The application of longitudinal wave propagation theory in case of impedance transformers utilized for ultrasonic welding.

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Abstract. The ultrasonic vibrations, having frequency in (20-40) kHz range and high energy (1-2) kW are utilized in non-conventional welding technology. With the help of this technology, the aluminum – aluminum metal can weld. The method presented in this paper starts from the general form of plane longitudinal waves propagation equation through bars with different sections and thickness. The mathematical expressions were finding for the mechanical tensions variations and for the amplitude vibration in function of the shape of the bars and the material from which are they made. The curves traced help us to design the impedance transformer, the component that it is finding between the ultrasound vibration generators (formed by piezoelectric elements) and the welding head. The derived equations were certifying in case of a welding machine TELSONIC-MPS-2 type. The paper proposes to realize a program in order to calculate and then experimental verify in case of an acoustic chain intended to substitute the defective acoustic chain, which exists on welding machine TELSONIC-MPS-2. The paper presents the principle of the calculus, the calculus mode and experimental results obtained by taken in account from a few models selected for this purpose.

Key - Words: Welding, ultrasound, high power, transformers, acoustic chain, propagation equation.

1. Introduction
In order to substitutes the defect acoustic chain, which exists on welding machine, it is necessary to find a calculus method able to assure the same efficiency and performances in case of a new acoustic chain in report with the original acoustic chain.

In addition, it must have the possibility to adapt to the existing work regimen, in our case of the welding machine TELSONIC-MPS-2.

A mathematical program that may apply in this case must be finding. The program starts from the propagation equation of waves in solid materials, theory presented in paper [1].

A calculus program was created, where it was studied the mechanical tensions and vibration amplitudes variation in case of a few acoustic chain shapes. It was analysed the influence of geometrical and material parameters on ensemble acoustic chain which works on welding machine [2].

The optimum acoustic chain [3] obtained by the calculus was matching the required work regime.

The acoustic chain it was obtained and verified experimentally. The certainty of the calculus and the method were certifying by the good function of the welding machine.

2. Operating principle
The operating principle consists of taking over, by an intermediate acoustic chain, of mechanical vibrations generated from a piezoelectric transducer. The ultrasonic vibration [3] it is generates by piezoelectric pastille when is applies on it a sinus tension having great amplitude (2000Vvv).

The acoustic chain [5] must transmit these vibrations and it must realize the following: the amplification of vibrations, the adaptation of impedances between piezoelectric the transducer and acoustic charge and the realization of a strong mechanical catch of solder head for good operating.

In the welding operation time, the welding head must go through the following phases: to dispose the welding head on welding place, to press on welding place, to apply the necessary ultrasonic vibrations for obtaining the welding operation, to cool the solder and to elevate the welding head from welding place.

The ultrasonic vibrations propagate through acoustic chain by stationary waves [4]. The acoustic chain realizes the necessary vibration amplitude on welding contact, which it is place between the welding head and the materials that are being soldered.

The piezoelectric transducer it was build with the help of piezoelectric rings, type SL-4040W-W and
of an acoustic chain that corresponds, by point of view of geometrical dimensions and performances with a defect acoustic chain on welding machine TELSONIC-MPS-2.

The plane longitudinal wave’s propagation through bar [1], having variable section, it makes by 

\[
\frac{\partial}{\partial x} \left[ A(x) \cdot \frac{\partial \zeta}{\partial x} \right] = \frac{A(x)}{c^2} \cdot \frac{\partial^2 \zeta}{\partial t^2} \tag{1}
\]

with condition:

\[
\left. \frac{\partial \zeta}{\partial x} \right|_{x=0} = \left. \frac{\partial \zeta}{\partial x} \right|_{x=l}
\]

That is to say that the amplitude vibrations has maximum amplitudes at ends of bar (for \(x=0\) and for \(x=l\)).

In above relation it is noted:
- \(\zeta(x,t)\) - represents the amplitude vibration in Ox direction;
- \(A(x)\) - represents the transversal section area at distance 
- \(c = \frac{E}{\rho}\) - represents the propagation velocity of signal through bar;
- \(E\) - represents the elasticity module of bar material;
- \(\rho\) - represents the density of bar material.

The calculus for (1) equation it was presented in [7].

It looks for a solution by form: \(\zeta(x,t) = X(x)\cdot \cos \omega_1 t\).

It substitutes in (1) and it obtains:

\[
\frac{\partial^2 X}{\partial x^2} + \mu_1^2 \cdot A(x) \cdot X(x) = 0 \tag{2}
\]

where: \(\mu_1 = \frac{\omega_1}{c}\)

having maximum vibration condition at ends of bar. Therefore, we have:

\[
\left. \frac{\partial X(x)}{\partial x} \right|_{x=0} = \left. \frac{\partial X(x)}{\partial x} \right|_{x=l} = 0
\]

Equation (2) admits an infinity solutions by form \(X_\mu(x)\) \(\mu=1,2,\ldots\). They are named proper functions and correspond with positive series numbers \(\mu_\mu\) \(\mu=1,2,\ldots\)

which are named proper values. The proper functions \(X_\mu(x)\) are by form \(C^1\) class and so they may develop in Fourier series converged to \(X_\mu(x)\) series.

For resolves this it determines the proper values \(\mu_\mu\) and proper functions \(X_\mu(x)\).

The solution is the research by the Galerkerk method, using the complete system of functions (1, \(\cos \pi x/1, \ldots, \cos \pi x/l, \ldots\)). It must satisfy, also, the limit conditions \(\frac{\partial X(x)}{\partial x}\bigg|_{x=0} = \frac{\partial X(x)}{\partial x}\bigg|_{x=l} = 0\).

For \(X(x)\), it is searching the solution by form:

\[
X(x) = \sum_{n=0}^{s} a_n \cdot \cos \frac{n \pi x}{l} \tag{3}
\]

where: \(s\) - determines the calculus precision to finds of \(X(x)\);

\(a_n\) - are indeterminate constants solutions of a homogeneous system.

We have the solutions, for \(a_n\), by resolving the system of equations:

\[
\sum_{n=0}^{s} a_n \cdot \left( S_{mn} - \mu_n^2 \cdot C_{mn} \right) = 0 \quad m=0,1,2,\ldots,s \tag{4}
\]

where we have, for \(S_{mn}\) and \(C_{mn}\), the values given by the following relations:

\[
\left. \begin{align*}
S_{mn} &= mn \cdot \left( \frac{\pi}{l} \right)^2 \\
C_{mn} &= \int_0^l A(x) \cdot \sin \left( \frac{m \pi x}{l} \right) \cdot \cos \left( \frac{n \pi x}{l} \right) \cdot dx
\end{align*} \right\} \tag{5}
\]

The homogeneous system (4) has no common solution, if and only if we have the conditions:

\[
| S_{mn} - \mu^2 \cdot C_{mn} | = 0 \quad m,n=0,1,2,\ldots,s \tag{6}
\]

The (6) equation has, for \(\mu\), a number of \(s+1\) positive solutions, having property: \(\mu_1 < \mu_2 < \mu_3 < \ldots < \mu_{s+1}\).

It introduces \(\mu_i^\prime\) \(i=1,2,\ldots,s+1\) in (4) system and resolves it for obtains the solutions:

\[
\frac{a_1}{a_0}, \frac{a_2}{a_0}, \ldots, \frac{a_s}{a_0} \tag{7}
\]

where \(i=1,2,\ldots,s+1\) and \(n=1,2,\ldots,s\)

These values are introduced in (3) equation and it obtains:

\[
X(x) = a_0^\prime \cdot \cos 0 + \sum_{n=1}^{s} a_n^\prime \cdot \cos \frac{n \pi x}{l} \quad i=1,2,\ldots,s+1 \tag{8}
\]

It takes \(a_0^\prime = 1\) and it obtains:

\[
X_i(x) = 1 + \sum_{n=1}^{s} a_n^\prime \cdot \cos \frac{n \pi x}{l} \quad i=1,2,\ldots,s+1 \tag{9}
\]
It is use the following procedure:
- it takes μ=μ₁;
- it calculates the values: \( \frac{a_{1}^{0}}{a_{0}^{i}} \), \( \frac{a_{1}^{0}}{a_{0}^{j}} \), ..., \( \frac{a_{1}^{0}}{a_{0}^{n}} \)
  where: \( i=1 \) and \( n=1,2,...,s \)
- it introduces these values in (9) equation and it obtains: \( X^{1}(x) = 1 + \sum_{n=0}^{s} a_{n}^{1} \cdot \cos \frac{m \pi}{l} x \)

3. Experimental results
To calculate the shape and functional parameters of acoustic chain it is using the relations given in [7]. It is chosen [5] that a calculus precision \( s \), for finds of \( X(x) \) having an optimum value for \( s=2 \). Therefore, we have:

\[
X(x) = \sum_{n=0}^{2} a_{n} \cdot \cos \frac{n \pi}{l} x = \]
\[
= a_{0} + a_{1} \cdot \cos \frac{n \pi}{l} x + a_{1} \cdot \cos \frac{2n \pi}{l} x
\]

The homogeneous system (4) becomes:

\[
\sum_{n=0}^{2} a_{n} \cdot (S_{mn} - \mu^{2} \cdot C_{mn}) = 0 \quad ; \quad m=0,1,2
\]

This homogeneous system has no common solution if and only if the determinant has a zero value:

\[
| S_{mn} - \mu^{2} \cdot C_{mn} | = 0 \quad \text{with} \quad m,n=0,1,2
\]

From this it is found the proper values \( \mu^{1} = \mu_{1}, \mu_{2}, \mu_{3} \). For each proper value \( \mu_{1}, \mu_{2}, \mu_{3} \) it is obtain 3 sets values for \( a_{0}, a_{1}, a_{2} \):

\[
\mu_{1} \rightarrow a_{0}^{1}, a_{1}^{1}, a_{2}^{1} \ldots ; \quad a_{0}^{1}, a_{1}^{1}, a_{2}^{1} ;
\]
\[
\mu_{2} \rightarrow a_{0}^{2}, a_{1}^{2}, a_{2}^{2} \ldots ; \quad a_{0}^{2}, a_{1}^{2}, a_{2}^{2} ;
\]
\[
\mu_{3} \rightarrow a_{0}^{3}, a_{1}^{3}, a_{2}^{3} \ldots ; \quad a_{0}^{3}, a_{1}^{3}, a_{2}^{3} ;
\]

It takes \( a_{0}^{1} = 1 \) and it obtains:

\[
\mu_{1} \rightarrow 1, a_{1}^{1}, a_{2}^{1} ;
\]
\[
\mu_{2} \rightarrow 1, a_{1}^{2}, a_{2}^{2} ;
\]
\[
\mu_{3} \rightarrow 1, a_{1}^{3}, a_{2}^{3} .
\]

For to find the solution it takes \( \mu=\mu_{1} \) and it resolves the system:

\[
\sum_{n=0}^{2} a_{n} \cdot (S_{mn} - \mu^{2} \cdot C_{mn}) = 0 \quad ; \quad m=0,1,2
\]

In conditions when the system has no banal solution, the value of \( \mu \) results from equation \( | S_{mn} - \mu^{2} \cdot C_{mn} | = 0 \). From this it results the values for \( a_{1}^{1} \) and \( a_{2}^{1} \). The solution it is determined by matrix

\[
| a^{1} |
\]

\[
= | B |
\]

\[
= | D |
\]

Therefore, it obtain the new matrix with elements:

\[
D_{k,l} = S_{k+1,l+1} - \mu^{2} \cdot C_{k+1,l+1}
\]

\[
B_{k} = -S_{k+1,0} + \mu^{2} \cdot C_{k+1,0}
\]

where \( \lfloor l \rfloor = 0.1 \quad \text{and} \quad k=0.1 \), in general: \( l, k = 0, 1, 2, \ldots, s-1 \).

Having the values for \( a_{1}^{1} \) and \( a_{2}^{1} \), it is possible to calculate \( X^{1}(x) \) with \( i=1 \):

\[
X^{1}(x) = 1 + a_{1}^{1} \cdot \cos \frac{\pi}{l} x + a_{2}^{1} \cdot \cos \frac{2\pi}{l} x
\]

Using the relations presented it is possible to calculate:
1. The vibration amplitude in Ox direction with relation: \( \xi'(x,t)=X^{1}(x) \cdot \cos \omega_{0}t \), where vibration amplitude at moment \( t=0 \) will be \( \xi'(x,0) = X^{1}(x) \);
2. The amplification through bar, which is given by ratio between vibration amplitude at the end of bar \( (x=1) \) and vibration amplitude at begin of bar \( (x=0) \). So we can write:

\[
G = \frac{\xi'(1,0)}{\xi'(0,0)}
\]

3. The areas ratio, in the case of a bar having different sections. It is given by relation:

\[
\frac{A(0)}{A(l)}
\]
where \( A(x) \) represents the equation which defines the transversal section area at \( x \) distance for the bar having different sections along Ox axis.

4. If it is note \( X_0 = q \), the distance when the vibration amplitude \( \zeta(x,t) \) it is null, will have: \( \zeta'(q,0) = 0 \)

\[ \Rightarrow X_0 \]

5. It can trace the curve for mechanical tensions along bar - noted \( T_m \) - starting from relation

\[ T_m(x) = \rho \cdot c \cdot v_m(x) \]

where: \( \rho \) -represents the bar density material;
\( c \) -represents the signal velocity propagation along the bar;
\( v_m(x) \) - represents the velocity particles propagation vibration through the bar;

The velocity of the particles propagation vibration through the bar - \( v_m(x) \) - is direct proportional with the differential quotient of the vibration amplitude \( \zeta'(x,0) \). So, it can write:

\[ T_m(x) \sim v_m(x) \sim \frac{d}{dx} \left[ \zeta'(x,0) \right] \]

For a higher precision of the calculus, when it is find \( X(x) \), it can take, for \( s \), a higher values than \( s=2 \). For \( s \) having a higher value, it obtains a higher precision, but the calculus volume rises too much. It has observed [7] that to choice a higher values for \( s \), more than 7, 9, it isn’t justifier in report with the rising of calculus volume and necessary time to it calculates.

With the help of this program, it can trace the curves that define the acoustic chain, by point of vibration amplitude and mechanical tensions along its length, as seen in Fig.1. For these the relations and methodology that it was explain uses in this paper.

**Fig.1** The curves of vibration amplitude and mechanical tensions that exist along of acoustic chain in case of a rise diameter before interface connector salt.

**Fig.2** The curves of vibration amplitude and mechanical tensions that exist along of acoustic chain in case of a constant diameter before interface connector salt.

The presented program it was calculate for the resonance frequency of the acoustic chain and it find the value of 36,5kHz, frequency that is very close to the necessary frequency for a correct function of the original welding machine.

The calculus and the laboratory measurements obtained it were verifier by the correct function of welding machine and by the quality of the welding obtained with that new acoustic chain. The acoustic chain, which it is use instead of original acoustic chain, it is present in fig. 3. It is form from the following components:

- a reflector, having \( \varnothing=41\)mm diameter, realized from steel;
- a pair of piezoelectric rings, by SL-4040W-W type;
- a director, realized from aluminum alloy, having \( \varnothing=41\)mm diameter;
- a transmitted chain, realized from aluminum alloy, having \( \varnothing=49\)mm diameter;
- a transmitted chain, having a fixture system from welding machine, realized from aluminum alloy and having \( \varnothing= 55\)mm diameter. This part of
acoustic chain has an important contribution for to cool the solder head;
- a solder head, having a variable section for amplification of vibration, made from titanium.

Fig. 4 The welding machine equipped with the new acoustic chain.

The theory expose [6] it was verify in many experimental cases.

In table no.1 it is synthesized the main characteristics of a few frequently utilizing resonant elements like acoustic transformer. It takes the following types:
1) resonant elements with section jump having exponential interface connector – fig.6;
2) resonant elements with exponential section jump fig.7;
3) resonant elements with conic section jump fig.8;
4) resonant elements with degree section jump fig.9;

Fig. 5 The new acoustic chain realized and mounted on welding machine.

Fig.6. Resonant elements with section jump having exponential interface connector

Fig.7. Resonant elements with exponential section jump

Fig.8. Resonant elements with conic section

Fig.9. Resonant elements with degree section jump
All of the acoustic elements have the same dimensions and they are realizing from the same material – aluminum alloy.

<table>
<thead>
<tr>
<th></th>
<th>-fig.1-</th>
<th>-fig.2-</th>
<th>-fig.3-</th>
<th>-fig.4-</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Diameter [m]</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Minimum Diameter [m]</td>
<td>0.02</td>
<td>-</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Total length [m]</td>
<td>0.065</td>
<td>0.065</td>
<td>0.065</td>
<td>0.065</td>
</tr>
<tr>
<td>Resonant frequency [kHz]</td>
<td>45.02</td>
<td>42.42</td>
<td>40.62</td>
<td>41.87</td>
</tr>
<tr>
<td>Total gain</td>
<td>3.43</td>
<td>1.654</td>
<td>1.396</td>
<td>3.932</td>
</tr>
<tr>
<td>Maximum value of vibration</td>
<td>4.628</td>
<td>6.76</td>
<td>8.642</td>
<td>4.318</td>
</tr>
<tr>
<td>Maximum value of mechanical stress</td>
<td>200.41</td>
<td>264.48</td>
<td>357.34</td>
<td>212.76</td>
</tr>
<tr>
<td>Longitudinal position where the mechanical stress is maximum</td>
<td>0.046</td>
<td>0.038</td>
<td>0.034</td>
<td>0.044</td>
</tr>
<tr>
<td>Longitudinal position where beginning section to vary</td>
<td>0.0325</td>
<td>0.0216</td>
<td>-</td>
<td>0.0325</td>
</tr>
</tbody>
</table>

It observes that the fig. 4 have the bigger value for gain realized with a passive resonant element.

From longitudinal position where the mechanical stress has a maximum value we can find the place where the mechanical aggregate demands for resonant element.

All physical elements have the same dimensions for a good comparison.

**4. Conclusions**

It was verifies the propagation theory for longitudinal plane waves through bars having variable section presented in [7].

This theory was applied in the case of an acoustic chain used on the welding machine TELSONIC - MPS - 2 by:
- finding of an optimum acoustic chain by point of view of shape, of component materials and of energy transfer;
- finding the mathematical relations which define this acoustic chain, relations introduced in calculus program;
- writing and putting into practice of calculus program;
- analysis of obtained results and influence study of different parameters on final result;
- realization of acoustic chain, with the help of this analysis;
- measuring and verification of the acoustic chain in our laboratory and on the welding machine using the new acoustic chain realized.

**References:**


