The Gait Analysis for Modular Walking Robot MERO
Walks on the Slope

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Abstract: - The performance of a legged system is closely related to the adopted gait. The main characteristic of the modular walking robots is that they are able to move away on not arranged, horizontal and rough terrain. The performance of walking robots is closely related to the adopted gait.

The movement of the walking robots can be divided in two modes:
- condition of the static stability;
- condition of the dynamic stability.

During walking, the legs move according to the wave gait and two forelegs are adjusted to avoid forbidden areas. Different methods of leg adjustments and body adjustments are integrated into the strategy. In the work are analyzed the possibilities of determination of the limit conditions for the stable displacement of the walking robots. Finally, this strategy is verified by using computer graphics simulations.

Key-Words: - Modular Walking Robot, Walk, Gait, Support phase, Transfer phase.

1 Introduction
The selection of the type of gait is a very complicated matter, especially in the real conditions of walking on the unarranged. Therefore, it is necessary that the terrain surface to be selected before the type of gait is chose.

The walking robot alternatively leans upon some of its legs and moves the others in a new position, ensuring to itself a stable support. To achieve and control a walking robot, one must know all its walking capabilities, as the choice of the number of legs and their structures depend a lot on the selected walking type. The selection of walking gait type depends on a string of factors such as:
- shape and constituency of the ground the robot walks upon;
- the gait’s stability;
- the way of guiding and controlling the movement of the shift system elements;
- attainment of the velocity and mobility that the motion requires.

It is quite sophisticated a job to choose the motion type, the more under the real field walking circumstances.

The gait of a walking robot is a sequence of movements by its legs, coordinated to a sequel of movements of its body, whose final goal of the robot’s moving to different places.

In order to visualize the motion of a modular walking robot, models of the walking robot and the terrain must be established. Figure 1 shows the model of a modular hexapod robot.

Recently, the gaits for walking on rough terrain have drawn more attention from researchers. For example, some obstacle-crossing gaits of a hexapod and quadruped were studied in [4],[5]. A computer-generated free gait was developed in [8]. A discontinuous follow-the-leader gait was developed and successfully implemented to control a hexapod in walking over uneven terrain [4]. [7] A continuous follow-the-leader gait was formulated and studied in [6],[3]. Two terrain-adaptable free gaits were developed to enable a quadruped to walk on a rough planar terrain [2].

Among the above-mentioned gaits, the gaits which have the follow-the-leader feature seem to be most suitable for rough terrain in walking. The discontinuous follow-the-leader gait developed in
followed a special leg moving sequence and allowed only one leg to be lifted at a time so that good stability could be maintained.

The body movements were inserted into two designed leg movements and the overall body motion was discontinuous [8]. This type of walking mode provides good mobility on rough terrain.

This type of walking mode provides good mobility on rough terrain. However, it suffers a slow speed due to the discontinuous body motion.

This may not be a problem for walking in severely rough terrain, but it certainly limits the performance of the walking machine in mildly rough terrain.

Compared to the discontinuous follow-the-leader gait, a continuous follow-the-leader gait can reach a higher speed and maintain smoother body motion on mildly rough terrain due to its periodic nature[6], [8].

During walking, the walking machine has at least two legs in the air at one time and the stability is reduced.

As a result, the mobility of the continuous follow-the-leader gait is not as good as the discontinuous follow-the-leader gait.

2 Mathematical modeling of gait for modular walking robots

2.1 The walk as a sequence of states

A cycle of the movement of the leg of a modular walking robot has two phases: the support phase and the transfer phase. In the first phase, the leg’s support part has a direct contact with the walking surface area. In the transfer phase, the leg of the robot is above the walking surface and is moving so that it realizes the stability state on the whole of the walking robot. The walk of the robot is characterized by the order raise and seat of the legs and by the trajectory form of the theoretical support point in comparison with the platform. To establish the walking order it is needed to number the legs. The state of the leg (i) at a given time [5], [6], [3]. is described by a state’s function $q'(t)$, that has only two values, 0 and 1, as it follows:

$$
q'(t) = \begin{cases} 
0 & \text{for the support phase;} \\
1 & \text{for the transfer phase.}
\end{cases}
$$

On the interval $[0, t_1]$, the leg is in the support phase. On the interval $[t_1, t_2]$, the leg is not leaning upon the support surface and it is in the transfer phase. On the interval $[t_2, t_3]$, the leg is on the support surface again etc.

At a moment of time, the state of the walking robot with $N$ legs is defined by a $N$-dimensional vector $q$, named the vector of the legs states. The vector’s components $q^i, i = 1, N$, are formed by the functions of the legs’ states, ordered by their numbering:

$$
q = \left[ q^1, q^2, \ldots, q^N \right]^T, \tag{2}
$$

so that the first component of the vector define the state of the leg 1, the second one is the state of the leg 2 etc.

It is assumed that in any finite interval of time there is a finit number of moments that defines the values of the functions $q^i(t)$. The $q$ states, that appear at every change of the value of the function $q^i(t)$, are numbered chronologically as they are carried on. As a result, the walk of a walking robot is described by a succession of states $(q_j), j = 1, 2, \ldots$.

An example of the succession of the states for a walking robot with 4 legs is:

$$
q_1 = [0,0,0,0]^T, q_2 = [1,0,0,0]^T, q_3 = [0,0,0,0]^T, q_4 = [0,1,0,0]^T, q_5 = [0,0,0,0]^T, q_6 = [0,0,1,0]^T, q_7 = [0,0,0,0]^T, q_8 = [0,0,0,1]^T, q_9 = q_1. \tag{3}
$$

It is assumed that, at the initial moment, all the walking robot’s legs are in the support phase. After this, the leg number 1 is raised and moved down, followed by the raise of the leg number 2 and its move down etc. The walk is cyclically if the succession of the states $(q_j)$ is periodical.

The total amount of the realized states in a time period is named the walk cycle. In the above example, the walk is cyclic if after the state $q_9$, determined above, follows the state $q_2$, state $q_3$ etc, in that order.
The support area separated through the mentioned planes is called the \textit{obstacle’s area} (\textit{zone}), where it is forbidden to place the support points of the robot’s legs and feet. The width of the obstacle’s area depends on the ratio of the obstacle’s dimensions, on the geometrical parameters and the technical features of the walking robot.

The presence of the obstacle on the support area can lead to the change in the movement direction, in the height of the movement of the body of the walking robot or its orientation in space as well as to the appropriate redefining of the steps’ order and sequence and in the regime of the legs’ movement.

\subsection*{3.1 Stepping over isolated obstacle while preserving static stability}

The presence of the obstacles on the support surface may lead to changing the direction, the height of the body’s movement and its orientation in space as well as the appropriate reorganization of the sequel of watching the regime of the legs’ movement.

This paragraph will follow, in compliance with \cite{2}, the case when to prevent the obstacles, it is enough only to change the sequel of following and maybe the height of the body’s center of gravity (mass point) preserving the same the other movement features of the robot.

Like before we will assume that the legs’ suspension points are symmetrically placed to the vertical plane including the body’s center and the speed parallel to it. The paths followed for the right and left legs of the robot we assume to be rectilinear and parallel to the speed vector. We are given the body’s mass point to be projected on the center line between the follow paths.

We will name the obstacle ‘isolated’ if we can include it in the field between two vertical planes, erect on the body’s movement direction so that the follow paths beyond this field lack the points forbidden to advance. On the support area the above mentioned planes separate the area called the \textit{obstacle’s zone} where it is forbidden to place any follow points. Its ends are erect on the follow paths.

The zone’s width depends on the ratio of the obstacle’s dimensions and the geometrical parameters characteristic to the robot as well as its control system. It occurs the problem of establishing the sequence of follow, which enables the robot to step over the area without disturbing its static stability.

\subsection*{3.2 Movement of walking robots on uneven surfaces}
3.2.1 Waking robots step over obstacles with defined configuration

The obstacle’s geometry is defined by one or two parameters. Fig 2 shows the main types of obstacles such as a slope in fig 2a, a ditch in fig 2b, a step in fig 2c, and an insulated wall in fig 2d.

3.2.2 The walking robot moves on a slope

The main difference between walking on a slope and walking on a horizontal plane surface consists in the fact that the projection of the robot’s center of gravity changes position against the sides of the support polygon, if the slope exceed a certain limit, the projection no longer lies inside the support polygon. If the periodical walking on a plane, flat surface is symmetric to the longitudinal and lateral axes of the body, the limit of the anterior (fore) longitudinal stability is equal to the limit of the posterior (back) longitudinal stability.

A sensor is used to emphasize the walking robot’s stance a sensor that measures the platform’s inclination in two planes, a sagittal and a frontal one.

To improve the robot’s stability when it moves along a slope, there are two strategies such as:
- its body’s height is diminished and its position adjusted;
- the step’s length is also curbed.

3.2.3 The walking robot moves on a slope, through adjusting the height it steps at and its body’s position

3.2.3.1 The walking robot moves along a maximally inclined slope

First, it is analyzed the efficiency of diminishing the body’s height when the robot walks along a slope.

The movement happens when the body’s longitudinal axis is parallel to the maximal slope’s line. \( \theta \) is the slope’s angle (fig 4) and \( H \tan \theta \) is the deviation of the projection of the center of gravity on the support surface. You can calculate the limit of the longitudinal stability by:

\[
S_l = S_0 \cos \theta
\]  

Instability occurs when \( S_l < 0 \) at any moment of the full locomotion cycle.

Fig 4 The slope’s inclination angle

Instability occurs when \( S_l < 0 \) at any moment of the full locomotion cycle.

Therefore we have to take in consideration only the minimal limit of the longitudinal stability.

No matter if the robot goes up or down the slope, its stability is similarly affected, because of the movement of the projection of the center of gravity, as undulated walk types are symmetrical to the body’s side axis, at a similar inclination. \( S_0 \) is the limit stability of the horizontal walk (slope angle \( \theta =0 \)). You can calculate stability limit at a slope angle \( \theta \), through the relation:

\[
S = (S_0 - H \tan \theta) \cos \theta
\]
The maximal height the robot can move at, \( H_m \) can be calculated putting the condition that stability limit
\[
S = 0 \quad \text{and} \quad H_m = S_0 \cdot \tan \theta
\]  
(5)

For a given height \( H \) of the body, the maximal inclination is given by the relation:
\[
\theta_{\text{max}} = \theta_m = \arctan(S_0 / H).
\]  
(6)

As the body’s minimal height is \( H_0 - R_{Z0} \), the maximal inclination is found by the relation:
\[
\theta_m = \arctan(S_0 / (H_0 - R_{Z0})).
\]  
(7)

For the precise walk, the maximal stability is obtained lifting the robot’s both back legs at an utmost height on the side where the legs go down, then:\( S_0 = P \cdot R_{Z0} / 2 \)
(8)

Taking into account the use factors \( \beta = 3/4 \) and \( P = 11/12 \), the limit stability of the undulated walking is determined by the equation [20]:
\[
S = (1/2) \cdot (P / R) \beta + 3/4,
\]
And the growth in the step’s length leads to the rise in its stability. This is why the walking robot’s stability limit is
\[
S_0 = P / 2 + (1 - 3/4) R
\]  
(9)

For walking on a slope it is advisable a use factor whose value is closer to the upper limit \( \beta = 11/12 \).

Replacing the values of the limit \( S_0 \) in equation (7) we find out the inclination’s maximal value.

Adjusting the platform’s position, the projection of the center of gravity does not move if the platform keeps horizontal and the same level. Thus, the stability limit is the same as that of waking on a plane and horizontal ground.

The body can be kept horizontally only if the slope’s angle is smaller than the limit angle \( \phi_m \). Figure 5 shows that the limit angle is
\[
\phi_m = \arctan(2R_{Z0} / L)
\]  
(10)

For this robot, the authors built and tested, the angle \( \phi_0 \) that equals 15°.

The stability limit \( S''_0 \) for a walk on a surface inclined at such an angle is
\[
S''_0 = S_0 / \cos \phi_0.
\]  
(12)

The distance between the center of gravity and the hypotenuse \( AB \) is:
\[
OC = (H_0 - R_{Z0} / 2) \cos \phi_0.
\]  
(13)

If the slope’s angle \( \theta \) is bigger than the angle \( \phi_0 \), the robot’s body cannot be perfectly horizontal (fig. 6). The body’s angle is \( \theta' = \theta - \phi_0 \).

Point \( E \) where the axis \( OZ \) and the slope’s surface intersect is called the geometrical center of the support area. As the supports are symmetrical to the body’s lateral axis that runs through point \( E \), a deviation of the center of gravity must be measured contingent to the position of point \( E \).

Point \( D \) is the center of gravity’s vertical projection (elevation) on the ground’s sloped surface. The projection’s deviation is \( DE = DC - EC \), or:
\[
DE = OC \tan (\theta - \phi_0).
\]  
(14)

In this case the limit stability on the slope is:
\[
S'' = S''_0 - DE,
\]  
(15)

and the walking’s limit stability becomes:
\[
S = S'' \cos \theta.
\]  
(16)

Replacing the equations (12), (14) and (15) in the equation (16) we get:
\[
S = [S_0 / \cos \phi_0 - OC \tan (\theta - \phi_0)] \cos \theta.
\]  
(17)

The slope reaches its maximal angle when \( S \) becomes zero. This is calculated through the equation
\[
\theta_m = \arctan[(S_0 / \cos \phi_0 + OC \tan \phi_0) / OC],
\]  
(18)

and for a walking robot it becomes
\[
\theta_m = \arctan[(1.2S_0 + 12.46) / 51.21].
\]  
(19)

Comparing the results of the previously used method where it has been diminished the height the body stands at, we notice that the previous method is more efficient. Sometimes we can combine the two methods specific to walking on a slope. First it is adjusted the body’s position so that to reach the desired \( \theta' \) angle.

Thus, the body’s height is diminished till the upper front edge of the workspace touches the ground in this case the body’s new height \( OC \) is:
\[
OC = OE \cos (\theta - \theta') = {H_0 [R_{Z0} - (P \cdot R_{X0} / 2) \tan (\theta - \theta')] \cos (\theta - \theta')}
\]  
(20)

The walk’s geometrical center is the point \( E \) and the OE line and the line OC intersect at the angle \( \theta - \theta' \).

Replacing the difference \( \phi_0 = \theta - \theta' \) in the equation (19), the stability limit becomes:
\[
S = (S_0 / \cos (\theta - \theta') - OC \tan (\theta - \theta')) \cos \theta
\]  
(19)
3.2.3.2 Movement along a slope whose angle is zero

If a walking robot crosses a sloped area whose inclination angle is θ and if it moves along a line whose slope’s angle is zero, and the walking robot’s body keeps parallel to the slope’s surface, the projection of the center of gravity runs laterally to the descending side, at a distance $H \tan \theta$ if the legs keep a normal position against the ground. summarizes such a situation.

For a precise walk, the maximal slope to be adjusted through such a strategy comes up when the deviation of the center of gravity equals $W/2$ and thus:

$$H \tan \theta = 0.5W.$$  \hspace{1cm} (20)

If the body’s height is maximally reduced, the equation (20) becomes

$$\theta_m = \arctan(0.5W/H).$$  \hspace{1cm} (21)

If the legs stepping on the descending slope are fully extended, the maximal slope is

$$\theta_m = \arctan[(W + R_T)/2],$$  \hspace{1cm} (22)

where $R_T$ is the stroke/haul of a lateral step for a maximal workspace value.

For the undulatory walk the stability limit for a movement on the slope is

$$S' = S' - H \tan \theta/\tan \gamma,$$  \hspace{1cm} (23)

where $\gamma$ is the angle for the minimal stability limit along the body’s longitudinal axis.

The moment when the limit of the longitudinal stability for an undulatory walk has a minimal value is that where one of the back legs is risen [7], [4].

Here they are the positions for feet 3 and 6, for a walking robot, at the moment when foot 5 is risen

$$P(3) = R/2 - (2\beta - 1)R/\beta$$  \hspace{1cm} (24)

$$P(6) = -P + R/2 - (\beta - 1/2)R/\beta$$  \hspace{1cm} (25)

The size of the angle $\gamma$ is:

$$\gamma = \arctan[W/[P(3) - P(6)]] = \arctan[W/[P + (1/(2\beta) - 1)R]],$$  \hspace{1cm} (26)

where $W$ is the platform’s width measured between the positions where the legs hang up.

As the body’s longitudinal axis is parallel to the horizontal plane $S' = S$ then :

$$S = S_0 - H \tan \theta/\tan \gamma$$  \hspace{1cm} (27)

For a slope having a given angle $\theta$, the body’s maximal height $H_{mB}$:

$$H_m = S_0 \tan \gamma/ \tan \theta.$$  \hspace{1cm} (28)

As the body’s minimal height is $H_0 - R_2$, the maximal inclination in this case is the smallest of those resulting from the equation (23) and of the following one:

$$\theta_m = \arctan[S_0 \tan \gamma/(H_0 - R_2)].$$  \hspace{1cm} (29)

For the second situation, the body’s position is adjusted namely it is brought to level, through stretching the legs on one side and bending those on the opposite side.

The projection of the center of gravity is kept on the central line so that it should not alter the body’s stability limit. The slope’s maximal inclination when the body can be completely brought to level is the following:

$$\alpha_0 = \arctan(R_{20}/W).$$  \hspace{1cm} (30)

If the slope’s angle is bigger than $\alpha_0$, the body cannot be fully abducted to the level.

![Fig 6. Robot’s movement on the slope to a direction whose inclination is null, through reducing the body’s height.](image)

The distance between the extremities’ positions, measured parallel to the slope’s surface is:

$$W' = W/\cos \alpha_0.$$  \hspace{1cm} (31)

The support’s geometric center is in point $E$. The deviation of the projection of the center of gravity is:

$$DE = DC - E = OC(\tan \theta - \tan \alpha_0),$$  \hspace{1cm} (32)

and:

$$OC = (H_0 - R_{20}/2) \cos \alpha_0.$$  \hspace{1cm} (33)

For the precise walk the slope’s angle is maximal when the deviation equals $W/2$ and therefore:

$$\theta_m = \arctan[0.5W'/OC + \tan \alpha_0].$$  \hspace{1cm} (34)

If the legs on the ascending side rise, and those on the descending side go down by the same distance $R_T$, the projection of the center of gravity goes upwards by the distance $d = R_T \cos \alpha_0$.

Then:

$$DE = OC(\tan \theta - \tan \alpha_0) - 0.5R_T \cos \alpha_0;$$  \hspace{1cm} (35)

$$OC = (H_0 - R_{20}/2) \cos \alpha_0 - 0.5R_T \sin \alpha_0.$$  \hspace{1cm} (36)

From equation (35) and using $W'$, you can calculate maximal inclination angle $\theta_m$:

$$\theta_m = \arctan[(W'/2 + (0.5R_T \cos \alpha_0)/OC + \tan \alpha_0)].$$  \hspace{1cm} (37)

Undulatory walk’s stability limit on a slope is:

$$S' = S_0 - DE/\tan \gamma.$$  \hspace{1cm} (38)
where angle $\gamma$ defines the minimal limit $S_l$ related to the body’s longitudinal axis.

As the body’s longitudinal axis runs parallel to the horizontal plane, $S = S'$ and $S_0 = S'_0$. The walk’s stability limit in this case is:

$$S = S_0 - \frac{DE}{\tan \gamma}, \text{ for } DE \leq \frac{W}{2}. \quad (39)$$

Replacing the expression $DE$ in the equation (30), in equation (36) we get:

$$\theta_m = \arctan\left(\frac{S_0 \tan \gamma + OC \tan \alpha_0}{OC}\right). \quad (40)$$

The slope’s maximal angle is the minimal values that the equations (34) and (40) give.

**Exemple.**
The Modular Mobil Walking Robot it is necessary to know all the walking possibilities, because the selection of the legs number and its structure depends on the selected type of the gait. The selection of the type of gait is a very complicated matter, especially in the real conditions of walking on the rough terrain.

The longitudinal stability margin, $S_l$ is the shorter of the distances from the vertical projection of the center of gravity to the front and rear boundaries of the support pattern, as measured along the direction of motion (see figure 7).

If certain obstacles occur on the walking surface, a special crossing gait must be used, after learning the dimensions of such obstacles.

Depending on the type of the obstacle, its surpassing can be made by the precise arrangement of the legs in the permitted areas around the obstacle. In such a case, the a periodic gait, named “follow the leader” is highly recommended. In case of walking on an unarranged terrain, due to the great diversity of the obstacle dimensions and forms, precise walk is not recommended.

Figure 8 shows the model of a modular walking robot. The body coordinate system $x$-$y$ is attached to the body center and the $x$-axis is aligned with the body longitudinal axis. The center of gravity is coincident with the body center. Each leg is assigned a number as is shown in figure 8. Each leg is represented by a line segment which connects the foot point and hip point is considered unlimited (i.e., the workspace of each leg is unlimited).

The terrain used in the study is two-dimensional, unarranged terrain. The terrain is divided into many cells and each cell is about the size of a footprint.

The cells are classified into two types: a permitted cell and a forbidden cell. A forbidden cell is not suitable for a foot to tramp on it due to weakness of the soil structure, a ditch, or other reasons.
4. Conclusion

The MERO modular walking robot (fig.1) was developed at University "Politehnica" of Bucharest. Such modul robot has two/four/six legs with three degrees of freedom each. The body of modular walking robot carries a gyroscopic attitude sensor to measure the pitch and roll angles of the body. The legs are powered by hydraulical drives and are equipped with joint angle potentiometer transducers. Each leg has three degree of freedom and a tactile sensor to measure the contact which consists of lower and upper levels. The MERO type tansducers used in walking robots offer both force control and robot protection. Each of the feet is equipped with stain gauged force sensing device optimized by finite element analysis. Each of the rotational pairs are closed-loop controlled by software servocontrolled by an external computer.

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