New Trends in Analytical and Numerical Computation

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Abstract: Any dynamical system evolution is analyzed by a mathematical model attached to physical model. A mathematical model is compound of the analytical and numerical phase of your deduction. In the paper, the possibilities of replacing the manual analytical calculation that intervene in the mechanical modeling, by an isomorphic numerical calculation which can be performed on digital computers are described. An original algorithm for performing the greatest common divisor of two polynomials with several variables to determine the analytical inverse matrix for a matrix of such polynomials used in a mathematical modeling of mechanical phenomena is précised. A new definition of the Euclidean ring is proposed and new possibilities of the theoretical algebraic research such are underlined. A possibility of applying our algorithm for performing the inverse matrix of differential polynomial operator matrix that intervene in the modeling for enlarger the analytical phase in the mathematical model deduction is also emphasized.

Key-Words: mechanical modeling, formal (analytical) calculation, coding of operations, shells, matrix of polynomials.

1 Introduction
A mathematical model, for study any dynamical system evolution, is needed to be attached. The analytical phase and the numerical phase in deduction of the mathematical model are needed to be performed. The analytical phase consist in specification of the model parameters and unknowns, performing of the model equations, other conditions as initial conditions, boundary conditions or restrictions of the model, using parameters and unknowns established. The numerical phase consists in assign of the numerical values to the parameters, numerical method selection (development in series, finite difference method, finite element method, boundary finite element method, etc) and deduction of the unknown values.

A competition between analytical and numerical phases of the mathematical model for performing them there exist. A large analytical phase is desired for increasing precision of the final calculus. Both phases through manual operations or computer operations are going. The possibilities of replacing the manual analytical calculation that intervene in the modeling by an isomorphic numerical calculation which can be performed on digital computers are used ([8], [11]). For coding the summing and the product operations in the set of polynomials of several variables with real or integer coefficients that intervene in the modeling, we start with the coding of the algebraic operations for two monomials. These codifications are précised. The order of independent or dependent variables that intervene in the mathematical model must be established, by order pattern, for codify the algebraic operations or other operations desired by the mathematical model. The possibilities of codify the operations of differentiation of single or several variable function in the model are described. We are tempted to lead the analytical stage, in mathematical model performing, so far away is possible. The analytical inverse of the matrix of polynomials with several variables [9], in the case if it is operational in the modeling, is very attractive. Analytical inverse matrix, performed in the modeling, enlarge the analytical phase in deduction of the mathematical model and increase precision of the numerical calculation in the numerical phase of the mathematical model deduction.

The analytical inverse matrix performed in the mathematical model for deduction of the unknowns, permits us to analyze, through the parameters of the system, the stability of the system dynamic evolution. Other results in the higher algebra domain can be deduced by the research developed in this direction. Some of the above considerations are applied to perform on the computer of the elastic revolution thin plates model [11]. A possibility of applying our algorithm for performing the inverse matrix of differential polynomial operator matrix that intervenes for modeling is analyzed.
2 Notions about coding

We start the coding of the summing and the product operations in the set of polynomial of several variables with real or integer coefficients with the coding of the algebraic operations for two monomials as follows [11]:

\[ C_{a_1...a_m}X_1^{a_1}...X_m^{a_m} + C_{b_1...b_m}X_1^{b_1}...X_m^{b_m} \rightarrow \{(C_{a_1...a_m}, a_1, ..., a_m), (C_{b_1...b_m}, b_1, ..., b_m)\} \]

\[ C_{a_1...a_m}X_1^{a_1}...X_m^{a_m} * C_{b_1...b_m}X_1^{b_1}...X_m^{b_m} \rightarrow \{(C_{a_1...a_m} * C_{b_1...b_m}, a_1 + b_1, ..., a_m + b_m)\} \]

\[ C * C_{a_1...a_m}X_1^{a_1}...X_m^{a_m} \rightarrow (C * C_{a_1...a_m}, a_1, ..., a_m) \]

For coding the summing and the product operations of two polynomials we codify the summing and the product polynomials of these polynomials.

The order of independent or dependent variables that intervene in the mathematical model must be established, by order pattern, for codify the algebraic operations or other operations in mathematical model performing. In the following we describe the possibilities of codify the differentiation of any monomial expressed by single variable functions in the model. For instance, a monomial of the type:

\[ (\sin(x))^\alpha \cos(x)^\beta \]

The order pattern

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<td>c</td>
<td>(sin(x))^\alpha</td>
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give the following differentiation rule:

\( (c, \alpha, \beta)' = \{(c\alpha, \alpha - 1, \beta + 1), (-c\beta, \alpha + 1, \beta - 1)\} \)

In this manner we see that it is possible to replace the manual analytical calculation by an isomorphic numerical calculation which can be performed on digital computers.

3 On the set of polynomials

A unitary and commutative ring \( K \) without divisors of zero is named an integral domain (i.d.).

Let \( K \) a factorial ring (f.r.), therefore an integral domain with the property that every non zero and non invertible element of \( K \) is a product of prime elements of \( K \) [4], [6], [10].

If \( a, b \in K \) we say that \( a \) divide \( b \) if \( b = ac \) with \( c \in K \). Will be denoted by \( a | b \).

A non zero and non invertible element \( p \in K \) is named “prime” if for any \( a, b \in K \) with \( p | ab \) it follows \( p | a \) or \( p | b \).

An element \( c \in K \) (if exist) is named a greatest common divisor (g.c.d.) of \( a \) and \( b \), denoted \( c = (a, b) \), if \( c | a, c | b \) and if \( d | a, d | b \) then \( d \leq c \).

The elements \( a, b \in K \) such that \( (a, b) = 1 \) are named relatively prime.

Two elements \( d_1, d_2 \in K \) such that exists \( u \in K \) invertible, with \( d_1 = u d_2 \) are named adjoins in divisibility.

The ring of polynomials of one variable with coefficients in \( K \) is denoted by \( K[X] \) and the ring of polynomials of several variables \( X_1, ..., X_n \) with coefficients in \( K \) is denoted by \( K[X_1, ..., X_n] \).

Let \( K \) i.d. and \( f \in K[X] \) of the form

\[ f = a_0 + a_1X + ... + a_nX^n \]

(2)

The (g.c.d.) for the coefficients of the polynomial \( f \) is denoted by \( c(f) \in K \).

If \( f \in K[X] \) is of the form (2.1) and \( a \in K \) with \( a | f \) then \( a | a_i \), where \( a_i \in K, i = 1, ..., n \).

If \( g \in K[X] \) and \( c(g) = 1 \) we say that \( g \) is a prime polynomial.

The ring of polynomials in indeterminate \( X_m \), \( m \leq n \), over ring \( K_0 \) is denoted \( K_0[X_m] \), where \( K_0 = K[X_1, ..., X_{m-1}, X_{m+1}, ..., X_n] \).

A polynomial \( g \in K_0(X_m) \) is of the form

\[ g = b_0 + b_1X_m + ... + b_nX_m^n \]

(3)

The coefficients \( b_0, b_1, ..., b_n \) of polynomial \( g \) are polynomials in the ring of polynomials \( K[X_1, ..., X_{m-1}, X_{m+1}, ..., X_n] \).

If \( K \) i.d. then \( K[X] \) i.d. and if \( K \) f.r. then \( K[X] \) f.r.

4 A theorem of division

Let \( K \) factorial ring, \( 0 < m \leq n \), with \( m, n \in N \) and \( K[X_1, ..., X_n] \) the factorial ring of polynomials with several variables and with coefficients in \( K \).

We formulate bellow the following [9]:

Theorem 1

If \( p_1, p_2 \in K[X_1, ..., X_n], p_1 \neq 0, p_2 \neq 0 \), for fixed \( m \), there exists \( q_1, q_2, r \in K[X_1, ..., X_{m-1}, X_{m+1}, ..., X_n] \), polynomials unique without an adjoins in divisibility, such that

\[ p_1 q_1 = p_2 q_2 + r \]

(4)
The polynomial $r$ have the alternative $r = 0$ or $\deg r < \deg p_2$, with degree referred to variable $X_\nu$, and the polynomials $q_1, q_2, r$ are relatively prime, $q_1 \neq 0$.

Let now $p_1, p_2 \in K(X_1, \ldots, X_n)$, $p_1 \neq 0, \ p_2 \neq 0$.

The set of divisors with zero remainder for both polynomials $p_1$ and $p_2$ is denoted by $D(p_1, p_2)$ and briefly the set of divisors for polynomials $p_1$ and $p_2$ is named.

There is the following property [9]:

Theorem 2

In the conditions of first theorem, for fixed $m$, the equality $D(p_1, p_2) = D(p_2, r)$ is true, where the remainder of the division of the polynomials $p_1$ and $p_2$ is $r$.

This theorem permits to give Euclid’s type algorithm for performing the greatest common divisor of two polynomials of several variables with coefficients in factorial ring.

5 A new definition of Euclidean ring

N. JACOBSON, in his treatise [6], gives the following definition of Euclidean ring:

A domain of integrity $D$ is called Euclidean if there exists a map $\delta : D \to N$, of $D$ into the set $N$ of non-negative integers, such that if $a, b \neq 0 \in D$ then there exist $q, r \in D$ such that $a = b q + r$ where $\delta(r) < \delta(b)$.

We propose the following definition:

A factorial ring $D$ is called Euclidean if there exists a map $\delta : D \to N$, of $D$ into the set $N$ of non-negative integers, such that if $a, b \neq 0 \in D$ then there exist $q_1, q_2, r \in D$ such that $a \ q_1 = b \ q_2 + r$ where $\delta(r) < \delta(b)$ and $q_1, q_2, r$ are relatively prime.

6 The model of thin plates and coding

The thin plate is supposed homogeneous, isotropic and elastic linear. The thickness of the plate is supposed constant and also the hypothesis Love-Kirchoff of the “non-deformed normal element” is considered [3]. We analyze the thin plate in hypothesis that the middle surface is of revolution.

The revolution thin plate is described by the vector of position for the point $P(\theta, z)$ situated on the middle surface:

$$\vec{R}(\theta, z) = r(\theta) \cos(\theta) \vec{i} + r(\theta) \sin(\theta) \vec{j} + z \vec{k} \quad (5)$$

The vector of displacements with his applied point on the surface is considered of the form:

$$\vec{U} = u \vec{i} + v \vec{j} - w \vec{k} \quad (6)$$

The vectors $\vec{i}_z$, $\vec{i}_0$ are the unitary vectors attached to coordinate curves concerning point $P(\theta, z)$.

The first fundamental form of the surface is of the form: $ds^2 = r(z)^2 d\theta^2 + (1 + r_z^2) \ dz^2$ with the coefficients $A = r(z), \ B = (1 + r_z^2)^{1/2}$.

The coefficients of the second fundamental form of the surface are: $D = -r/B, \ D' = 0, \ D'' = r_{zz}/B$, where $r_{zz} = d^2 r(z)/dz^2$.

The building of the thin plate model on computer is based on coding of the differentiation operation.

For deduction of the codified differentiation of the vector $\vec{U}$, firstly the codified formulas of differentiation for the unitary vectors $\vec{i}_z$, $\vec{i}_0$, $\vec{n}$ are deduced. For example:

$$\partial \vec{i}_z / \partial \theta = -r_z/B^2 \vec{i}_0 - 1./B^2 \vec{n} \quad (7)$$

The notation $r_z = \partial r / \partial \theta$ is used.

The vector equilibrium equations of the Goldeneizer model [4], attached to any middle surface point of the plate and to appropriate three dimensional local system of axis, are:

$$-\partial (B \vec{F}^{(z)})/\partial z = \partial (B \vec{F}^{(0)})/\partial \theta + rB \vec{P} = 0 \quad (8)$$

$$-\partial (B \vec{C}^{(z)})/\partial z = -\partial (B \vec{C}^{(0)})/\partial \theta - rB \vec{F} \times \vec{i}_z - rB \vec{F} \times \vec{i}_0 + rB \vec{C} = 0 \quad (9)$$

The vector $\vec{P}$ is the external force which action on the plate, $\vec{C}$ is the external resultant moment and

$$\vec{F}^{(z)} = N^0 \vec{i}_z - N^0 \vec{i}_0 + Q^0 \vec{n}, \ \vec{C}^{z} = M^0 \vec{i}_z - M^0 \vec{i}_0$$

The coding of the operations for the deduction of the model of the revolution thin plates, in displacements, is performed in the following fixed order of successive variable which intervenes in the model [7]:

$$E, \ h, \ \frac{1}{1+v}, \ \frac{1}{1-v^2}, \ \frac{1}{r}, \ r, r_z, r_{zz}, \ \frac{1}{B}, \ B, \ u, u_0, \ u_z, \ u_0, \ u_z, \ u_{zzz}, \ v_{zz}, \ v_{zzz}, \ w, \ w_{zzzz}$$

Above, the modulus of elasticity is $E$, the shell half-thickness is $h$, Poisson’s coefficient is $v$ and
the components of the displacement of a point of the middle surface of shell are \(u, v, w\).

Vector and scalar codified operations of addition, multiplication and differentiation subroutines have been performed.

All the results needed for the mathematical model, expressed in displacements, are deduced on the computer by decoding the analytical expressions and are used as input data for the program of static or dynamic calculus of thin plates [8].

We deduce the expression of the generalized forces which are used to impose the boundary conditions at the end \(z=\text{const.}\) by minimizing the energy of the forces and of the moments which action on the plate. The relations performed are:

\[
N^{z*} = N^{z}, \quad N^{z0*} = N^{z0} + \frac{1}{rB} M^{z0}, \quad Q^{z*} = Q^z + \frac{1}{r} \frac{\partial M^z}{\partial \theta}, \quad M^{z*} = M^z
\]  

(10)

From the hypothesis of linearity the constitutive equations are deduced:

\[
N^0 = \frac{2Eh}{1-\nu^2}(\varepsilon^z + \nu \varepsilon^u), \quad N^z = \frac{2Eh}{1-\nu^2}(\varepsilon^0 + \nu \varepsilon^z),
\]

\[
N^{z0*} = N^{z0} = \frac{Eh}{1+\nu},
\]

\[
M^{z0} = -\frac{2Eh^3}{3(1-\nu^2)}(\chi^z + \nu \chi^0),
\]

\[
M^z = -\frac{2Eh^3}{3(1-\nu^2)}(\chi^0 + \nu \chi^z),
\]

\[
M^{z0*} = -M^{z0} = \frac{2Eh^3}{3(1-\nu^2)}\chi^z
\]

(11)

(12)

The coding of the operations for the deduction of the model of the revolution thin plates, in displacements, is performed in the following fixed order of successive variable which intervenes in the model [8], [11]:

\[E, h, \frac{1}{1+\nu}, \frac{1}{1-\nu^2}, \frac{1}{r}, r, \ldots, r_z, \frac{1}{B}, u, u_0, u_z, u_{00}, u_{0z}, u_{zz}, u_{000}, \ldots, u_{zzz}, v, \ldots, v_{zzz}, w, \ldots, w_{zzz}\]

where modulus of elasticity is \(E\), the shell half-thickness is \(h\), Poisson’s coefficient is \(\nu\) and the components of the displacement of a point of the middle surface of shell are \(u, v, w\).

Vector and scalar codified operations of addition, multiplication and differentiation subroutines have been performed.

The boundaries conditions for the generalized displacements and forces are applied to:

\[
u, v, w, \gamma^0, N^z, N^{z0}, Q^z, M^z
\]

(13)

We describe some of the results, expressed in displacements, deduced on the computer by decoding the analytical expressions [8].

\[
\gamma^0 = r_z B^2 v - B^1 w_z
\]

\[
N^{z*} = -2 E^1 r^1 u_0 + 2 E^2 r^2 u_0 - 0.66 E^2 h^2 r_z r_z B^7 v + 2 E^2 h^2 r_z r_z 3 B^9 v - 2 E^1 r^1 r_z B^1 v + 2 E^2 r^2 r_z B^2 v - 0.66 E^2 h^2 r^2 r_z B^5 v + 2 E^2 h^2 r^2 r_z 2 B^7 v + 2 E^2 B^1 v_z + 0.66 E^2 h^2 r_z^2 3 B^9 w - 2 E^2 r^1 B^1 w + 2 E^2 r^2 B^2 w + 0.66 E^2 h^2 r^2 r_z B^7 w - 0.66 E^2 h^2 r^2 r_z 2 B^5 w_z - 0.66 E^2 h^2 r_z^2 B^7 w_z + 0.66 E^2 h^2 r^2 B^3 w_{zz} + 0.66 E^2 h^2 r_z^2 B^5 w_{zz}.
\]

\[
M^z = 0.66 E^1 h^2 r^2 B^1 u_0 - 0.66 E^2 h^2 r^2 B^1 u_0 + 0.66 E^2 h^2 r^2 r_z r_z B^4 v - 0.66 E^1 h^2 r^2 r_z r_z B^4 v + 0.66 E^2 h^2 r_z^2 B^4 v - 2 E^2 h^2 r_z r_z 2 B^6 v - 0.66 E^2 h^2 r^2 B^2 v_z - 0.66 E^2 h^2 r^2 r_z B^2 w - 0.66 E^2 h^2 r_z^2 B^2 w_z - 0.66 E^2 h^2 B^2 w_{zz}.
\]

(14)

(15)

Above are used the notations:

\[
E^1 = \frac{E}{1+\nu}, \quad E^2 = \frac{E}{1-\nu^2}.
\]

The codified scalar equilibrium equations in displacements deduced on the computer are used as input data for the program of static or dynamic calculus of the thin plates.

The numerical method used for calculus of the revolution thin plates takes into account the boundary conditions at the extremities \(z=z_1\) and \(z=z_2\) as well as the development in series of vector displacement components concerning coordinate variables which are considered of the form [5], [8]:

\[
u = \sum_{n=0}^{n_1} v_{n_1}u_n(z)\sin(n\theta), \quad v = \sum_{n=0}^{n_2} v_{n_2}v_n(z)\cos(n\theta), \quad w = \sum_{n=0}^{n_3} w_{n_3}w_n(z)\cos(n\theta)
\]

(16)
In (16) the functions $u_{ns}(z)$, $v_{ns}(z)$, $w_{ns}(z)$ are developed in series by a complete system of polynomials of single variable.

We use a complete system of polynomials, derived from Legendre polynomials and named “modified Legendre polynomials”, defined below, for development in series of the unknown functions $u_{ns}(z)$, $v_{ns}(z)$, $w_{ns}(z)$.

The recursive definition of the “modified Legendre polynomials” is [5]:

$$P_{n}^{m}(t) = (P_{n+1}^{m-1}(t) - P_{n-1}^{m-1}(t))/(2n + 1),$$

$$n \geq m, m, n = 1, 2,...$$

(17)

$$P_{n}^{0}(t) = \frac{1}{2^{n} n!} \frac{d^{n}}{dt^{n}}(t^{2} - 1)^{n}, n = 0, 1, 2,...$$

In (17) Legendre polynomials $P_{n}^{0}(t)$ and the modified Legendre polynomials $P_{n}^{m}(t)$, on the interval $[-1, 1]$, are defined. For the interval of definition $[z_{1}, z_{2}]$ we consider a variable change as $z = \frac{z_{2} - z_{1}}{2}t + \frac{z_{2} + z_{1}}{2}$, that is applied from the interval $[-1, 1]$ to the interval $[z_{1}, z_{2}]$.

The Legendre polynomials $P_{n}^{m}(t)$ are orthonormal on the interval $[-1, 1]$, and $P_{n}^{0}(-1) = (-1)^{n}$, $P_{n}^{0}(1) = 1$, $P_{0}^{0}(t) = 1$. It is known that the modified Legendre polynomials $P_{n}^{m}(t)$, $n \geq m; n, m = 1, 2,...$ are zero at the extremities of the interval $[-1, 1]$, that facilitate achievement of the boundary conditions.

Another expression of the polynomials $P_{n}^{m}(t)$ is:

$$P_{n}^{k}(t) = \frac{1}{2^{n} n!} \frac{d^{n-k}}{dt^{n-k}}(t^{2} - 1)^{n}, n \geq k, n, k = 1, 2,...$$

(18)

We see that the sequence of polynomials:

$$P_{0}^{0}, P_{1}^{0}, P_{1}^{1}, P_{2}^{0},..., P_{s+1}^{m-1}, P_{s}^{m-1},$$

$$P_{s+1}^{s+1},..., P_{k}^{s+1}, k > s + 1$$

for fixed $s$, define a complete system of polynomials [5], using the expression of polynomial $P_{k}^{s}$ as a linear expression of Legendre polynomials and using the completeness of the sequence of Legendre polynomials.

Taken into account the order of differentiation of the unknowns $u_{s}, v_{s}, w_{s}$ that intervene in the system, we search for the developments of the form [5], [8]:

$$g_{n}(z) = C_{0}^{0}P_{0}^{0}(z) + C_{1}^{0}P_{1}^{0}(z) + C_{1}^{1}P_{1}^{1}(z) +$$

$$+ C_{2}^{1}P_{2}^{1}(z) + C_{2}^{2}P_{2}^{2}(z) + C_{2}^{3}P_{2}^{3}(z) +$$

$$+ C_{3}^{3}P_{3}^{3}(z) + C_{4}^{4}P_{4}^{4}(z) + \sum_{k} C_{k}^{k}P_{k}^{k}(z), k \geq 4$$

(19)

The number of the terms fixed for development of the unknown $u_{s}(z), v_{s}(z), w_{s}(z)$, with $z$ into the interval $[z_{1}, z_{2}]$, for fixed $n$, are denoted by $NU$, $NV$, $NW$, such that the total number of unknowns of the mathematical model is $NT = NU + NV + NW$.

The three scalar equations of equilibrium, in displacements are performed for the model of revolution shell.

We describe symbolic, one of them, by the relation $EC1 = 0$.

The “projection” of this equation on the Legendre polynomial $P_{k}^{s}$, $k \geq 0$, is defined by the following true relation [5]:

$$\int_{z_{1}}^{z_{2}} P_{k}^{s}(EC1) dz = 0$$

(20)

The semi-analytical method used for performing the unknown displacements of the closed shell take into account a decomposition of the revolution surface in modules concerning direction of revolution axis. We consider a clamped extremity at the level $z = z_{1}$ and free extremity at the level $z = z_{2}$.

There are four boundary conditions in generalized displacements $u_{s}, v_{s}, w_{s}, \gamma_{s}'$ on the each module and four boundary conditions in generalized forces $N_{s}^{zz}, N_{s}^{zv}, Q_{s}^{z}, M_{s}^{z}$. On the each module from discretization we consider eight conditions of continuity between modules and a number of projections of equilibrium equations of the module such that the total number of algebraic equations to be equal with the total number of unknown coefficients from the development of unknown displacements on the each module. For the dynamic problems we take into account initial conditions that assure the uniqueness of the solution.

In this mathematical model we see that the research is compound of two stages, the analytical stage where are defined the equations of the model using the parameters of the physical model and the numerical stage where the values of the parameters are defined and a numerical method of performing the solution is applied.

We are tempted to lead the analytical stage so far
away is possible. The analytical inverse of the matrix of polynomials with several variables, in the case if it is operational, is very attractive.

7 On the differential operators
A system of linear partial differential equations with constant coefficients, non homogeneous, is supposed in the mathematical model, such as:

\[
\sum_j P_j \frac{\partial}{\partial x^j} \phi_j = f_i, \quad i, j = 1, 2, \ldots, n, \tag{21}
\]

The notations used are:

\[
[P_v \left( \frac{\partial}{\partial x} \right)] = P, \quad P \neq 0, \quad \{\phi_1, \ldots, \phi_n\}^T = \Phi \tag{22}
\]

For \( \det(P) \neq 0 \), in fact \( \det(P) \) operator non identical zero, there is an operator matrix \( Q \) such that:

\[
PQ = QP = (\det(P))E \tag{23}
\]

Matrix \( E \) is a unity matrix.

The following relationship may be written:

\[
QP\Phi = (\det(P))\Phi \tag{24}
\]

On the components we can write:

\[
\sum_{i,j} Q_{ij} P_j \phi_j = (\det(P))\phi_i, \quad i, j, l = 1, 2, \ldots, n \tag{25}
\]

But \( \sum_j P_j \phi_j = f_i, \quad i, j = 1, 2, \ldots, n \), such that:

\[
\sum_l Q_{il} f_i = (\det(P))\phi_i, \quad i, l = 1, 2, \ldots, n \tag{26}
\]

The initial differential system is substituted by the system where the unknown functions appear in separate equations. For each equation of the system is necessary to simplify a set of polynomial operators \( \{Q_1, Q_2, \ldots, Q_m, \det(P)\} \) with the greatest common divisor of them, using our algorithm.

8 Conclusion
Deduction on the computer of the mathematical model of physical phenomena, using analytical and numerical phase, is analyzed. The possibilities of replacing the manual analytical calculation that intervene in the mechanical modeling by an isomorphic numerical calculation, which can be performed on digital computers are investigated. A division with a remainder theorem in the set of polynomials of several variables with coefficients in factorial ring (as the integers ring) is described, that permit us to perform on the computer the analytical expression of the inverse matrix of polynomials with several variables used in the modeling. A question about how much can lead the analytical (formal) calculation in the modeling up to replacing with a numerical method of the solutions calculation appeals. Possibilities of intervening with analytical calculation on the differential operators of the mathematical model for simplifying deduction of the solution are underlined. A competition between analytical and numerical phase in the mathematical model deduction is discovered. A large analytical phase developed in the modeling permit us to increase the precision of the final numerical calculus.

Many authors (Knuth 1981, Buchberger et al. 1982, Davenport et al. 1987, etc) have analyzed the symbolic computation, but this research is not yet exhausted [1], [2], [7].

References:
3. Goldenveizer AL, Theory of elastic shells (in Russian), Gostehizdat, Moscow, 1953
5. Iovanevici VR, Stress in shells with middle surface of mixed type (in Romanian), Ph.D. thesis, Bucharest University, 1980