Triple Selection Diversity over Exponentially Correlated Nakagami-*m* Fading Channels Desired Signal and Cochannel Interference

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Abstract: - In this paper, system performances of selection combining over correlated Nakagami-*m* channels are analyzed. Selection diversity based on the signal to interference ratio (SIR) is a very efficient technique that reduces fading and cochannel interference influence. Fading between the diversity branches and between interferers is correlated and Nakagami-*m* distributed with exponential correlation model.Very useful closed-form expressions for the output SIR's probability density function (PDF), cumulative distribution function (CDF), and outage probability are obtained, which is main contribution of this paper.

Key-Words: - Cochannel Interference; SIR; Nakagami-*m* Exponential Correlation Fading; Selection Combinig

1 Introduction

In wireless communication systems various techniques for reducing fading effect and influence of cochannel interference are used. Upgrading transmission reliability and increasing channel capacity without increasing transmission power and bandwidth is the main goal of diversity techniques. Diversity reception, based on using multiple antennas at the receiver (space diversity), is very efficient methods used for improving system's quality of service (QoS), so it provides efficient solution for reduction of signal level fluctuations in fading channels [2]. Multiple received copies of signal could be combined on various ways, and most popular of them are maximal ratio combining (MRC), equal gain combining (EGC) and selection combining (SC). MRC is the optimal combining scheme, but also the most complicated. EGC provides comparable performances to MRC technique but has lower implementation complexity, so it is an intermediate solution. With SC receiver, the processing is performed at only one of the diversity branches, which is selectively chosen, and no channel information is required. That is why SC is much simpler for practical realization. In general, selection combining, assuming that noise power is equally distributed over branches, selects the branch with the highest signal-to-noise ratio (SNR), that is the branch with the strongest signal [1–3]. In fading environments as in cellular communications systems where the level of the cochannel interference is sufficiently high as compared to the thermal noise, SC selects the branch with the highest signal-to-interference ratio (SIR-based selection diversity) [4]. This type of SC in which the branch with the highest SIR is selected, can be measured in real time both in base stations and in mobile stations using specific SIR estimators as well as those for both analog and digital wireless systems (e.g., GSM, IS-54) [5]. Most of the recently the published papers assume independent fading between the diversity branches and also between the cochannel interferers. However, in practice due to insufficient spacing between antennas, when diversity system is applied on small terminals with multiple antennas, correlation arises between branches [6].

While the Rayleigh and Rice distributions can be indeed used to model the envelope of fading channels in many cases of interest, it has been found experimentally, that the Nakagami distribution offers a better fit for a wider range of fading conditions in wireless communications[7]. The effect of correlated fading has been extensively analyzed on the performance metrics of wireless communication system [8,9]. In paper [10] analysis of signal combining for Nakagami-*m* distributed with exponential correlation model of fading has been given, but the influence of interference was not considered. More general case is when, the correlated interference is also present. Moreover to the best author's knowledge, no analytical study of multibranch selection combining involving assumed exponentially correlated Nakagami-*m* fading for both desired signal and co-channel interference, has been reported in the literature.

2 Problem Formulation

In this paper, we consider diversity system with triple correlated Nakagami-*m* fading channels with exponential correlation model in the presence of mutually correlated interferences. We model fading and interference by Nakagami-m distribution with exponential correlation model, which is an adequate for the scenario of multichannel reception from equispaced diversity antennas, in which the correlation between the combined signals decays as the spacing between the antennas increases[10]. In order to study the effectiveness of any modulation scheme and the type of diversity used, it is required to evaluate the system's performance over the channel conditions. In this paper, for proposed system model, expressions for probability distribution function (PDF) and cumulative distribution function (CDF) of the output SIR for selection combining diversity are derived. Numerical results for PDF and CDF are graphically presented. Furthermore, closed-form expressions for important performance measures such as the outage probability is obtained. Outage probability is shown graphically for different system parameters. Nakagami fading (*m*-distribution) describes multipath scaterring with relatively large delay-time spreads, with different clusters of reflected waves [11]. In provides good fits to collected data in indoor and outdoor mobile-radio environments and is used in many wireless communications applications. In this paper, wireless communication system with triple SIR-based SC diversity is considered. The desired signal received by the *i*-th antenna can be written as [12]:

$$D_i(t) = R_i e^{j\phi_i(t)} e^{j[2\pi f_c t + \Phi(t)]} \qquad i = 1, 2, 3 \tag{1}$$

where f_c is carrier frequency, $\Phi(t)$ desired information signal, ϕ_i (t) the random phase uniformly distributed in [0.2π], and $R_i(t)$, a Nakagami-*m* distributed random amplitude process given by [7]:

$$f_{R_i}(t) = \frac{2t^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(-\frac{t^2}{\Omega}\right); \qquad t \geq 0 \tag{2}$$

where $\Gamma(\bullet)$ is the Gamma function, $\Omega = t^2/m$, with t^2 being the average signal power, and m is the inverse normalized variance of t^2, whitch must satisfy

$$m \geq 1/2,$$

describing the fading severity. The resultant interfering signal received by the *i*-th antenna is:

$$C_i(t) = r_i(t) e^{j\theta_i(t)} e^{j[2\pi f_c t + \psi(t)]} \qquad i = 1, 2, 3 \tag{3}$$

where $r_i(t)$ is also Nakagami-*m* distributed random amplitude process, θ_i (t) is the random phase, and $\psi(t)$ is the information signal. This model refers to the case of a single cochannel interferer.

The performance of the triplebranch SC can be carried out by considering, as in [13], the effect of only the strongest interferer, assuming that the remaining interferers are combined and considered as lumped interference that is uncorrelated between antennas. Furthermore, $R_i(t)$, $r_i(t)$, ϕ_i (t), and $\theta_i(t)$ are assumed to be mutually independent is sufficiently high for the effect of thermal noise on system performance to be negligible (interference-limited environment) [13]. Now, due to insufficient antennae spacing, both desired and interfering signal envelopes experience correlative multivariate Nakagami-*m* fading with joint distributions.

We are considering exponential correlation Nakagmi-m model of distribution. We are assuming arbitrary correlation coefficients between fading signals and between interferences, because correlation coefficients depend on the arrival angles of the contribution with the broadside directions of antennas, which are in general case arbitrary [14]. The exponential correlation model [15] can be obtained from by setting

$$\Sigma_{i,j} \equiv \rho^{|i-j|}$$

for in correlation matrix, for both desired signal and interference. Now joint distributions of pdf for both desired and interfering signal correlated envelopes for multi-branch signal combiner could be expressed by[10]:

$$p_{R_1,R_2,R_3}(R_1,R_2,R_3) = \frac{R_1^m R_2 R_3^m}{2^{m-1}\Gamma(m)\rho_d^{2(m-1)}\left(1-\rho_d^2\right)^2} \exp\left[-\frac{R_1^2 + \left(1+\rho_d^2\right)R_2^2 + R_3^2}{2\left(1-\rho_d^2\right)}\right]$$

$$\times I_{m-1}\left(\left(\frac{\rho_d}{1-\rho_d^2}\right)R_1 R_2\right) I_{m-1}\left(\left(\frac{\rho_d}{1-\rho_d^2}\right)R_2 R_3\right) \tag{4}$$

$$p_{r_1,r_2,r_3}(r_1,r_2,r_3)=\frac{r_1^m r_2 r_3^m}{2^{m-1}\Gamma(m)\rho_c^{2(m-1)}\left(1-\rho_c^2\right)^2}\exp\left[-\frac{r_1^2+\left(1+\rho_c^2\right)r_2^2+r_3^2}{2\left(1-\rho_c^2\right)}\right]$$

$$\times I_{m-1}\left(\left(\frac{\rho_c}{1-\rho_c^2}\right)r_1r_2\right)I_{m-1}\left(\left(\frac{\rho_c}{1-\rho_c^2}\right)r_2r_3\right) \qquad (5)$$

I_{m-1} (.) is modified Bessel function of the first kind and m-1 order, ρ_d is the power correlation coefficient defined as

$$\rho_d = cov(R_i^2, R_j^2)/(var(R_i^2)var(R_j^2))^{1/2}$$

with $0 \le \rho_d \le 1$ and ρ_c is interference correlation coefficient defined as

$$\rho_c = cov(r_i^2, r_j^2)/(var(r_i^2)var(r_j^2))^{1/2}$$

with $0 \le \rho_c \le 1$. Moreover, without loss of generality and for simplification purposes of the matrix Σ, it is assumed into (4) and (5) that $\Omega_{di}=2\sigma_{di}^2$, $\Omega_{ci}=2\sigma_{ci}^2$ i=1,2,3, with $\sigma_{di}=1$ and $\sigma_{ci}=1$ being the variances of desired signal and interference, respectively. Instantaneous values of SIR at the diversity branches input can be defined as $\lambda_1=R_1^2/r_1^2$, $\lambda_2=R_2^2/r_2^2$ and $\lambda_3=R_3^2/r_3^2$. The selection combiner chooses and outputs the branch with the largest SIR.

$$\lambda = \max\left(\lambda_1,\lambda_2,\lambda_3\right) \qquad (6)$$

Joint probability density function of instantaneous values of SIR in three output branches λ_1, λ_2 and λ_3 is as in [4]:

$$p_{\lambda_1,\lambda_2,\lambda_3}(t_1,t_2,t_3)=\frac{1}{8\sqrt{t_1t_2t_3}}\int_0^\infty\int_0^\infty\int_0^\infty p_{R_1R_2R_3}\left(r_1\sqrt{t_1},r_2\sqrt{t_2},r_3\sqrt{t_3}\right)p_{r_1r_2r_3}(r_1,r_2,r_3)r_1r_2r_3dr_1dr_2dr_3 \qquad (7)$$

Substituting (4) and (5) in (7), $p_{\lambda1,\lambda2,\lambda3}(t_1,t_2,t_3)$ can be written as

$$p_{\lambda_1,\lambda_2,\lambda_3}(t_1,t_2,t_3)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\Bigg[\;G_1\frac{t_1^{i+m-1}}{\left(t_1\left(1-\rho_c^2\right)+\left(1-\rho_d^2\right)\right)^{i+j+2m}}$$

$$\times\frac{t_2^{i+l+m-1}}{\left(t_2\left(1+\rho_d^2\right)\left(1-\rho_c^2\right)+\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)\right)^{i+j+l+n+2m}}$$

$$\times\frac{t_3^{l+m-1}}{\left(t_3\left(1-\rho_c^2\right)+\left(1-\rho_d^2\right)\right)^{l+n+2m}}\;\Bigg] \qquad (8)$$

$$G_1=\frac{1}{\Gamma(m)^2}\frac{\left(1-\rho_d^2\right)^{2j+2n+4m}\left(1-\rho_c^2\right)^{2i+2l+4m}\rho_d^{2i+2l}\rho_c^{2j+2n}}{i!j!l!n!}$$

$$\times\frac{\Gamma(i+j+2m)\Gamma(i+j+l+n+2m)\Gamma(l+n+2m)}{\Gamma(i+m)\Gamma(j+m)\Gamma(l+m)\Gamma(n+m)} \qquad (9)$$

3 Problem Solution

3.1 Cumulative distribution function

For this case cumulative distribution function can be written as [4]

$$F_\lambda(t)=\int_0^t\int_0^t\int_0^t p_{\lambda_1,\lambda_2,\lambda_3}(x_1,x_2,x_3)dx_1dx_2dx_3 \qquad (10)$$

Substituting expression (8) in (10), and after triple integration joint cumulative distribution function becomes:

$$F_\lambda(t)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\Bigg[\;G_2\times\frac{\left(\frac{t}{\left[\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t\right]}\right)^{i+m}\left(\frac{t}{\left[\frac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_c^2\right)\left(1+\rho_d^2\right)}+t\right]}\right)^{i+m+l}\left(\frac{t}{\left[\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t\right]}\right)^{l+m}}{(i+m)(i+m+l)(m+l)}$$

$$\times\,{}_2F_1\left(i+m,1-j-m;i+m+1;\frac{t}{\left[\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t\right]}\right)$$

$$\times\,{}_2F_1\left(i+l+m,1-j-n-m;i+l+m+1;\frac{t}{\left[\frac{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}{\left(1-\rho_d^2\right)\left(1+\rho_c^2\right)}+t\right]}\right)$$

$$\times\,{}_2F_1\left(l+m,1-n-m;l+m+1;\frac{t}{\left[\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t\right]}\right)\;\Bigg]; \qquad (11)$$

$$G_2=\frac{1}{\Gamma(m)^2}\frac{\left(1-\rho_d^2\right)^m\cdot\left(1-\rho_c^2\right)^m\cdot\rho_d^{2i+2l}\cdot\rho_c^{2j+2n}}{i!\cdot j!\cdot l!\cdot n!\cdot\left(1+\rho_d^2\right)^{i+l+m}\cdot\left(1+\rho_c^2\right)^{j+n+m}}$$

$$\times\frac{\Gamma(i+j+2m)\cdot\Gamma(i+j+l+n+2m)\cdot\Gamma(l+n+2m)}{\Gamma(i+m)\Gamma(j+m)\Gamma(l+m)\Gamma(n+m)} \qquad (12)$$

and $_2F_1(u_1,u_2;u_3;x)$, being the Gaussian hypergeometric function [16, (9.100)].

3.2 Probability distribution function

Probability density function (PDF) of the output SIR can be obtained easily from previous expression:

$$p_\lambda(t)=\frac{d}{dt}F_\lambda(t)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\Bigg[\;G_2 t^{2i+2l+3m-1}$$

$$\times\Bigg\{\Bigg[\frac{\left(\frac{1-\rho_d^2}{1-\rho_c^2}\right)^{j+m}}{\left(\frac{1-\rho_d^2}{1-\rho_c^2}+t\right)^{i+j+l+3m}\left(\frac{(1-\rho_d^2)(1+\rho_c^2)}{(1-\rho_c^2)(1+\rho_d^2)}+t\right)^{i+l+m}(i+l+m)(l+m)}$$

$$\times{}_2F_1\left(i+l+m,1-j-n-m;i+l+m+1;\frac{t}{\frac{(1-\rho_d^2)(1+\rho_c^2)}{(1-\rho_c^2)(1+\rho_d^2)}+t}\right)$$

$$\times{}_2F_1\left(l+m,1-n-m;l+m+1;\frac{t}{\frac{(1-\rho_d^2)}{(1-\rho_c^2)}+t}\right)\Bigg]$$

$$+\Bigg[\frac{\left(\frac{(1-\rho_d^2)(1+\rho_c^2)}{(1-\rho_c^2)(1+\rho_d^2)}\right)^{j+m+n}}{\left(\frac{1-\rho_d^2}{1-\rho_c^2}+t\right)^{i+l+2m}\left(\frac{(1-\rho_d^2)(1+\rho_c^2)}{(1-\rho_c^2)(1+\rho_d^2)}+t\right)^{i+j+l+n+2m}(i+m)(l+m)}$$

$$\times{}_2F_1\left(i+m,1-j-m;i+m+1;\frac{t}{\frac{(1-\rho_d^2)}{(1-\rho_c^2)}+t}\right)$$

$$\times{}_2F_1\left(l+m,1-n-m;l+m+1;\frac{t}{\frac{(1-\rho_d^2)}{(1-\rho_c^2)}+t}\right)\Bigg]$$

$$+\Bigg[\frac{\left(\frac{1-\rho_d^2}{1-\rho_c^2}\right)^{n+m}}{\left(\frac{1-\rho_d^2}{1-\rho_c^2}+t\right)^{i+l+n+2m}\left(\frac{(1-\rho_d^2)(1+\rho_c^2)}{(1-\rho_d^2)(1+\rho_c^2)}+t\right)^{i+l+m}(i+l+m)(i+m)}$$

$$\times{}_2F_1\left(i+l+m,1-j-n-m;i+l+m+1;\frac{t}{\frac{(1-\rho_d^2)(1+\rho_c^2)}{(1-\rho_c^2)(1+\rho_d^2)}+t}\right)$$

$$\times{}_2F_1\left(i+m,1-j-m;i+m+1;\frac{t}{\frac{(1-\rho_d^2)}{(1-\rho_c^2)}+t}\right)\Bigg]\Bigg\}\;\Bigg]$$

(13)

Probability density function of output signal to interference ratio for of SIR at the input of the branches and various values of correlation coefficient is shown at Fig. 1

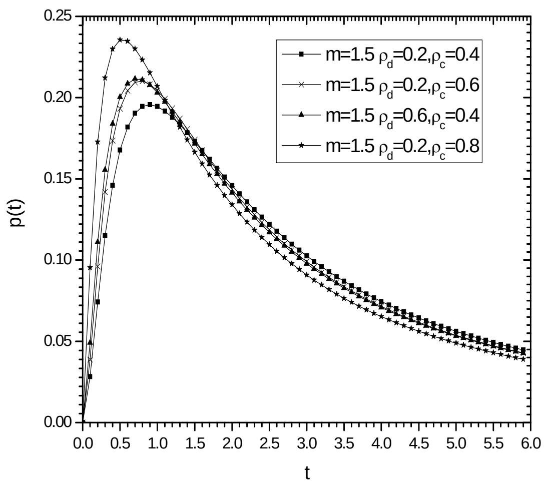

Fig. 1. Probability density function of output SIR for various values of correlation coefficients

3.3 Outage probability

One of the accepted performance measure for diversity systems operating in fading environments is the outage probability P_{out} . This performance measure is very useful in wireless communication systems design especially for the cases when cochannel interference is present.

In the interference limited environment, P_{out} is defined as the probability that the output SIR of the SC falls below a given outage threshold γ also known as a protection ratio.

$$P_{out}=P_R(\xi<\gamma)=\int_0^{\gamma}p_\xi(t)dt=F_\xi(\gamma)\quad .$$

(9)

Protection ratio depends on modulation technique and expected QoS.

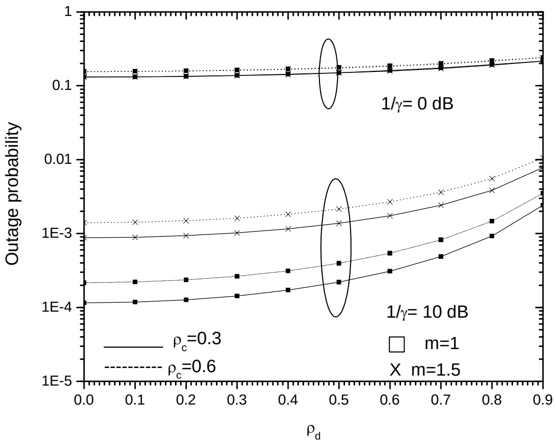

Fig. 2. Outage probability versus ρ_d

Fig. 2. shows the outage probability versus the power correlation coefficient ρ_d for desired signal and several values of m. It is evident that for strong interference the outage probability increases slowly as the correlation coefficient increases, while small increase in m does not have significant effect on outage probability. For higher values of normalized SIR the influence of m and ρ_d becomes stronger.

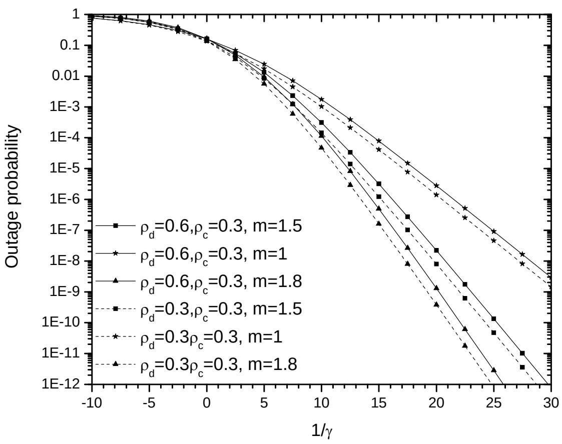

Fig. 3. Outage probability versus average SIR normalized by the protection ratio for various values of correlation coefficients and fading severity

Fig. 3. shows the outage probability versus average SIR normalized by the protection ratio for various values of the power correlation coefficient ρ_d for desired signal and several values of m. Average SIR's at the i-th input branch of the triple-branch selection combiner is defined as $S_i = \Omega_{di} / \Omega_{ci}$. Without loss of generality, it is assumed that $\Omega_{di}=\Omega_{ci}=1$. It is very interesting to observe that for lower values of $1/\gamma$ (< 3dB) outage probability deteriorates when the fading severity decreases due to interference domination. For higher values of $1/\gamma$ (when desired signal dominates), interference fading severity increase leads to outage probability decrease. Also, it's shown that as the fading severity index m increases and normalized outage threshold decreases, P_{out} slightly decreases as well. However, as the signal correlation coefficient, ρ_d increases and normalized outage threshold decreases, P_{out} increases.

4 Conclusion

In this paper, the performance of system with selection combining over exponentially correlated Nakagami-m channels in the presence of interference, was studied. Fading was modelled as exponentially correlated Nakagami-m process which is flexible model providing very good fit to experimental fading channel measurements for both indoor and outdoor environments. Channel interference is also modelled as exponentially correlated Nakagami-m process. The complete statistics for the SC output SIR is given in the closed form, i.e, PDF, CDF. Using these new formulae, outage probability was efficiently evaluated. As an illustration of the mathematical formalism, numerical results of these performance criteria are presented, describing their dependence on correlation coefficient and fading severity. The main contribution of this analysis of triplebranch signal combiner is that it has been done for the first time considering general case when correlated cochanel interference is present.

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