A Software Tool for Study Stability and Equilibrium on Financial Derivates Market

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Abstract: The stability and equilibrium of the stock market system constitutes a permanent preoccupation of the management system. The stock market manager is interested in obtaining certain stability and a real equilibrium of the market which he organizes in the purpose of making transactions with financial assets. Starting from this idea, in this paragraph we want to model the activity of the stock market system with the help of certain general models of stability and economic equilibrium.

Keywords: Stability, equilibrium, system, dinamic, Hicks model, software tool

1. Introduction

The structure of the system and his connections determine a certain evolution of it and measured at a certain moment (the value of the parameters, which are instruments of regulation of the system), represents the system’s state.

If it’s admitted that any behavior is a form of manifestation of the system – environment interaction, than the equilibrium state of the system is the expression of compatibility of the decisions taken by the management system, following a relative compatibility between the two, which can be maintained for a certain period of time.

Looking at it from the manifestation point of view we can observe the static and dynamic equilibrium. Considered as a work apotheosis, so with no effective correspondent in reality, the static equilibrium can be characterized thru the manifestation of almost unseen changes, insignificant between the different processes or structures of the stock market, in such a way that the general state of it remains unchanged.

2. Problem formulation

The general models of stability and equilibrium, of the stock market system are versions of classical models of regulation of the economic processes. From all of them we have stopped at the model proposed by Hicks [1]. The Hicks model is a complex model; it is an accelerator type of dynamical equilibrium to of the national product, based on the following relations:

\[ Y(t) = C(t) + I(t) + G(t) \]  \hspace{1cm} (1)
\[ C(t) = a Y(t-1) \]  \hspace{1cm} (2)
\[ I(t) = b (Y(t-1) - Y(t-2)) \]  \hspace{1cm} (3)

At the level of system object – Market of financial derivates – the parameters of the Hicks model modify the variables of the system are:

- \( Y(t) \) = offer assets
- \( C(t) \) = values transactions
- \( I(t) \) = investments
- \( G(t) \) = demand assets

Starting from the Hicks [8], we exemplify a model of economic dynamics described by the ordinary differential equation (II order):

\[ \dot{Y} + (1 + s - v) Y + s Y = G \]  \hspace{1cm} (4), which can be put under the form of a dynamic system with the

help of the relation \( \dot{Y} = \dot{X} \) \hspace{1cm} (5).

Using the relation above, we obtain the following system of two equations, representing the processes that take place in the same time, \( t \) defining a structure of relations between the variables \( Y \) and \( X \) of the stock market system.

\[
\begin{align*}
X &= Y \\
Y &= -sX -(1+s-v)Y+G
\end{align*}
\]  \hspace{1cm} (6)

where : \( Y \) = the offer of financial assets;
\( G \) = the demand for financial assets;
In the case of the stock market system, as in any real system the simple comparison of the two system variables (Y and X) is difficult, being perturbed by the effects other variables which occur within the system. Because of this, in this model:

- \( l/s \) are called multiplier – tendency towards transitioning;
- \( v \) accelerator.

It is imposed to exemplify the necessity of introducing the two parameters (\( l/s \) and \( v \)) because these will influence the general behavior of the system. The correlation which is formed in the stock market time and space between demand, offer, investments, profit, and transactions is under the direct influence of the principals of the multiplier and accelerator. We have identified the multiplier \( l/s \) as being tendency towards investment starting from the principals of the multiplier [2], according to the economic theory and used frequent in the activity of modeling a system. Thru it’s meaning, the multiplier, proves the direct connection between the entries in the stock market – investments– and it’s exists, under the form of income for the participants at the stock market activity.

The principal of the multiplier expresses the interaction which is formed between the income growth (\( \Delta Y \)) and the growth of stock market investment (\( \Delta I \)), under the form of a coefficient of amplification which shows the size of the growth in income, obtained thru transactions at the growth with one unit of the sums invested. The stock market investments compel the transactions in the stock market ring, generating incomes – but attention – only in the conditions of stability and equilibrium.

The principal of the accelerator expresses the effect that income growth has over investment in the sense that the growth with one unit of the demand of financial assets (\( AG \)) begins to force a direct growth of the offer. Between the demand of assets and investments there is an acceleration relationship. Calculated as a fraction between the size of investment during one year \( I(t) \) and the variation of the income during the previous period, the accelerator tries to give a theoretic explication of the variation in income growth [3].

We will study mathematically the behavior in time of this system remembering that economically looking at it: \( s \in (0,1) \) and \( v > 0 \). (9)

The dynamic of the economical system supposes determining the points of equilibrium and the zones of stability inside the space of the parameters \( (s, v, G) \).

Points of equilibrium

The points of equilibrium are the solutions of the algebraic system:

\[
\begin{align*}
X &= 0 \\
Y &= 0 \\
\Rightarrow sX - (1 + s - v)Y + G &= 0
\end{align*}
\]

\[
X = \frac{G}{s}
\]

where \( s \neq 0 \) (10)

So, for \( s \neq 0, G \neq 0 \) the system admits only one point of equilibrium \( \left( \frac{G}{s}, 0 \right) \). For \( s = 0, G = 0 \) admits an infinity of equilibrium points and if \( s = 0, G = 0 \) the system does not admit equilibrium points [11].

We will study the system behavior around the equilibrium \( u_0 = \left( \frac{G}{s}, 0 \right) \). The initial system is an affine system and his linearity around the equilibrium point is:

\[
\begin{align*}
X &= Y & (7) \\
Y &= -sX - (1 + s - v)Y
\end{align*}
\]

The attached matrix will be

\[
A = \begin{bmatrix} 0 & 1 \\ -s & -(1 + s - v) \end{bmatrix}
\]

and the characteristic equation is:

\[
\lambda^2 + (1+s-v)\lambda + s = 0 \quad (11), \quad \text{with discriminant}
\]

\[
\Delta = (1+s-v)^2 - 4s = s^2(2(1+v)s + (v-1)^2) \quad (8).
\]

We will discuss the sign of \( \Delta \), considering \( \Delta = 0 \), equation in \( s \):

\[
\Delta' = 4(1+2v+v^2) - 4(v^2 - 2v + 1) = 16v \quad (9)
\]

If \( v > 0 \) \( \Rightarrow \Delta'>0 \) and the equation \( \Delta = 0 \) has the following solutions:

\[
\begin{align*}
s_1 &= \frac{2(1+v)-4\sqrt{v}}{2} = 1 + v - 2\sqrt{v} = \left(1-\sqrt{v}\right)^2, \\
s_2 &= \frac{2(1+v)+4\sqrt{v}}{2} = 1 + v + 2\sqrt{v} = \left(1+\sqrt{v}\right)^2 > 1 \quad (11)
\end{align*}
\]

The two parameters \( s \) and \( v \) determine the stability of \( Y_t \), according to the nature and signs of \( \lambda_1 \) and \( \lambda_2 \), therefore in the three cases in which the determinant \( \Delta \) is positive, null or negative.
Table 1

<table>
<thead>
<tr>
<th>S</th>
<th>The solutions of the characteristic equation</th>
<th>Type of equilibrium point</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>0&lt;s&lt;s₁</td>
<td>$\Delta&gt;0, \quad \lambda_{1,2} = \frac{1 + s - v \pm \sqrt{\Delta}}{2} \in \mathbb{R}$</td>
<td>$P = s&gt;0$ 1+s-v&lt;0 node repulse</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1+s-v&lt;0 node repulse</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1+s-v&gt;0 node attractive</td>
<td></td>
</tr>
<tr>
<td>S=s₁</td>
<td>$\Delta = 0, \quad \lambda_{1} = \lambda_{2} = 1 - \sqrt{v}$</td>
<td>v&gt;1 node attractive</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>v&lt;1 node repulse</td>
<td></td>
</tr>
<tr>
<td>s₁&lt;s&lt;s₂</td>
<td>$\Delta &lt; 0, \quad \lambda_{1,2} = \frac{-1(1 + s - v) \pm i\sqrt{\Delta}}{2}$</td>
<td>1+s-v&lt;0 focal point repulse</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1+s-v=0 solution periodic</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1+s-v&gt;0 focal point attractive</td>
<td>C</td>
</tr>
<tr>
<td>S=s₂</td>
<td>$\Delta = 0, \quad \lambda_{1} = \lambda_{2} = 1 + \sqrt{v} &gt; 0$</td>
<td>node attractive with only one direction asymptotic</td>
<td>3</td>
</tr>
<tr>
<td>s&gt;s₂</td>
<td>$\Delta&gt;0, \quad \lambda_{1,2} = \frac{1 + s - v \pm \sqrt{\Delta}}{2} \in \mathbb{R}$</td>
<td>$P = s &gt;0$ node repulse</td>
<td>D</td>
</tr>
</tbody>
</table>

3. Problem solution

3.1. Mathematical Modeling

The solutions offered by the model have been synthesized with the help of Table 1. These results allow the determination of the type of movement around the equilibrium point, underlining the complex situations that may occur. For an easier interpretation we will consider Fig.1 on which we have identified the four regions:

![Fig.1 Delimitation of the regions in the plane (v, s)](image)

**Region A** – the stability condition is not satisfied because: $1 + s - v > 0$ (12) and $s > (1 + \sqrt{v})^2$ (13).

The movement is momentarily explosive.

region B – In this case $1 + s - v > 0$ (15), $(1 - \sqrt{v})^2 < s > (1 + \sqrt{v})^2$ (16), the stability condition being satisfied, the roots $\lambda_{1}$, $\lambda_{2}$ are complex with the real part not null but positive. An explosive oscillation will take place around the equilibrium point.

![Fig.2](image)

![Fig.3](image)
Region C – has $1 + s - v < 0$ (17) and $(1 - \sqrt{v})^2 < s > (1 + \sqrt{v})^2$ (18), the roots being complex, with the real part not null, negative. Therefore the stability condition is satisfied, obtaining an amortization oscillation around the equilibrium point $n_0$.

Region D – is determined by the following conditions $1 + s - v < 0$ (19) and $0 < s < (1 - \sqrt{v})^2$ (20). The stability condition is satisfied, the roots $\lambda_1, \lambda_2$ are real and negative. The stock market system will have a monotone motion converge towards the equilibrium value.

Fig. 4

Particular case 1 – in this situation, the movement is dominated by a monotone movement infinite. Particular case 2 – For the points situated on the separation line between region B and region C, $\lambda_1, \lambda_2 \in C$ with $\text{Real} \lambda_1 = \text{Real} \lambda_2 = 0$ therefore the characteristic roots are purely imaginary, which a constant magnitude of the oscillation. In this case the movement is neutrally stable and the delimitation line separates the stability zone (region B) from the instability zone (region C). Particular case 3 – For the points on the separation line between region A and region B we will have $\lambda_1 = \lambda_2 = \lambda < 0$ and the movement converge monotone towards equilibrium.

3.2. Software Platform

XTrade

XTrade is a software tool, of great help for the identification and measurement of the relations between different dimensions of data and especially of those that have in common certain sets of values that can forecast in their turn, other sets of values. Thus, with this software tool can be obtained complex relations between data allowing a qualitative and quantitative analysis of the data necessary for the development of an efficient management of the operations with financial derivatives products.

The system has incorporated the application WinPP, which allows the verification of the stability and equilibrium of the stock-exchange system analyzed. The application constitutes a true tool of analysis that can supply information at any moment regarding to ‘how stable is the market?’ and ‘what must be done taking into account the signals indicated?’

In Figure 6 it is presented through a print screen the application WinPP in execution and the result that is obtain after the establishment of the phase trajectories by this. The dialog box Fixed Points indicates STABLE, which means the stability of the balance according the data introduced. At the same time, there are supplied to the user the coordinates of the equilibrium point as well as the proper corresponding values ($\lambda_1, \lambda_2$ see Table 1).

Fig. 6

Also, WinPP facilitates the visualization of the variables in time as well as the evolution of a variable in report with the other one. For this, it is appealed to the menu Graphics and the command button View so that, to be determined the axis of...
coordinates \((x_1, x_2 \text{ or } t)\) and to be obtained a tridimensional graphic according these variables (see Dialog box from Fig.7 below).

\[\text{Fig. 7}\]

4. Conclusions

The dynamic of the operations with financial derivates products and the informational explosion from the ‘stock-exchange ring’, make the utilization of XTrade a software tool, a necessity. Despite the limits imposed by the impossibility of perfect reproduction by the computer of the human reasoning, XTrade represents the indispensable instrument of the modern manager of portfolio, because the majority of the decisional activities of routine, the total volume of the information necessary for the decisional model together with the techniques of research and re-finding of the information are completely overtaken by this.

The functionalities offered by XTrade to the management of operations, are detached from the general advantages that spring from the introduction of soft systems in the assistance of decisions, regarding to the: access of pertinent information and the control of access, manipulation of the information and presentation of the results, structuring of the information as models: relations between variables, systems of relations, decisional trees, making simulations, generation of alternatives (scenarios) and their evaluation, conservation as library of concepts and useful information in the interpretation of results.

References:
[9] Ștefănescu L., Managementul operațiunilor cu produse financiare derivate asistat de calculator, Editura Didactica și Pedagogică, București, 2005, in Romanian