Computer-aided model of the dynamic behavior of the feed drive systems

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Abstract: This paper presents a new computer-aided modeling way of the feed drive system (FDS) from modern machine tools [7, 8], based on a general method elaborated especially for multi-bodies branched systems [1, 2, 3]. It was developed in the welcoming frame of Computational Mechanics, suitable to the mathematical simulation of their dynamic behavior, in order to reduce the perturbations provoked by vibrations. The paper presents an 8-DOF new dynamic model of an FDS that allows to obtain the mathematical model as second order nonlinear differential equations with some non-constant coefficients. The flexibility of this mathematical model synergistically completed by the Mathematica® software symbolic calculus capabilities, allow us to determine the values of design parameters that optimize the dynamic behavior, according to predefined criteria.

Key-Words: Computer-aided modeling, elasto-dynamic behavior, vibrations, machine tools, multi-bodies systems, Mathematica.

1 Introduction
This paper presents a new computer-aided (CA) modeling way of the feed drive systems (FDS), taking into account the new demands.
(a) Machining precision, which has always been an important issue, has drastically increased due to the nano-science studies that demand the characterization of system behavior under micro/nano-scales.
(b) During the last years, as the increase in speed and acceleration became a constant trend, the need for high performance FDS became stringent in aerospace, semiconductor industry, manufacturing industries, etc.
(c) The general way of considering a machine tool as a massive structure leads to high structural stiffness, desired for reducing deformation under the influence of machining forces and static weight of the machine structure and work-piece. The structure deflection, which can be regarded as a structural loop deformation, leads to errors at the interface between the tool edge and the work-piece. Machine tool stiff structures tend to transmit vibrations at higher frequencies than compliant (un-stiff) ones [5,7,8]. The transmitted vibrations will cause structure time-varying deflections, which can be amplified in the work-piece, if the vibrations are near an eigenvalue of the machine tool. These time varying deflections cannot be easily predicted [5]. Their prediction is one of our goals.
(d) The flexibility regarding the structure elements has to be taken into consideration and thus, the study of dynamic behavior of FDS plays a very important role for part dimensioning and also for design control [7].
(e) The capacity to dampen vibrations, which depends on the component materials, influences the performances of the machine tool [6].
(f) Studies concerning the design and performance improvement of FDS are performed based on the component design methodology, which focus solely on the design or optimization of each subsystem. However, performance of feed drive systems depends upon not only the characteristics of each subsystem, but also on the interaction among the subsystems, and this is one of our targets.
Having all these in view, this paper considers the FDS (generally described in Fig. 1) as a group of interacting flexible systems and focuses on the study of its elasto-dynamic behavior.

Fig. 1. A general scheme of FDS.
2 Problem Formulation

2.1 An FDS general description

Although machine tool designs vary immensely, the mechanical configuration of their feed drive is largely standardized [5]. In almost all cases, the recirculating ball-screw has established itself as the solution for converting the rotary motion of the servomotor into linear slide motion. The ball-screw is normally fixed in axial direction at only one end with a preloaded angular-contact ball bearing, which takes up the axial forces of the slide. The servomotor and ball-screw drive are usually directly coupled. Toothed-belt drives are also widely used to achieve a compact design and better adapt the speed. The first step in any machine tool design process is to establish a simplified scheme of its mechanical structure. The FDS structure can be “broken” into the base (which should be able to support the heavy weight and withstand the cutting action), the transmission motion mechanism (ball-screw, nut) and the table. The main components of the FDS are shown in Fig. 1 and the physical characteristics are listed in Table 1.

Table 1

<table>
<thead>
<tr>
<th></th>
<th>Formula</th>
</tr>
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<tbody>
<tr>
<td>$F$ - cutting force</td>
<td>$[N]$</td>
</tr>
<tr>
<td>$M_o$ - motor torque</td>
<td>$[N m]$</td>
</tr>
<tr>
<td>$J$ - the mass moment</td>
<td>$[Kg m^2]$</td>
</tr>
<tr>
<td>of inertia of the main</td>
<td></td>
</tr>
<tr>
<td>shaft</td>
<td></td>
</tr>
<tr>
<td>$J_o$ - the mass moment</td>
<td>$[Kg m^2]$</td>
</tr>
<tr>
<td>of inertia of the rotor</td>
<td></td>
</tr>
<tr>
<td>$d_i$ - ball-screw</td>
<td>$[m]$</td>
</tr>
<tr>
<td>diameter</td>
<td></td>
</tr>
<tr>
<td>$L$ - ball-screw</td>
<td>$[m]$</td>
</tr>
<tr>
<td>length</td>
<td></td>
</tr>
<tr>
<td>$p$ - lead of ball-screw</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$R$ - transformation</td>
<td>$[m/rad]$</td>
</tr>
<tr>
<td>ratio</td>
<td></td>
</tr>
<tr>
<td>$c$ - longitudinal</td>
<td>$[N/m]$</td>
</tr>
<tr>
<td>stiffness of the nut</td>
<td></td>
</tr>
<tr>
<td>$M_p$ - table mass</td>
<td>$[Kg]$</td>
</tr>
<tr>
<td>$E$ - Young’s modulus</td>
<td>$[Pa]$</td>
</tr>
<tr>
<td>$G$ - Shear modulus</td>
<td>$[Pa]$</td>
</tr>
</tbody>
</table>

To reach a model for the vibrations behavior of this machinery, we use a special CA method to synthesize the dynamic equations in a matrix form [1, 2, 3], which was developed for a special kind of multi-body system, but which can be extended to other types of mechanical systems.

2.2 The assumptions frame

The method was developed in the following assumptions frame:

i. The inertial properties of the model, $J$ and/or $m$, are concentrated in points or across sections.

ii. These points or cross-sections are connected by elastic, dissipative or kinematic links, described by the coefficients $c$ and $b$, or the position function $\Pi$, respectively, which are free of inertial properties (the elastic, dissipative and kinematic elements are mass-less).

iii. The reduction (the motion transmission) procedure must respect the condition of invariability of the system's energy (the balance of potential, kinetic and dissipated energy fractions).

iv. The absolute motion (displacement or rotation) of an inertial element is represented as the sum of the ideal motion (assuming that the system contains only the rigid members) and the dynamic error (perturbation) caused by real deformations of the connecting links. By "real" we mean that the parts possess limited stiffness. The "rigid body system" assumption does not hold.

v. The friction is proportional to the relative speed between two neighboring inertial bodies; the energy dissipation in the elastic elements is proportional to the relative speed of the masses connected by the element.

vi. The elasto-dynamic behavior is defined by the torsion, compression-extension and bending deformations of the parts of the mechanism, due to the forces of inertia and other forces acting on them and to the limited stiffness of these real parts.

vii. The bending and the torsional vibrations of the main shaft are separately treated and the effects are superimposed to compute the driving force for each driven branch. The bending deflection is defined as the vertical (only) displacement of the neutral axis of the deformed beam.

Our CA method, elaborated especially for multi-bodies branched systems, can be applied for any multi-body system for which the dynamic model is a collection of vibratory elements that can be grouped in four subsystems (see &2.4 in [1]) and its descriptor matrix DE can be defined (see &2.5 in [1]). In the followings we build the dynamic model for the FDS and its descriptor matrix that respects these applicability conditions.

3 The Dynamic model for an FDS

In our view, the dynamic model is a collection of vibratory elements from subsystems S1-S4, connected in series and parallel under the assumption that the vibratory element provides all the necessary information to describe the dynamical behavior by three components: an inertial one - $J$ or $m$, a kinematic one - $\Pi$ and a deformability one - $c$ (optionally with damping, $b$). Since, in the FDS normal function mode, the elastic main shaft (composed of the ball-screw and the bearings) undergoes mainly torsion and bending and the active mass (composed by table and nut) undergoes translation, we need a type of vibratory element for each type of motion. Therefore, to describe the FDS dynamic behavior, three subsystems are to be used:
S₁ describes the torsion of the main shaft; S₂ describes the bending of the main shaft, and S₃ describes the translation of the active mass. The dynamic model of the FDS is shown in Fig. 2.

![Dynamic model of FDS](image)

Fig. 2. The dynamical model for FDS.

As any dynamic model, the dynamic model of an FDS must be equivalent kinetically and dynamically to the real system. To do this, the discretization method has been used. "Dividing" the shaft into segments and "lumping" the mass associated with each segment, the inertial behavior is described by \( J_k \) - mass moment of inertia of the rigid disks. The elastic behavior is described by the mass-less shafts of torsional rigidity \( c_{t_k} \), optionally with damping \( b_{t_k} \) (defined according to assumption \( v \)). The element \( \{J_0, c_{t_0}/b_{t_0}\} \) represents the motor and the elements \( \{J_k, c_{t_k}/b_{t_k}\}, k = 1, 4 \) represent the main shaft which is divided into three segments that are delimited by the ends of the shaft, its middle point and the variable position \( x \) of the active mass. As a consequence, some of the physical coefficients are not constant. They will be functions of the variable position \( x(t) \) of the active mass. The cases: \( 0 < x(t) < L/2 \) and \( L/2 < x(t) < L \) have to be separately treated. In the followings, we’ll discuss the first case. So, noting by \( J \) the main shaft mass moment of inertia and by \( c_t \) the torsional rigidity, the mass moments of inertia of the four rigid disks are:

\[
J_1 = J_2 = J_3 = \frac{2}{9}J; \quad J_4 = \frac{1}{3}J
\]

The torsional rigidities of the three segments (constant, \( c_{t3} \), and variables, \( c_{t1} \) and \( c_{t2} \)) are:

\[
c_{t3} = \frac{1}{2}c_t
\]

\[
c_{t1}(x(t)) = \frac{32x(t)}{\pi GD^2}
\]

\[
c_{t2} = (2/c_t - 1/c_{t1})^{-1} = \frac{32c_t x(t)}{64x(t) - \pi c_t G D^2}
\]

The bending of the main camshaft (considering only the vertical displacement, according to \( vii \)) is described by vibratory elements from S₄ of \( \{mb, ce/be\}\)-type, where \( mb \) is the bending mass and \( ce/be \) are the stiffness bending coefficients (computed from the influence coefficients [4]) and, respectively, the damping coefficients computed according to assumptions \( v \) and \( vii \). Under the assumption that \( 0 < x(t) < L/2 \), the mass \( mb \) represents the active mass, and the mass \( mb \) represents the main shaft drive mass and \( ce_{22} \) has a constant value:

\[
ce_{22} = \frac{L^4}{48EI}
\]

The others are variable:

\[
ce_{11} = \frac{x(t)^2 (L - x(t))^2}{3EI L}
\]

\[
ce_{12} = \frac{x(t)(L - x)^3}{6EI L} \left[ \frac{3}{4}L^2 - x(t)^2 \right]
\]

The subsystem \( S_2 \) describes the branches driven by the main shaft. Since, in the FDS case, exists only one active mass, \( mp \), a single branch appears, in our case. So, the vibratory element from the branch is \( \{mp, c(b), \Pi \} \). \( \Pi \) (the position transfer function of the mechanism of the branch) transforms the rotation \( \varphi_2 \) at section \( J_2 \) in a translation motion, \( x = x(t) \), of the active main mass, \( mp \). The motion of the active main mass, \( x = x(t) \), describes the positioning process and the milling process, as well. The positioning process is a very short, but very speedy one. The constant advance speed, in this process, is noted by \( v_p \) with a value around 5 m/min.

At the end of the positioning process, the main mass gets its operating position, \( x_p = x(t_p) = v_p t_p \), where \( t_p \) is the necessary time to reach it. The milling process is composed of a heavy duty milling process and a finish milling process. During the milling process, the advance speed is constant, but has different value at each phase: about 0.15 m/min at the heavy duty milling process and about 0.3 m/min at the finish milling process. At each phase, \( x(t) \) follows a trajectory deduced from the piece contour with variable appropriate advance speed. Noting by \( t_d \) the end of the heavy-duty milling process, the operating time \([t_0, t_{end}]\) is composed as follows:

\[
[t_0, t_p] \cup [t_p, t_d] = [t_d, t_{end}]
\]

Then, the position function \( x(t) = \Pi(\varphi_2(t)) \), must be a piecewise defined function:

\[
x(t) = \begin{cases} 
    v_p t, & \text{if } t_0 \leq t < t_p \text{ and } 0 < x(t) \leq L/2 \\
    x_p(t), & \text{if } t_p \leq t \leq t_{end}
\end{cases}
\]

At its turn, \( x_p(t) \) is a piecewise defined function:
\[ x_{op}(t) = \begin{cases} x_p(t), & \text{if} \ t_p \leq t < t_d \\ x_f(t), & \text{if} \ t_d \leq t \leq t_{end} \end{cases} \] (6)

Generally, the functions \( x_p(t) \) and \( x_f(t) \) describe the movement along piece contour with different speeds and are to be introduced by operator for each type of piece. \( x_{op}(t) \) is repeated several times for each piece and then the other piece comes in the milling process.

4 Mathematical Model (MM)

4.1 The absolute/generalized coordinates relations

In the general case (see (2.4.18) in [1]), the global vector of the generalized coordinates is defined as:

\[ q_0 = q_{1:0} = \Phi_0, \]

\[ q_k = q_{1:k} = \Phi_k - \Phi_{k-1}, \quad 1 \leq k \leq n_1 \]

\[ q_{k+k} = q_{2:k} = Y_k - [\Pi_k(\Phi_{k-1}) + Z_k], \quad 1 \leq k \leq n_2 \] (7)

\[ q_{k+k+n_k} = q_{k:k} = X_k - Y_k, \quad 1 \leq k \leq n_3 \]

\[ q_{k+k+n_k+n_k} = q_{k:k} = Z_k, \quad 1 \leq k \leq n_4 \]

where \( q_{s,k} \), \( s = 1, 4 \), and \( k = 1 \), \( n_1 \) (or, respectively, \( n_2, n_3, n_4 \)) are components of the vectors of generalized coordinates in each subsystem. As we have seen in sect. 3, \( n_1=4, n_2=1, n_3=0, n_4=1 \), in our case only, three subsystems are used. According to Fig. 2, the absolute coordinates for the FDS are:

\[ \Phi_0, \Phi_1, \Phi_2, \Phi_3, \Phi_4, \]

and denote the rotations of the drive sections \( J_0 - J_4 \), \( Y \) denotes the translation of the active mass, and \( Z_i, Z_k \), the bending deflections of the drive. So, the global vector of the generalized coordinates for the FDS is:

\[ q = \{q_{0,0}, q_{1,1}, q_{1,2}, q_{1,3}, q_{1,4}, q_{2,2}, q_{4,1}, q_{4,2}\} = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\} = \{\varphi_0, \varphi_1 - \varphi_0, (2\pi/p)x - \varphi_0, \varphi_0 - (2\pi/p)x, \varphi_0, \}

\[ \varphi_2, [Y - (Z_1 + x)], Z_1, Z_2 \] (8)

4.2 The descriptor matrix for an FDS

Since the dynamic model for the FDS is composed of four vibratory elements from subsystem \( S_0 \), one from subsystem \( S_2 \) and two elements from subsystem \( S_4 \), the descriptor matrix for the FDS is:

\[ DE = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \] (9)

Based on our method, this descriptor matrix allows us to obtain the MM in a special matrix.

4.3 The matrix form of MM of the FDS

The core of the method is based on the observation that for each subsystem, the equations of motions can be written under the same matrix form:

\[ [M_s]_k + [B_s]_k + [C_s]_k q_k = F_k, \]

(10)

where: \( s \) is the subsystem number (\( s = 1, 4 \)); \( n_s \) is the number of elements in subsystem \( s \) (\( n_1=4, n_2=1, n_3=0, n_4=1 \), in our case); \( q_k \) is the generalized coordinates vector in subsystem \( s \); \( F_k \) is the generalized forces vector in subsystem \( s \); \( [M_s] \) is the matrix inertia of the subsystem \( s \); \( [C_s]([B_s]) \) describe the stiffness (damping) behavior of the subsystem, respectively.

Using the notation \( \{0\}_k \) - a \( k \times k \) dimension zero-matrix and \( \{0\}_k \) - a \( k \) dimension zero-column vector, the MM matrix form for the entire system is:

\[ [M_1]_0 = [0], \quad [B_1]_0 = [0], \quad [C_1]_0 = [0] \]

\[ [M_2]_0 = [0], \quad [B_2]_0 = [0], \quad [C_2]_0 = [0] \]

\[ [M_4]_0 = [0], \quad [B_4]_0 = [0], \quad [C_4]_0 = [0] \]

The column vector \( \{F_1, F_2, F_4\} \) can be decomposed as follows:

\[ \text{Eq} + \]

\[ \left\{ \begin{array}{l}
\{0\}_3 \\
\{-m_c q_{1,1}\}
\end{array} \right\} + \left\{ \begin{array}{l}
\{0\}_3 \\
\{0\}_2
\end{array} \right\} + \left\{ \begin{array}{l}
\{0\}_3 \\
\{0\}_2
\end{array} \right\} \]

(12)

where \( \text{Eq} \) is the following column vector:

\[ \text{Eq}_0 = M_0 \]

\[ \text{Eq}_1 = 0, \quad \text{Eq}_2 = c \Pi', \quad \text{Eq}_3 = 0, \quad \text{Eq}_4 = 0 \]

\[ \text{Eq}_0 = - (mp \Pi + g), \quad \text{Eq}_0 = b \dot{q}, \quad \text{Eq}_7 = 0 \]

\( \Pi_k', \Pi_k'' \) - the derivatives in respect with time, contain hidden derivative of the variable that can be unhidden if \( \Pi_k', \Pi_k'' \) are expressed in respect with \( \Pi_k', \Pi_k'' \) (the derivatives in rapport with the absolute coordinate). In our case:

\[ \Pi = \Pi'\dot{q}_2' + \Pi''\dot{q}_2 = \Pi'\dot{q}_0 + \Pi'\dot{q}_1 + \Pi'\dot{q}_2 = \Pi'(\dot{q}_0 + \dot{q}_1 + \dot{q}_2)^2 + \Pi'\dot{q}_0 + \dot{q}_1 + \dot{q}_2 \]

(14)
Combining Eqs. (12) - (14), the global force vector from (11) becomes:

\[
F = \begin{bmatrix}
0 \\
0 \\
c_t \bar{I}
\end{bmatrix}
\]

\[
0 \\
0 \\
- \Pi + c_t \phi_2 + b \phi_2 + g
\]

\[
0 \\
0
\]

(15)

Then, the matrix form of the mathematical model is:

\[
[M]q + [B]q + [C]q = F
\]

(16)

\[F\] was defined in (16). \([M]\) is a diagonal matrix which has as diagonal the values of the inertial elements: \(J_{01}, J_{12}, J_{13}, J_{45}, m_p, m_b_1, m_b_2\).

\([C]\), the global stiffness matrix is:

\[
\begin{bmatrix}
0 & -ct_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & ct_0 & -ct_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & ct_1 & -ct_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & ct_2 & -ct_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & ct_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & c_{e_11} & c_{e_12} \\
0 & 0 & 0 & 0 & 0 & 0 & c_{e_12} & c_{e_22}
\end{bmatrix}
\]

(17)

\([B]\) - the global damping is deduced from Eq. (17) using the assumption \(v\).

Using a Mathematica®-software especially created, the extended explicit form of the mathematical model of the FDS is obtained for any given milling contour. The Appendix shows the principal lines of the Mathematica®-software and the computed second order nonlinear differential equations with some non-constant coefficients that describe the dynamic behavior of the FDS when a circular milling contour is given.

5 Problem Solution

We choose MATHEMATICA as a tool to solve the set of nonlinear second-order differential equations (its compact form is Eq. (17)) that describes the elasto-dynamic behavior of the FDS. Once the MM is solved, a very rich set of results can be obtained:

- the dynamic perturbations (velocities and accelerations, too) for each inertial point, in numeric and/or graphic form;
- the main characteristics of the solution;
- the mean frequency;
- the stability information.

6 Conclusions

Our new computer-aided modeling way for the FDS, considered as an 8-DOF system, allows us to obtain a symbolic form of its mathematical model - eight second order nonlinear differential equations, which can be used for the analysis of the dynamic behavior of the FDS and for its optimization, as well.

References:

APPENDIX

(*This package provides the extended form of second order nonlinear differential equations with some non-constant coefficients of the Mathematical Model for a FDS*)

(*:Requirements: Here :)*)

(*:Requirements for independent running: The requested input data are to be given: radius, p. w., L, 4, 6, 8, c, t, mb, nr, c, g, fi, *)

\[ q_0(t) = \phi_0(t); \quad q_1(t) = \phi_1(t) - \phi_0(t); \quad q_2(t) = x(t)/\tau; \quad q_3(t) = x(t)/\tau^2; \quad q_4(t) = \phi_4(t) - \phi_3(t); \quad q_5(t) = \phi_5(t); \quad q_6(t) = Z(t); \quad q_7(t) = Z(t); \]

\[ u(t) = \phi(t); \quad q_1(t); \quad q_2(t); \quad q_3(t); \quad q_4(t); \quad q_5(t); \quad q_6(t); \quad \text{ud}[t] = \phi_\text{ud}[t]; \quad \text{ud}[t] = \phi_\text{ud}[t]. \]

Clear[mq, mb, nr, c];

\[ m_q = \text{Table}[0, \{i, \text{do}, \}, \{j, \text{do}, \}]; \quad mb = \text{Table}[0, \{i, \text{do}, \}, \{j, \text{do}, \}]; \]

\[ m_q[1, 1] = J_0; \quad m_q[2, 2] = J_2 / 2; \quad m_q[3, 3] = J_2 / 2; \quad m_q[4, 4] = J_2 / 2; \quad m_q[5, 5] = J / 2; \quad m_q[6, 6] = \text{mp}; \quad m_q[7, 7] = m_b; \]

\[ ctt1[t] = \frac{32}{(\pi^2 \cdot c^4)} \cdot x(t); \quad ctt2[2]; \quad ctt2[2] = \frac{1}{c^2}; \quad ctt2[t]; \]

\[ ctt2 = \frac{1 \cdot 4}{48 \cdot \text{EI}}; \quad ctt1[t] = \frac{x(t)}{c^2} \left( L - x(t)^2 \right) \cdot 3 \; \text{EI}; \quad ctt1[t]; \quad ctt2[t] = \frac{x(t)}{c^2} \left( L - x(t)^2 \right); \quad ctt2[t]; \]

\[ m_c[1, 2] = -c_0; \quad m_c[2, 2] = c_0; \quad m_c[3, 3] = \text{ctt1}[t]; \quad m_c[4, 4] = \text{ctt1}[t]; \quad m_c[5, 5] = -c_2[t]; \quad m_c[6, 6] = c; \quad m_c[7, 7] = \text{ctt1}[t]; \quad m_c[8, 8] = \text{ctt1}[t]; \quad m_c[9, 9] = \text{ctt1}[t]; \quad m_c[10, 10] = \text{ctt1}[t]; \quad m_c[11, 11] = \text{ctt1}[t]; \quad m_c[12, 12] = \text{ctt1}[t]; \quad m_c[13, 13] = \text{ctt1}[t]; \]

\[ f(t) = \{3000, 0, \text{c_0}, 0, 0, 0, 0, 0, 0, 0, \text{g} + \text{mp} \cdot \text{x}_d[t], \text{q}_5[t], \text{q}_5[t], \text{q}_5[t], \text{u}_d[t] + \phi_\text{ud}[t]; \quad \text{ud}[t] = \phi_\text{ud}[t]. \]

\[ \text{eq} = \text{mj} \cdot \text{ud}[t] + \text{nb} \cdot \text{ud}[t] + \text{nc} \cdot \text{u}[t] - \text{f}[t]; \quad \text{tmf} = \text{Table}[0, \{i, \text{do}, \} \}; \]

Mathematica - [ FDS eq generation.nb *]

(*The computed Mathematical Model for a circular milling contour* )

\[ x(t) = \text{radius} \cdot \text{Cos}(t \cdot \text{w}); \]

\[ \{3000, 0, \text{ph}_{1}(t) - \text{ph}_{0}(t) ; \quad 3000 \cdot \text{ph}_{0}(t) \} = 0, \]

\[ 32 \text{radius} \cdot \text{Cos}(t \cdot \text{w}) \left( \frac{2 \text{radius} \cdot \text{Cos}(t \cdot \text{w})}{\pi} - \text{ph}_{1}(t) \right) \]

\[ + \frac{2 \text{ct} \cdot \text{radius} \cdot \text{Sin}(t \cdot \text{w})}{32 \text{radius}} + \frac{2 \text{radius} \cdot \text{Cos}(t \cdot \text{w})}{\pi} - \text{ph}_{1}(t) \]