]. This transform has two important parameters: the wavelets mother and the number of iterations used in computation. Previous research [7] has shown that the BER performance of WOFDM systems is significantly influenced by the wavelets mother choice. Now, we will focus on the other parameter of the WOFDM transmission, the number of DWT iterations.

In the next section, we will review the principles of WOFDM, focusing on its practical implementation. The transmission chain it’s
described in section 3. In section 4, simulation results are presented and commented. Last section is dedicated to some conclusions and possible advances on the subject.

2 WOFDM Overview

In any multi-carrier modulation, the orthogonality of the multiple carriers is the key point that allows their separation at receiver. The multicarrier approach has the advantage of the long symbol duration, provided by the simultaneous transmission of several low-rate parallel streams. OFDM relies on this idea and employs complex exponential carriers having frequencies which are multiples of $f_0$ (1):

$$\left. \begin{array}{c} f = 2 j k_0 e^{k subc} \\ \end{array} \right\} \text{otherwise}$$

where $subc(k)$ denotes the $k$-th subcarrier used. The idea which connects OFDM and wavelets is that, in the same manner that the complex exponentials are orthogonal to each-other, the members of a “wavelet family” will satisfy the same property:

$$< \psi_{j,k}(t), \psi_{m,n}(t) > = \begin{cases} 1, & \text{if } j = m \text{ and } k = n \\ 0, & \text{otherwise} \end{cases}$$

Such a family can be obtained by translating and scaling a unique function called wavelet mother and denoted by $\psi(t)$ (3):

$$\psi_{j,k}(t) = s^{-j/2}_0 \psi(s^{-j}_0 \cdot t - k \tau_0)$$

Equation 3 corresponds to a sampled version of a wavelet family, the discrete variables being $s_0$ (the scale) and $k$ (the position within the scale). In the following we will consider we considered $s_0=2$ and $\tau_0=1$. If $\{\psi_{j,k}\}$ forms an orthonormal, family, we can state that any signal $s(t)$ can be written as a sum of wavelet functions (4), the weights being the wavelet coefficients ($w_{j,k}$):

$$s(t) = \sum_{j \in \mathbb{Z}} \sum_{k \in \mathbb{Z}} w_{j,k} \psi_{j,k}(t)$$

In the context of our discussion, the signal $s(t)$ can be interpreted as being a “WOOFDM symbol”. In practice, in order to restrict the number of scales $j$ to a finite value, some “scaling functions” $\phi(t)$ must be employed. In fact, any wavelet from a certain scale $j$ can be composed as a weighted sum of scaling functions from the previous scale $j-1$. Thus, a different form of signal synthesis will be:

$$s(t) = \sum_{j \leq L} \sum_{k} w_{j,k} \psi_{j,k}(t) + \sum_{k} a_{L,k} \phi_{L,k}$$

where $j=L$ is the coarsest level used for signal composition, and $a_L$ are called approximation coefficients. However, in practice, the WOFDM signal $s(t)$ is not directly computed using equation (5). Rather, its discrete version $s[n]$ is calculated by performing the Inverse DWT (IDWT). At each iteration, an upsampling operation is performed (with the factor 2), followed by a filtering. (fig. 1).

Figure 1 describes the IDWT transform for three iterations (decomposition levels). $g_i$ and $h_i$ are the impulse responses of the synthesis low-pass and high-pass filter respectively. Their concrete form depends on the wavelets mother which is employed. We can consider that the data we have to transmit is a set of approximation and wavelet (detail) coefficients, as follows:

$$\text{data} = \{[a_L],[w_L],[w_{L-1}],...,[w_1]\}$$

After each iteration (from right to left) the number of wavelet coefficients halves. Thus, considering $w_1$ detail coefficients,
that our data vector from (6) has \( N \) samples, than half of them will be stored in \([w_1]\), and they will be transmitted in the channel using the upper half of the dedicated bandwidth. Next, \([w_2]\) contains one quarter of the symbols to be transmitted. Finally, considering the last iteration performed, let it be \( L \), we will have \( 2^{L-L} \) symbols grouped in the approximations vector and an equal number stored in the details vector, where \( J \) stands for the maximum number of iterations, which equals \( \log_2 N \).

The relation between fig. 1 and equation 5 is not straightforward. Note however that the best time (and the poorest frequency) resolution is achieved after the first iteration. At this point, we can figure out that the wavelet coefficients at this scale will modulate the wavelet carriers which have the best time localization within the wavelet family. Note that, higher the number of iterations, less compacted will be the corresponding wavelet carriers in time and more concentrated in frequency. Now, taking into account the time-scale nature of the wavelet transform, we can state that at those scales where the number of symbols to be transmitted is smaller, the duration of each symbol is higher. The same tendency can be highlighted for the wavelet carriers at different scales. These remarks are of particular interest for our study. Since WOFDM relies on an IDWT modulator, it follows that the transmission performance can be influenced by the parameters of this modulator. As stated earlier, we will focus on the number of iterations used in the IDWT computation.

3  The transmission chain

An area of particular interest for any multi-carrier modulation technique is the transmission in the radio channels. We try to follow this interest, by considering a flat Rayleigh fading channel scenario. The transmission chain employed for simulation purposes is shown in fig. 2.

![Fig.2: Baseband implementation of a WOFDM system.](image)

3.1 The transmitter

The data source is a sequence of equally likely bipolar symbols (+1 and -1). This data is grouped in blocks of \( N=1024 \) samples, before being transferred to the IDWT modulator. The IDWT modulator is the key point of the transmitter. Data is processed as shown in the previous section, and next transmitted in the channel. Because our main goal was to assess the influence of the number of IDWT iterations, we considered two cases with respect to this parameter: \( n_0=1 \) and \( n_0=4 \). In the first case, we can view the data vector at IDWT entry as being a sequence of 512 approximation coefficients and 512 detail coefficients. In the second case, the structure of the data vector will be interpreted by the modulator as:

\[
data = \{a_4(64), d_4(64), d_3(128), d_2(256), d_1(512)\}
\] (7)

The subscript values from relation 7 represent the iteration number, whereas in the parenthesis we retrieve the number of symbols transmitted at that scale. The modulator output will be a signal \( s[n] \) of 1024 samples. If we consider that this signal is a sampled version of an analog signal, the sampling step being \( T_s \), then the duration of the symbol transmitted after \( j \) iterations will be:

\[
T_j = 2^j T_s
\] (8)

This formula will become meaningful when we will try to explain some simulation results, during the next section.

3.2 The channel

The radio channel exhibits small scale fading, which confers to this transmission environment two independent characteristics: its time variance and its frequency selectivity [8]. The variance in time of the radio channel's behavior can be expressed by the mean of the Doppler shift parameter, which depends on the relative motion between transmitter and receiver (assuming mobile communications) and on the carrier frequency used for transmission. The author uses in this paper a normalized version of this parameter:

\[
f_m = f_d \cdot T_s
\] (9)
where $T_S$ is the symbol duration, coinciding with the sample time from (8). In our simulations, we considered two values for $f_m$: 0.005 and 0.05. Higher the value of this parameter, more rapidly the channel changes in time. A more straightforward parameter which quantifies the variability of the channel is the coherence time, expressed as:

$$T_C = \frac{0.423}{f_d}$$ (10)

By extracting $f_d$ from (9) and replacing it in (10), the coherence time can be expressed as:

$$T_C = \frac{0.423T_S}{f_m}$$ (11)

For the two values taken into account for $f_m$ we obtain the following coherence time values: $84.6T_S$ for the lowest value of the normalized Doppler, and $8.46T_S$ for the other one. These values will gain a particular interest for the explanation of the simulation results illustrated in the next section. The small scale fading can be modeled using a Rayleigh distribution. Its impact is given by the multiplicative $ray[n]$. A white noise $p[n]$ is next added to the signal above, obtaining the sequence $r[n]$ to be processed by the demodulator:

$$r[n] = s[n] \cdot ray[n] + p[n]$$ (12)

The white noise variance is gradually changed in order to be able to plot BER versus SNR curves.

### 3.3 The receiver

The key point of the receiver is the DWT “demodulator”. The decision on the transmission symbol is made based on a simple zero threshold comparison. The BER is computed for 1 and 4 DWT iterations, corresponding to the value of this parameter used at the transmitter side.

### 4 The simulation results

The WOOFDM transmission was simulated using Matlab 7. The wavelet related issues of our simulations are implemented in Wavelab, a free Matlab toolbox. The main parameters employed are synthesized in table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blocks</td>
<td>10000</td>
</tr>
<tr>
<td>Number of wavelet carriers</td>
<td>1024</td>
</tr>
<tr>
<td>Wavelets used as carriers</td>
<td>Haar, Daubechies-4, 8</td>
</tr>
<tr>
<td></td>
<td>12 and 16</td>
</tr>
<tr>
<td></td>
<td>Coifflet-1, 2, 3, 4 and 5</td>
</tr>
<tr>
<td></td>
<td>Symmlet-4, 6, 8 and 10</td>
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<tr>
<td>SNR range</td>
<td>0 to 30 dB</td>
</tr>
<tr>
<td>Channel</td>
<td>AWGN and Flat fading</td>
</tr>
<tr>
<td>Normalized Doppler shift, $f_m$</td>
<td>0.005 and 0.05</td>
</tr>
<tr>
<td>DWT iterations</td>
<td>1 or 4</td>
</tr>
</tbody>
</table>

Table 1: Simulation parameters

As the multitude of the parameters from table 1 demonstrates it, an extensive set of simulations was conducted. In our study, we will mainly focus on the influence of the last parameter from table 1, the number of DWT iterations used in the modulator and demodulator. The two values of this parameter were chosen such as to provide a relevant view of how this choice can modify the BER performance of our system.

Besides the classical BER measure, we will introduce another related parameter, for comparison purposes. This parameter will be called Error Increase Ratio (EIR). At a fixed SNR, this measure can be computed as follows:

$$EIR = \frac{BER_4}{BER_1}$$ (13)

where the numerical values, 4 and 1, point to the number of iterations. Higher the value of EIR, higher will be the performance degradation when the number of iterations increases.

As a general remark, for all the tested wavelets, the increase of the number of iterations led to a BER performance degradation. This results lead to an EIR which is generally higher than 1, no meters what is the SNR value. Furthermore, our measurements showed that this parameter is monotonically increasing with SNR. This fact is illustrated in fig. 3, for one wavelet selected from each family, and $f_m=0.05$. 


The explanation of this result is that, at high SNRs most of the errors will be caused by the Doppler shift, and not by the noise. As shown in some previous studies [5,7], the amount of errors caused by the variability in time of the channel is related to the time localization of the wavelet carrier. Thus, higher number of iterations means longer duration wavelet carriers and longer symbol time, as illustrated by equation 8. After each iteration (at each scale), the coherence time of the channel will exhibit a different degree of influence on the transmitted symbols.

The most spectacular difference can be observed for the Haar wavelet, where the BER at 30 dB is three times lower when a single iteration is used than in the case with four iterations. The other extreme is Daubechies-12 wavelet, with barely one half BER improvement at one iteration, compared to the Haar case. The other wavelets are somehow at the middle of the two cases. The reason behind this result is that amongst all wavelets, Haar is the more compact in time. Consequently, this wavelet is the more affected by the choice of the number of iterations. As shown in previous studies, the Haar wavelet provides, by far, the best results in the flat fading scenario, especially when a single iteration is used. This explains the spectacular growth of the curve in figure 3, at high SNR. In general, the wavelets with weak time localization will not be so sensitive to the number of iterations. Otherwise stated, they provide poor results even for a single iteration. A compendium of the results is shown in table 2. This table contains, besides the EIR, the gain brought by the use of a single scale, for all the tested wavelets. Because our SNR resolution was of 1dB, it was not possible to compute an exact value of this parameter. Instead, a minimal gain is defined. For this purpose, the BER at SNR=30 dB and 4 iterations is considered as a starting point. Next, we identify the lowest SNR for the one iteration case which leads to better results. The minimal gain will be the difference between the two SNRs. Note however that this measure represents a minimal gain only for the highest simulated SNR.

Table 2 shows that there is a totally different behavior pattern for \(f_m=0.05\) and \(f_m=0.005\). Under certain assumptions, the two scenarios can be considered as fast fading and slow fading respectively.

Thus, the number of iterations impact is much higher in the fast fading scenario. In this case, the time variability of the channel has a predominant role in the errors occurrence. In this case, the duration of the transmitted symbol after the third iteration (8T_s) is comparable with the coherence time (8.46T_s) and becomes much higher at the fourth scale (16T_s). This explains that, while at \(f_m=0.05\) spectacular gains are obtained (from 10 to 13 dB), the results for the slow fading case are rather irrelevant.

Effective BER results are shown in figures 4 and 5, for the extreme cases (bolded in table 2). We treated only the fast fading case, considering that in the slow fading the influence of the number of iteration is much smaller. The highest BER improvement at one iteration versus 4 iterations is obtained by the wavelet Daubechies-4. Thus, gain of over 13 dB is obtained in this case. Note that this wavelet is compact in time, and the measured

<table>
<thead>
<tr>
<th>Wavelet type</th>
<th>Min gain</th>
<th>EIR</th>
<th>Min gain</th>
<th>EIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coif1</td>
<td>12</td>
<td>2.35</td>
<td>3</td>
<td>1.02</td>
</tr>
<tr>
<td>Coif2</td>
<td>12</td>
<td>2.40</td>
<td>2</td>
<td>1.02</td>
</tr>
<tr>
<td>Coif3</td>
<td>12</td>
<td>2.14</td>
<td>4</td>
<td>1.05</td>
</tr>
<tr>
<td>Coif4</td>
<td>12</td>
<td>1.98</td>
<td>2</td>
<td>1.03</td>
</tr>
<tr>
<td>Coif5</td>
<td>12</td>
<td>1.87</td>
<td>2</td>
<td>1.03</td>
</tr>
<tr>
<td><strong>Daub4</strong></td>
<td><strong>13</strong></td>
<td><strong>3.40</strong></td>
<td><strong>0</strong></td>
<td><strong>1.00</strong></td>
</tr>
<tr>
<td>Daub8</td>
<td>13</td>
<td>2.30</td>
<td>4</td>
<td>1.02</td>
</tr>
<tr>
<td>Daub12</td>
<td>12</td>
<td>1.66</td>
<td>3</td>
<td>1.02</td>
</tr>
<tr>
<td>Daub16</td>
<td>11</td>
<td>1.38</td>
<td>6</td>
<td>1.08</td>
</tr>
<tr>
<td><strong>Daub20</strong></td>
<td><strong>10</strong></td>
<td><strong>1.19</strong></td>
<td><strong>5</strong></td>
<td><strong>1.09</strong></td>
</tr>
<tr>
<td>Symm4</td>
<td>12</td>
<td>2.35</td>
<td>2</td>
<td>1.00</td>
</tr>
<tr>
<td>Symm6</td>
<td>12</td>
<td>2.09</td>
<td>2</td>
<td>1.05</td>
</tr>
<tr>
<td>Symm8</td>
<td>12</td>
<td>1.93</td>
<td>1</td>
<td>1.02</td>
</tr>
<tr>
<td>Symm10</td>
<td>12</td>
<td>1.86</td>
<td>2</td>
<td>1.04</td>
</tr>
<tr>
<td>Haar</td>
<td>10</td>
<td>2.96</td>
<td>0</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Fig. 3: Error increase ratio for different wavelets, \(f_m=0.05\).
The parameters are very close to the Haar case. This supports our previous remarks about the better results of the short duration wavelets. By the other hand, even with an important gain of 10dB, the Daubechies-20 wavelet leads to very close curves in the two cases. This limits the practical significance of the 10dB gain. The result can be explained by the fact that Daubechies-20 is the more dilated in time amongst all the tested wavelets. In our simulations, we noticed that, from the BER point of view, this wavelet provides the worst performance. Thus, with a poor result even for one iteration, the WOFDM transmission based on this wavelet is not significantly impacted by increasing the number of transmission scales.

5 Conclusions and further work

The most important conclusion is that for the WOFDM transmission in the flat, time variant fading channel, using a single IDWT iteration leads to the best BER performance. This happens because when a single iteration is performed, the duration of the transmitted symbols (and of the carriers) is kept significantly shorter than the coherence time of the channel. This effect can be very well highlighted especially in the fast fading case, whereas in the slow fading scenario, the impact is rather limited.

Following the same logic, we proved that the wavelets which are the most sensitive to the choice of the number of IDWT iterations are the ones which, by their nature, are more concentrated in time (e.g. Haar, Daubechies-4).

For the continuation of this work, we intend to support our theory by performing an analysis of the way that the errors are distributed across the scales. Next, we will test the behavior of the WOFDM systems for channels which are not only time variant, but frequency-selective too.

References