Active control techniques for piezo smart composite wing

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Abstract: The objective of the present work is the investigating of the capabilities of piezoelectric actuators as active vibration control devices for wing structures. The first step of the study is that of providing the basic order two structural equation of the system by an ANSYS FEM structural modeling, as it was proceeded in recent works of the authors. The enhancing of the wing dynamic behavior is then based on the LQR, LQG/LQG and sliding mode control synthesis. Numerical simulations are presented, thus pointing out the efficacy of the piezo actuators in the developing of smart structures. On the side, a comparison of considered active control techniques from viewpoint of modes vibration attenuation is performed.

Key-Words: composite wing, piezo actuators and sensors, LQR control, LQG/LTR control, sliding mode control

1 Introduction
Certification regulations require that any certified aircraft is free of wing dangerous vibrations. The active control techniques enhance dynamic behavior of the wing, without redesign and adding mass for flutter suppression and structural load alleviation. Thus, herein our target concerns the performing of some active control laws for a piezo smart composite wing. In fact, the paper continues recent researches of the authors, see [1-3].

2 Mathematical model
The computational program ANSYS, performing FEM analysis of a wing physical model (Fig. 1) defined only in terms of geometrical and structural data, was applied to obtain the structural, order two, mathematical model

\[ M\ddot{x} + C\dot{x} + Kx = 0 \]  (1)

where \( x \) is the vector of nodal displacements, and \( M, C \) and \( K \) are mass, damper and stiffness matrices. The wing skin is built on composite material E-glass texture/orto-ophthalic resin with 3 layers and 0.14 mm thickness each of them. The wing spars, placed at 25%, respectively, 65% of chord, are performed of dural D16AT (1 layer, with 5 mm thickness). The interior of the wing is filled with a polyurethane foam. The ANSYS geometric model equipped with MFC actuators is given in Fig. 1.

The wing skin with MFC P1 actuators (see www.smart-materials.com) is built from 4 layers (3 layers for composite material and 1 layer for MFC materials).

Following a modal analysis using the full ANSYS model, the first four natural modes (see Fig. 2) and frequencies (Hz) – 8.60; 20.55; 94.38; 131.97 – were found. Then, by an ANSYS substructuring analysis, the mathematical model was completed in the form.
\[ M\ddot{\mathbf{x}} + C\dot{\mathbf{x}} + K\mathbf{x} = \mathbf{B}_2\mathbf{\xi} + \mathbf{B}_\mathbf{u}\mathbf{u} \quad (2) \]

where \( \mathbf{B}_1, \mathbf{B}_2 \) are the matrices of the influences of the perturbation \( \mathbf{\xi} \) and the control \( \mathbf{u} \). The operation assumed the static interaction cause-effect

\[ K\mathbf{x}_k = \mathbf{B}_2\mathbf{u}_k, \quad K\mathbf{x}_k = \mathbf{B}_2\mathbf{u}_k \quad (3) \]

\( x_k \) is the displacement vector corresponding to a unitary electric field \( u_k \) applied to the \( k \) MFC actuator, herein \( k = 1, 2 \) ; the two columns of the matrix \( \mathbf{B}_2 \) are so obtained. In principle, to calculate piezo action, we used the analogy between thermal and piezoelectric equations developed in [4], so introducing a thermal model for piezo material. Analogously one proceeds for obtaining of the matrix \( \mathbf{B}_1 \), by applying the unitary force \( \mathbf{\xi}_k \) in the point 3 noted in Fig. 1. The subsequent operations concern a) the recuperation in MATLAB of the matrices in system (2), codified in ANSYS as Harwell Boeing format and b) the modal transforming

\[ \mathbf{x} = \mathbf{V}\tilde{\mathbf{q}} \quad (4) \]

of the system (2) by using a reduced modal matrix (of order four) of eigenvectors \( \mathbf{V} \) of dimension \( 34281 \times 4 \) (34281 is the number of generalized coordinates in ANSYS, in connection with the number of the chosen FEM nodes)

\[ \mathbf{V}^\mathbf{T}\mathbf{M}\mathbf{V}\tilde{\mathbf{q}} + \mathbf{V}^\mathbf{T}\mathbf{C}\mathbf{V}\tilde{\mathbf{q}} + \mathbf{V}^\mathbf{T}\mathbf{K}\mathbf{V}\tilde{\mathbf{q}} = \mathbf{V}^\mathbf{T}\tilde{\mathbf{B}}_2\mathbf{\xi} + \mathbf{V}^\mathbf{T}\tilde{\mathbf{B}}_\mathbf{u}\mathbf{u} \quad (5) \]

Thus, modal quasidecentralized system, of four modes, is obtained

\[ \ddot{\mathbf{q}} + \text{diag}(2\zeta_1\omega_1)\dot{\mathbf{q}} + \text{diag}(\omega_1^2)\mathbf{q} = \mathbf{B}_2\mathbf{\xi} + \mathbf{B}_\mathbf{u}\mathbf{u} \quad (6) \]

as a basis for the first order state form system

\[ \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_2\mathbf{\xi}(t) + \mathbf{B}_\mathbf{u}(t)\mathbf{u}(t) \]
\[ \mathbf{z}(t) = \mathbf{C}_1\mathbf{x}(t), \quad \mathbf{y}(t) = \mathbf{C}_2\mathbf{x}(t) + \mathbf{u}\mathbf{y}(t) \quad (7) \]

where \( \mathbf{x}(t) \) is the state, \( \mathbf{z}(t) \) is the controlled output, \( \mathbf{y}(t) \) is the measured output, and \( \mathbf{u}(t) \) is the control input. The state vector is given by

\[ \mathbf{x}(t) = (q_4, q_3, q_2, q_1, q_4, q_3, q_2, q_1)^T \quad (8) \]

The components of the perturbations \( \zeta \) and \( \eta \) are the substitute of the aerodynamic disturbances and sensor noise vector, respectively. The controlled output \( \mathbf{z}(t) \) will concern the whole system state. As the measured output \( \mathbf{y}(t) \) will be taken the \( z \)-axis components of the generalized coordinates associated to nodes noted in Fig. 1. Consequently, the system matrices will be succinctly transcribed

\[ \mathbf{C}_1 = \begin{bmatrix} I_8 \end{bmatrix}, \quad \mathbf{C}_2 = \begin{bmatrix} C_2^T \\ C_0^T \end{bmatrix} = \begin{bmatrix} I_4 & 0 & 4 & 4 \end{bmatrix} \quad (9) \]

\section{Applying LQR and LQG/LTR Control Synthesis}

The Linear Quadratic Gaussian – LQG – control synthesis [5] concerns the system (7). The goal is to find a control \( \mathbf{u}(t) \) such that the system is stabilized and the control minimizes the cost function

\[ J_{\text{LQR}} = \lim_{T \to \infty} \frac{1}{2T} \mathbf{E}\left\{ \int_0^T [\mathbf{x}(t)^T \mathbf{Q}\mathbf{x}(t) + \mathbf{u}(t)^T \mathbf{R}\mathbf{u}(t)] dt \right\} \quad (10) \]

where the matrices \( \mathbf{Q} \) and \( \mathbf{R} \) are so defined herein

\[ \mathbf{Q} = \mathbf{C}_1^T\mathbf{Q}_\mathbf{z}\mathbf{C}_1, \quad \mathbf{R} = \rho\mathbf{R}_\mathbf{u} \quad (11) \]

\( \mathbf{Q}_\mathbf{z} \) is a weighting matrix. The solution consists in the building of a controller and a state-estimator (Kalman filter) having the filter gain \( \mathbf{K}_f \) and controller gain \( \mathbf{K}_c \). The state estimator is of the form

\[ \dot{\mathbf{\hat{x}}}_f = \mathbf{A}_f\mathbf{\hat{x}}_f(t) + \mathbf{K}_f\mathbf{y}(t), \quad \mathbf{A}_f = \mathbf{A} - \mathbf{B}_2\mathbf{K}_c - \mathbf{K}_f\mathbf{C}_2 \quad (12) \]

The controller makes use of this estimator and is defined by

\[ \mathbf{u}^*(t) = -\mathbf{K}_c\mathbf{\hat{x}}_f(t) \quad (13) \]

The Linear Quadratic Regulator – LQR – and LQG controls are built by solving the first of the following decoupled algebraic Riccati equations, while the estimator for LQG synthesis is obtained from the second of the these Riccati equations

\[ \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}_s\mathbf{R}^{-1}\mathbf{B}_s^T\mathbf{P} + \mathbf{C}_1^T\mathbf{Q}_\mathbf{z}\mathbf{C}_1 = 0 \]
\[ \mathbf{A}\mathbf{S} + \mathbf{S}\mathbf{A}^T - \mathbf{S}\mathbf{C}_2\mathbf{Q}_\mathbf{\eta}^{-1}\mathbf{C}_2\mathbf{S} + \mathbf{B}_s\mathbf{Q}_\mathbf{z}\mathbf{B}_s^T = 0 \quad (14) \]

\( \mathbf{Q}_\mathbf{z} \) and \( \mathbf{Q}_\mathbf{\eta} \) are state disturbances and sensor noise covariance matrices. The controller and filter gains are defined by, respectively

\[ \mathbf{K}_c = \mathbf{R}^{-1}\mathbf{B}_s^T\mathbf{P}, \quad \mathbf{K}_f = \mathbf{S}\mathbf{C}_2\mathbf{Q}_\mathbf{\eta}^{-1} \quad (15) \]

The LQR control supposes a state feedback while the LQG control is a estimates feedback (13). Using the state-estimator (12) and the control law (13), the system (7) becomes

\[ \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_2\mathbf{\xi}(t) - \mathbf{B}_2\mathbf{K}_c\mathbf{\hat{x}}_f(t) \]
\[ \dot{\mathbf{\hat{x}}}_f(t) = \mathbf{K}_f\mathbf{C}_2\mathbf{x}(t) + \mathbf{K}_f\mathbf{\mu}\mathbf{\hat{y}}(t) + \mathbf{A}_f\mathbf{\hat{x}}_f(t) \quad (16) \]
As comparison term for the active system in numerical simulations will be taken the LQR closed loop system

$$\dot{x}(t) = (A - B_2 K_f)x(t) + B_2^T \xi(t)$$

(17)

and the “passive” one (without control)

$$\dot{x}(t) = Ax(t) + B_2^T \xi(t)$$

(18)

It is well known that the LQR controller has good robustness properties, but these properties are usually lost when the LQR is used in conjunction with Kalman filter [6].

Now, in the following, the LQG/LTR procedure [7] will be applied to recover the lost robustness of the LQR system.

In view of this proceeding, both the $H_2$-type optimization perspective used in [7] and the open loop singular value perspective defined in [8] will be involved. Thus, the controller gain synthesis is performed such that

$$C_i(sI - A)^{-1} B_2 / \rho = W(s)$$

(19)

where $W(s)$ is a weight chosen such that its crossover frequency be at least the frequency of the first neglected mode (the five one, herein).

The filter gain synthesis will be performed such that

$$B_i = B_2, \mu \to 0$$

(20)

and also, such that

$$L_{L\text{QG}} (j\omega) \approx L_{\text{LQR}} (j\omega)$$

(21)

in a certain range, as large as possible $\omega \in [0, \omega_{\text{max}}]$, where $(j\omega = s)$

$$L_{L\text{QG}}(s) = \left[-(sI - A - K_f C_2 - B_2 K_c)^{-1} K_f C_2\right] \times(sI - A)^{-1} B_2$$

$$L_{\text{LQR}}(s) = K_c (sI - A)^{-1} B_2$$

(22)

Thus, the filter gain $K_f$ will be chosen so that the closed-loop LQG/LTR system (having open loop $L_{\text{LQG}}$) recovers internal stability and some of the robustness properties (gain and phase margins) of the LQR design (with open loop $L_{\text{LQR}}$). $\mu$ in (20) appears as a trading-off factor.

### 4 Applying sliding mode control synthesis

The sliding mode [9]-[13] with $m$ manifolds ($m$ is the dimension of the control vector $u$)

$$H_i : g_i^T \dot{x} = 0, i = 1, 2, ..., m$$

(23)

is thought so that the system be stable as long as the state remains on the hyperplanes (24). The equivalent control law to keep the state on these hyperplanes is given by

$$u_{eq} = -(GB_2)^{-1} \left[ G(A - K_f C_2) \dot{x} + GK_f y \right]$$

$$G = [g_1, g_2, ..., g_m]^T$$

(24)

which means: if the estimated states never leave sliding hyperplanes $H_i$, then $dH_i / dt = 0$ for $i = 1, ..., m$. To satisfy the reaching condition, the control law is chosen as

$$u = u_{eq} - (GB_2)^{-1} \text{diag}(\beta) \text{sgn}[H_1, H_2, ..., H_m]$$

(25)

which means: the Lyapunov function $V = H^2 / 2, H = [H_1, H_2, ..., H_m]$ has negative derivative, $dV / dt < 0$ and the state of the system is transferred on the hyperplanes $H_i$ (sgn is a notation for the signum function). diag($\beta$) is a diagonal matrix with the $i$th diagonal entry equal to a positive number $\beta_i$. To eliminate the chattering behavior, the saturation function will substitute the signum function in (25)

$$\text{sat}H_i = \begin{cases} \text{sgn}H_i & \text{if } |H_i| > \delta_i \\ H_i / \delta_i & \text{otherwise}, i = 1, ..., m \end{cases}$$

(26)

Thus, the effective linear control law can finally be written as ($n$ is the dimension of the state)

$$u = -(GB_2)^{-1} \left[ G(A - K_f C_2 + \rho I_n) \dot{x} + GK_f y \right]$$

(27)

where, without loss of generality, $\beta_i$ and $\delta_i$ have been assumed to be $\beta$ and $\delta$ and $\rho > 0$. The vectors $g_i$ in matrix $G$ will be sought to minimize a reduced versus (10) quadratic objective function

$$J = \int_{0}^{\infty} x^T Q x dt, Q = C_1^T C_2 \geq 0$$

(28)

Let the matrix $P$ be composed of basis vectors of the null space of $B_2^T$, ker($B_2^T$). Defining a similarity transformation

$$q(t) = Tx(t), T := [P \quad B_2]^T$$

(29)

and ignoring the disturbance $\xi$, the first equation in (7) can be recast as
\[
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u
\] (30)

Thus
\[
J = \int_0^T \left[ q_1^T (T^{-1})^T Q T^{-1} q_1 + 2q_1^T N q_2 + r q_2^2 \right] dt = \int_0^T (q_1^T Q q_1 + 2q_1^T N q_2 + r q_2^2) dt
\] (31)

The equation
\[
\dot{q}_1 = S_{11}q_1 + S_{12}q_2
\] (32)

and (31) represent a standard linear quadratic problem provided \( r > 0 \) (if not, a new \( Q \) will be chosen according to (9) and (19)). Now, if \( H \) is expressed as
\[
H = K q_1 + q_2
\] (33)

then \( H = 0 \) implies \( q_2 = -K q_1 \) and in view of equation (31), \( K \) is a state feedback gain vector. For a full-state regulator problem, the equation (23) will take the form of \( H = G x \); thus
\[
G = \begin{bmatrix} K & I_m \end{bmatrix} H, K = R^{-1} \begin{bmatrix} S_{11}^T P_2 + N^T \\
S_{12}^T P_2 + N^T \end{bmatrix}
\]

\[
P_2 \left( S_{11} - S_{12} R^{-1} N^T \right) + \left( S_{11}^T - NR^{-1} S_{12}^T \right) P_2 - P_2 S_{12} R^{-1} S_{12}^T P_2 + Q_1 - NR^{-1} N^T = 0
\] (34)

Finally, the dynamics of the closed loop system can be described as
\[
\begin{bmatrix}
\dot{x} \\
\dot{\xi}
\end{bmatrix} =
\begin{bmatrix}
A - U (A + \rho L_u) U (A + \rho L_w) - K_f C_2 \\
A - K_f C_2
\end{bmatrix} x \\
A - K_f C_2
\end{bmatrix} \begin{bmatrix}
\xi \\
\eta
\end{bmatrix} = A_{cl} \begin{bmatrix} x \\ \xi \end{bmatrix} + B_{cl} \begin{bmatrix} u \\ \eta \end{bmatrix}
\] (35)

where
\[
\varepsilon := x - \hat{x}, U := B_2 (GB_2)^{-1} G
\] (36)

### 5 Numerical simulations

The afore-described LQR, LQG/LTR and sliding mode control laws were tested in numerical simulations of the systems (17), (16) and (35).

The key LQG/LTR synthesis is presented in Fig. 3. In the figure it is shown the accuracy of the criterion (21) accomplishing.

We are then first interested in a passive (see Fig. 4) versus LQR (Fig. 5) and sliding mode closed loop comparison of the time histories of the four modes. As it is to expect, the modes time histories in the case of LQG/LTR control are very close to that of the LQR control and were avoided for space reasons. By a trail and error process, the following values of the LQG/LTR and sliding mode weighting matrices were chosen

\[
\begin{align*}
Q_{\xi} &= q_{csi} I_2, \\
Q_{\eta} &= \mu^2 I_3
\end{align*}
\]

LQR
\[
\begin{align*}
q_{csi} &= 5 \times 10^7, \\
\mu &= 10^{-9}, \\
\rho_R &= 500
\end{align*}
\]

LTR
\[
\begin{align*}
Q_j &= \text{diag}(10, 10^9, 10^{10}, 10^4, 10^{12}, 1) \\
Q_j &= \text{diag}(1, 10^3, 10^2, 10^{-2}, 1, 10^1)
\end{align*}
\] (37)

SLM
\[
\begin{align*}
q_{csi} &= 5 \times 10^7, \\
\mu &= 0.1, \\
\rho &= 1
\end{align*}
\]

The state perturbation
\[
\xi = \begin{bmatrix} 10^8 \cos(2\pi \times 20.55t), \\
10^6 \cos(2\pi \times 94.38t), 10^6 \cos(2\pi \times 131.97t) \end{bmatrix}
\] (38)

was considered as generating a system resonance for the most dangerous, bending type, mode two. Similar sensor noises were introduced

\[
\begin{align*}
\eta_1 &= 0.1 \cos(2\pi \times 50t), \\
\eta_2 &= 0.1 \cos(2\pi \times 100t), \\
\eta_3 &= 0.1 \cos(2\pi \times 175t)
\end{align*}
\] (39)

The above values, together with the values of the all involved matrices avoided here given space limitations, define the LQR, LQG/LTR and sliding mode nominal systems. Worthy noting, the entire designing process focused on the mode two damping, with attention on keeping the associated control in usual limits.

In the Table 1 are summarized the main results of the simulations as attenuations, in percents (%) and dB. The following relations were used

\[
\begin{align*}
delta(q_{i,P}) &= \frac{\text{std}(q_{i,P}) - \text{std}(q_{i,LQR})}{\text{std}(q_{i,P})} \times 100 \\
delta(q_{i,LTR}) &= \frac{\text{std}(q_{i,P}) - \text{std}(q_{i,LTR})}{\text{std}(q_{i,P})} \times 100 \\
delta(q_{i,SM}) &= \frac{\text{std}(q_{i,P}) - \text{std}(q_{i,SM})}{\text{std}(q_{i,P})} \times 100
\end{align*}
\] (40)

The indices \( "P" \) means RMS values obtained in the passive case and \( i = 1, ..., 4 \). The main conclusions concern a) close attenuations of the LQR and LQG/controllers (expected result, based on synthesis criterion (21)) and b) a better working, on the whole, of the sliding mode controller versus the LQG/LTR one.
Fig. 2 The accuracy of LQG/LTR synthesis measured by the proximity of LQR and LQG open loop matrices (see the relation (21)). In the top and the bottom we have maximal and respectively minimal singular values.

Fig. 3 Open loop histories of the four modes

Fig. 4 LQR closed loop time histories of the four modes.

Fig. 5 Sliding mode closed loop histories of the four modes.
### Table 1 Active controllers attenuations versus passive system

<table>
<thead>
<tr>
<th>controller mode</th>
<th>LQR % (dB)</th>
<th>LQG/LTR % (dB)</th>
<th>SM % (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mode 1</td>
<td>76.38</td>
<td>76.31</td>
<td>75.53</td>
</tr>
<tr>
<td></td>
<td>(–28.86)</td>
<td>(–28.80)</td>
<td>(–28.15)</td>
</tr>
<tr>
<td>mode 2</td>
<td>96.00</td>
<td>95.99</td>
<td>89.84</td>
</tr>
<tr>
<td></td>
<td>(–64.39)</td>
<td>(–64.33)</td>
<td>(–45.73)</td>
</tr>
<tr>
<td>mode 3</td>
<td>11.33</td>
<td>11.35</td>
<td>44.91</td>
</tr>
<tr>
<td></td>
<td>(–2.40)</td>
<td>(–2.41)</td>
<td>(–11.92)</td>
</tr>
<tr>
<td>mode 4</td>
<td>59.88</td>
<td>59.87</td>
<td>75.29</td>
</tr>
<tr>
<td></td>
<td>(–18.26)</td>
<td>(–18.26)</td>
<td>(–27.96)</td>
</tr>
</tbody>
</table>

### 6 Concluding remarks

The purpose of this study was to evaluate some active control techniques – LQR, LQG/LTR and sliding mode – as applied to a piezo smart composite wing mathematical model in view of vibrations attenuation.

In fact, the numerical results resumed in the Table 1 and in some graphs can be assumed as validating of the proposed techniques.

A future work must consider a detailed study on the validation of robustness properties of the LQG/LTR and sliding mode controllers.

### Acknowledgment

The work described above was supported (partially) by National Center of Programmes Management (CNMP) from Romania, Contracts No. 71-028, 81-031/2007, and by Ministry of the Education and Research, AMTRANS Programme, Contract No. X2C12/2006.

### References