

# DIAGNOSIS OF STRUCTURAL INTEGRITY USING THE NON-LINEAR VIBRATION TECHNIQUE

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**Abstract:** This paper refers to a study based on theoretical and experimental data regarding the impact of the vibration came from technological equipment functioning with intensive and various activities, environmental propagated (human and building). The aim of the presented work was to investigate the possibility of using hangs in the non-linear vibration characteristics to detect damage in mechanical structure. The nonlinearities were detected by examining the changes in time and frequency response, during period of time.

**Key-Words:** vibration, non-linear, linear, detect, damage, frequency

## 1 Introduction

The knowledge and evaluation of the shock and vibration influences on environment become a priority in European society sustainable development. In this way, European Directive 44/2002 establishes minimal requirements, in order to limit the level exposure of transmitted vibration on human or environment.

A multitude of technological equipments utilize shocks and vibration in the production process [4], such as forging hammer or press with eccentric. Due to peculiarity of production process, this equipment propagates shocks and vibration to environment, wherefore is necessary to implement a vibration protection system, able to decrease the effect impact on the environment. In time the vibration protection system, that is based on viscous-elastic type, suffering damage that produce system malfunction. These damages are recognized by non-linear characteristics of the viscous-elastic vibration protection system. The nonlinearities are identified through monitoring in time the fundamental frequency of the viscous-elastic system.

## 2 Theoretical problem formulations

In this chapter, will be develop a generalized theoretical model, capable to characterized both linear and non-linear characteristics of the viscous-elastic system in the mathematical approach.

### 2.1 Generalized model

It is considered a rigid body in the inertial system

OXYZ that is considered fix and a reference system attached on rigid [1], with the origin placed in its mass centre Cxyz, figure 1.

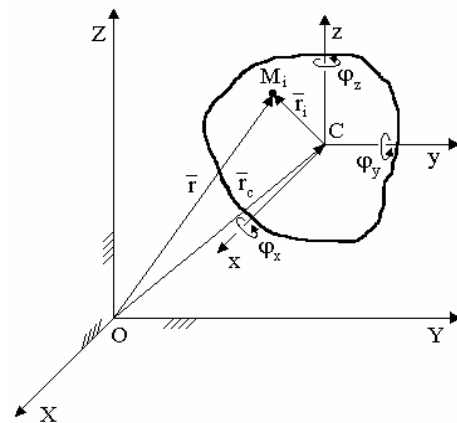


Fig. 1 The rigid in the inertial system OXYZ

The translational movements of the mass centre C are determinate by X, Y, Z coordinate toward fixed system OXYZ, and the rotary movements are describe by angular movements  $\varphi_x$ ,  $\varphi_y$  and  $\varphi_z$  of the Oxyz system.

In order, to calculate the movement of point A of the rigid toward Cxyz system when the rigid make an instantaneous rotation, as the case from figure 2. The rigid rotation  $\Delta\varphi$  can be the result of infinitesimal rotation sum.

Using the second kind Lagrange equation is obtained the differential equation system for the movement. The general form of the second kind Lagrange [1] equation is:

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}_i} \right) - \frac{\partial E}{\partial q_i} = Q_i^P + Q_i^F + Q_i^R, \quad i=1..6 \quad (1)$$

where  $Q_i^P = -\frac{\partial V}{\partial q_i}$  are generalized forces on potential kind,  $Q_i^R = -\frac{\partial D}{\partial \dot{q}_i}$  are generalized forces on viscous kind,  $Q_i^F = \frac{\partial L_{q_i}}{\partial q_i}$  are generalized forces on perturbation,  $\delta L_{q_i}$  - virtual mechanical work on perturbation which corresponds  $q_i$  coordinate.

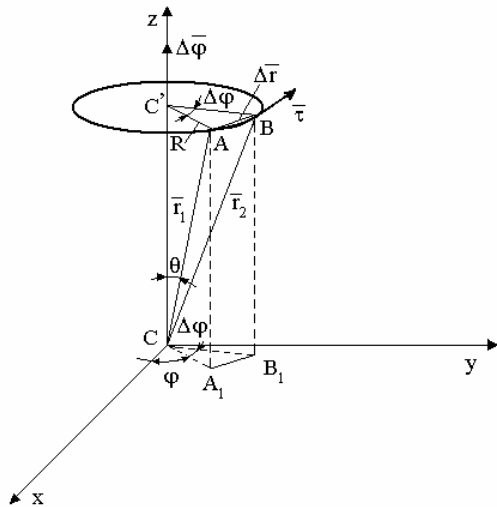


Fig. 2 Instantaneous rotation of the rigid

In this way will consider that foundation of the technological equipment like forging hammer is placed on the four identically viscous-elastic elements [4], and it has one plan of symmetry, figure 3.

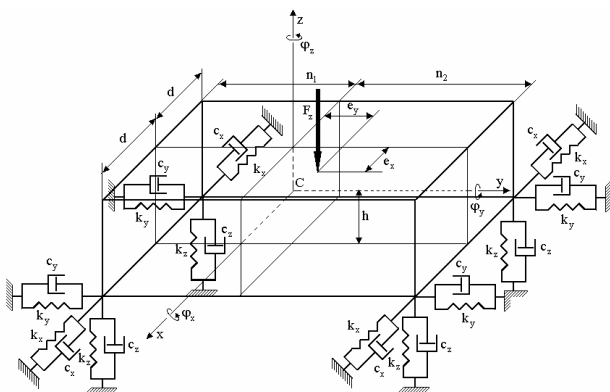


Fig. 3 The physical model

This presented model has a general character, and the possible rigid movements are: on direction OX – forcing lateral vibration, on direction OY - forcing longitudinal vibration, on direction OZ - forcing

vertical vibration,  $\phi_x$  - forcing pitching vibration,  $\phi_y$  - forcing rolling vibration,  $\phi_z$  - forcing turning vibration.

The principal axes of the elastic supports are parallel with the references axis. In this case, the movements corresponding to the six degree of freedom are decoupled in two possibilities: coupled movements that are characterized by the coordinate Y, Z and  $\phi_x$  variations and coupled movement that are characterized by the coordinate Y,  $\phi_y$  and  $\phi_z$  variations.

### 2.2 The coupled mode “YZ $\phi_x$ ”

Forwards, will be analyzed the coupled model characterized by the coordinate Y, Z and  $\phi_x$  variations because the movement on OZ direction is a very important factor in propagation vibration from technological equipment.

#### 2.2.1 The linear elastic characteristic hypothesis

The rigidity on OZ direction of the viscous-elastic element on which is the foundation placed of the technological equipments, have constant value. The mathematical model [4] that characterized this dynamically system is:

$$\begin{cases} m\ddot{Y} + 4c_y\dot{Y} + 4c_y h\dot{\phi}_x + 4k_y Y + 4k_y h\phi_x = 0 \\ m\ddot{Z} + 4c_z\dot{Z} + 2c_z(n_2 - n_1)\dot{\phi}_x + 4k_z Z + 2k_z(n_2 - n_1)\phi_x = -F_z \\ J_x\ddot{\phi}_x + 4hc_y\dot{Y} + 2c_z(n_2 - n_1)\dot{Z} + 2[2c_y h^2 + c_z(n_2^2 + n_1^2)]\dot{\phi}_x + 4hk_y Y + 2k_z(n_2 - n_1)Z + 2[2k_y h^2 + k_z(n_2^2 + n_1^2)]\phi_x = -e_y F_z \end{cases} \quad (2)$$

where m is foundation mass, k is rigidity of the viscous-elastic element, c is damping of the viscous-elastic elements, J is inertia moments of the foundation block.

Analyze of this system will be made by evaluating three cinematically measure: - acceleration, velocity, movement, and frequency response.

The excitation force is on OZ direction, applied point being eccentricly toward mass centre figure 3. The excitation of the system is half-sine shock pulse (figure 4), the applied being T=0.005 s.

The equation system was resolved with Runge – Kutta method with  $10^{-5}$  value of absolute error. Shape of curves from the three cinematically measures are presents in the next figures 5, 6 and 7.

The solving system was made in the next numerical value hypothesis:  $P=900 \cdot 10^4 \text{ N}$ ;  $k_0=2.5 \cdot 10^9 \text{ N/m}$ ;  $c_y=2.5 \cdot 10^6 \text{ Ns/m}$ ;  $m=100 \cdot 10^3 \text{ kg}$ ;  $k_z=8 \cdot 10^9 \text{ N/m}$ ;  $c_z=2.1 \cdot 10^6 \text{ Ns/m}$ ;  $J=77 \cdot 10^4 \text{ kgm}^2$ ;  $e=0.02 \text{ m}$ ;  $n_1=3\text{m}$ ;  $n_2=3\text{m}$ ;  $h=1.5\text{m}$ .

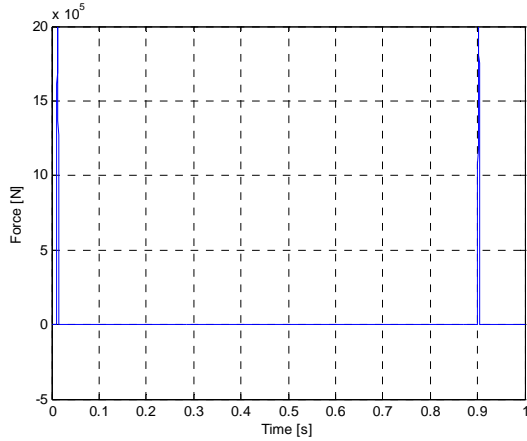


Fig. 4 The shape of half-sine shock

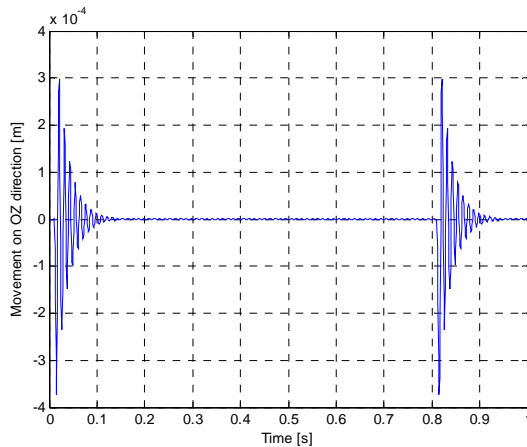


Fig. 5 Movement on OZ direction

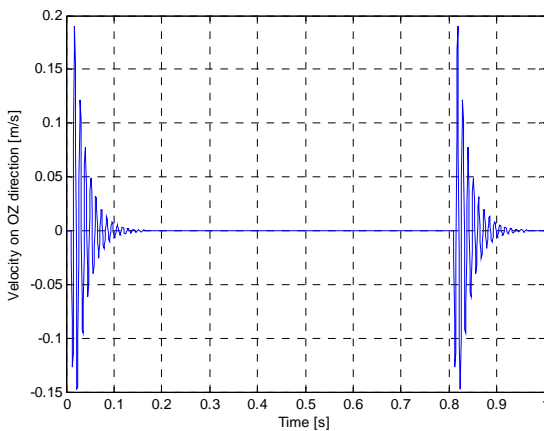


Fig. 6 Velocity on OZ direction

These three cinematically measures are quantitative criteria for evaluating the vibration effects on the human structure or on environment.

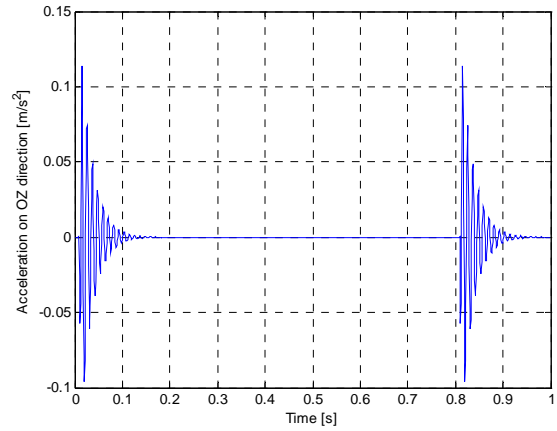


Fig. 7 Acceleration on OZ direction

Eliminating the time between movement and velocity expressions, it is obtained the characteristically curve or movement trajectory (figure 8).

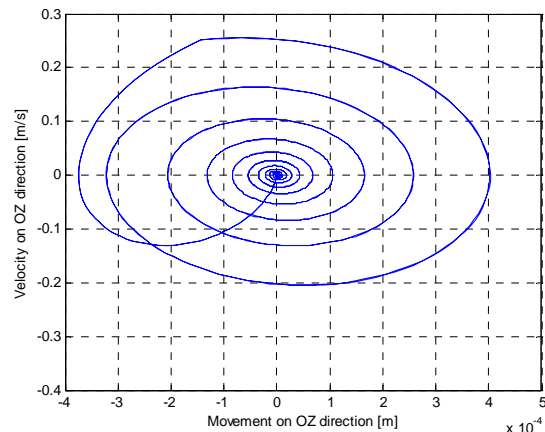


Fig. 8 The phase plan representation

From the figure 8 we observe that movement is damping and stabilized because the amplitude of movement don't have an increasing infinite value.

In the figure 9 is presented the movement on OZ direction in the frequency representation. From this representation we observe that dominant frequency domain is around on 97Hz value.

Another analyze in frequency response is power spectral density, figure 10). Because the elements on which is the foundation placed have viscous-elastic characteristic; these elements dissipate hysteretic energy with  $W=5093 \text{ J}$ , figure 11.

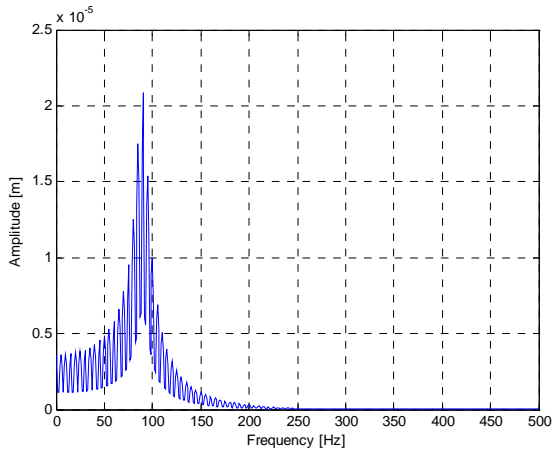


Fig. 9 The system response in frequency domain

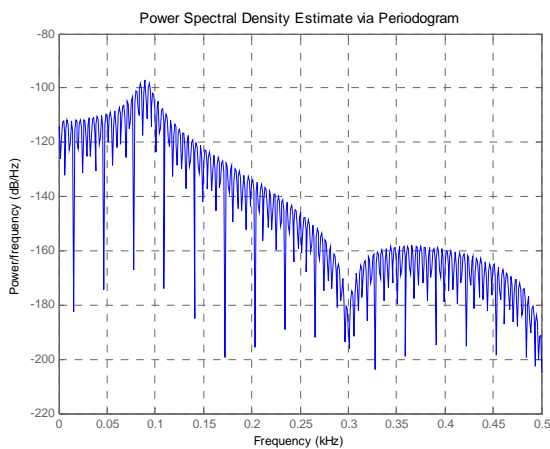


Fig. 10 Power spectral density

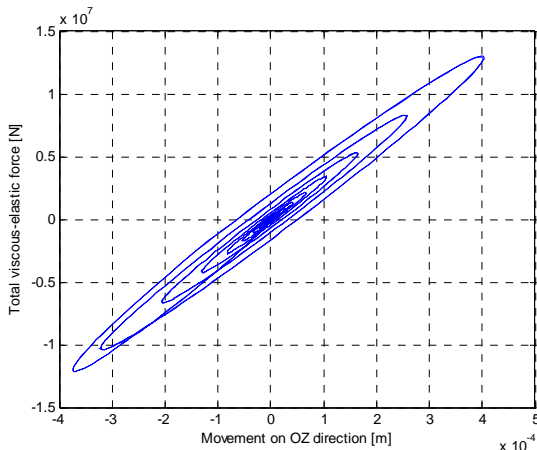


Fig. 11 The hysteretic characteristic

**2.2.2 The non-linear elastic characteristics hypothesis**

The rigidity on OZ direction of the viscous-elastic element on which is placed the foundation of the technological equipments, have the nonlinear expression [2] followed:

$$k_z = k_0(1 + \beta \cdot x_{OZ}^2) \tag{3}$$

The solving system was made in hypothesis of the next numerical values:  $P=900 \cdot 10^4 \text{N}$ ;  $k_0=2.5 \cdot 10^9 \text{ N/m}$ ;  $c_y=2.5 \cdot 10^6 \text{ Ns/m}$ ;  $m=100 \cdot 10^3 \text{ kg}$ ;  $k_z=8 \cdot 10^9 \text{ N/m}$ ;  $\beta =2 \cdot 10^8 \text{ 1/m}^2$ ;  $c_z=2.1 \cdot 10^6 \text{ Ns/m}$ ;  $J=77 \cdot 10^4 \text{ kgm}^2$ ;  $e=0.02 \text{ m}$ ;  $n_1=3\text{m}$ ;  $n_2=3\text{m}$ ;  $h=1.5\text{m}$ .

In hypothesis of the same numerical values, the solution of the system (2) leads to the evolution determination in time for three kinematics parameters: acceleration, velocity and movement - on OZ direction [3], figures 12, 13 and 14.

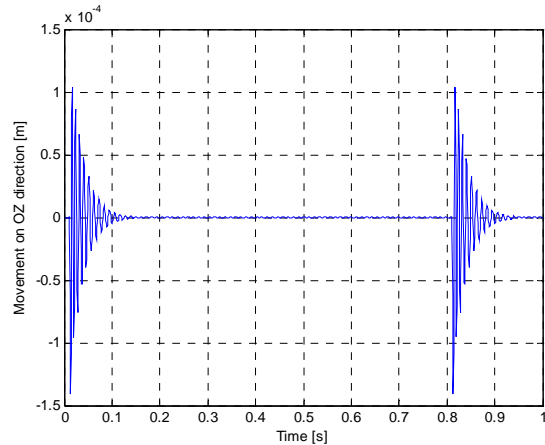


Fig. 12 Movement on OZ direction

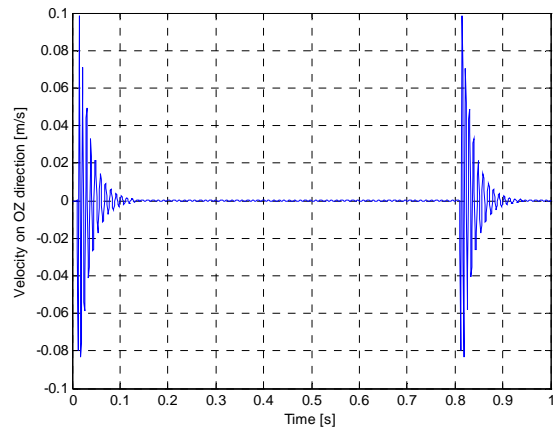


Fig. 13 Velocity on OZ direction

Time history of kinematics parameters enable effects characterization of transmitted vibration to the environment, comparative with established limit of effectual standard.

Eliminating time between velocity and movement permits obtaining - movement trajectory (fig. 15), that show the movement is damped and stabilized

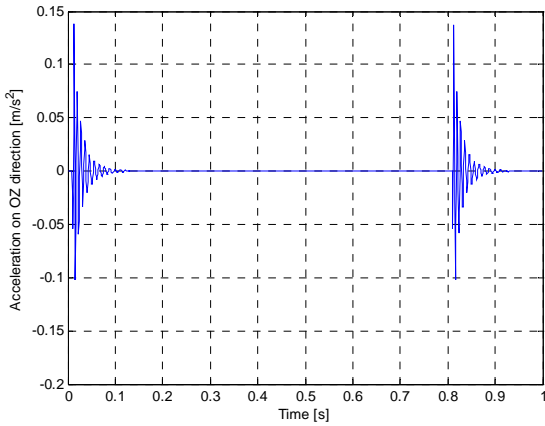


Fig. 14 Acceleration on OZ direction

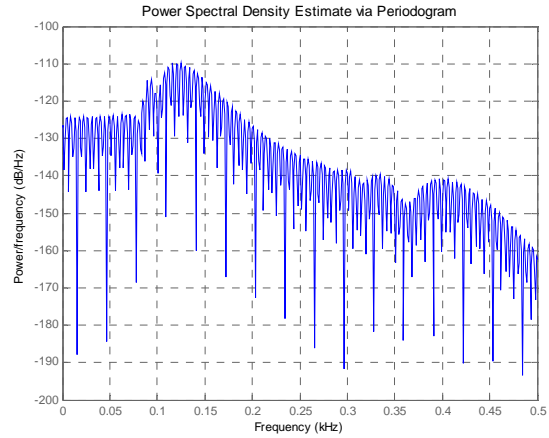


Figure. 17 Power spectral density

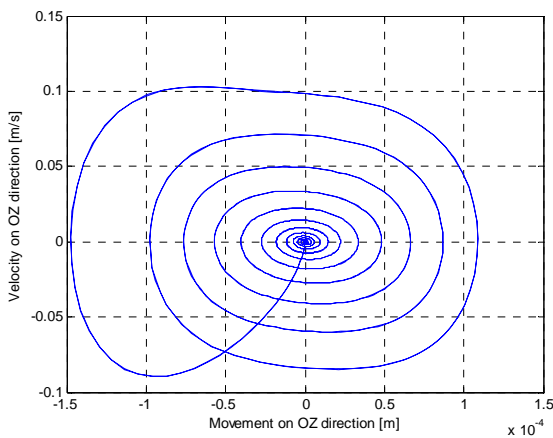


Fig. 15 The phase plane representation

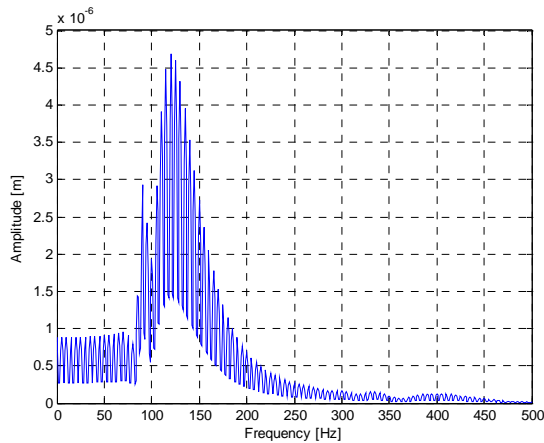


Fig. 16 The system response in frequency domain

So, frequency responses have spectral components around of 97 Hz value (as in the case of linear rigidity), but appear dominant spectral components around of 120Hz value, figure 16. Distribution of the energy of the shock on spectral components is noticed by plotting the power spectral density (figure 17).

The energy dissipation is made by viscous amortization that is emphasizing by plotting the total forces viscous-elastic function of movement (figure 18).

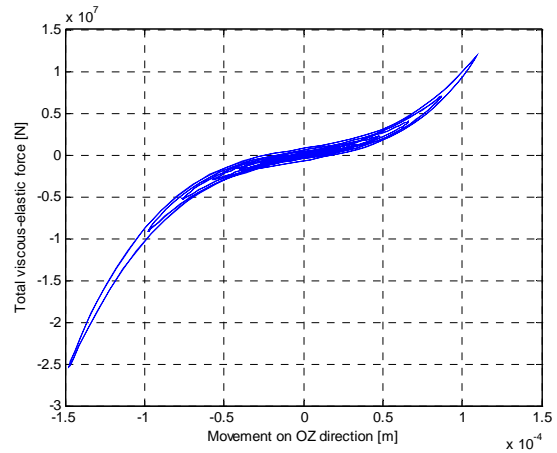


Fig. 18 The hysteretic characteristic

The value of dissipate energies on a loop of movement is of  $W = 1341J$ . Towards the case of linear rigidity we observe a diminution of dissipate energy, explained by the diminution of movement amplitudes on OZ direction.

### 3 Experimental researches

This chapter presents the result of the experimental determinations made on forging hammer (1250kg capacity) at Workshop in IUS – Brasov. The measurements were made simultaneous on anvil block and foundation vat between are placed the viscous-elastic systems for isolating and damping generated vibration during the technological process. Wave shape recorded on the anvil block and the spectral density are represented in figures 19 and 20, and wave shape recorded on the foundation vat and

the spectral density are represented in figures 21 and 22.

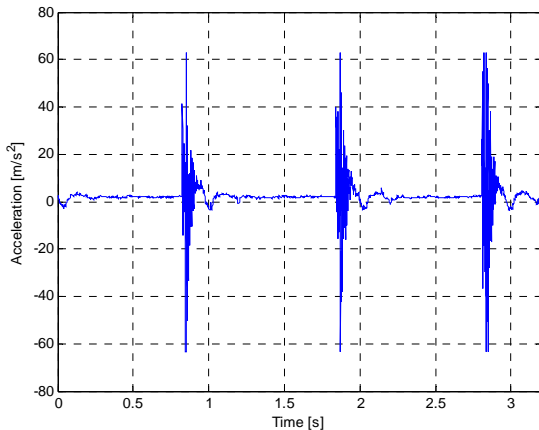


Fig. 19 Wave shape recorded on the anvil block

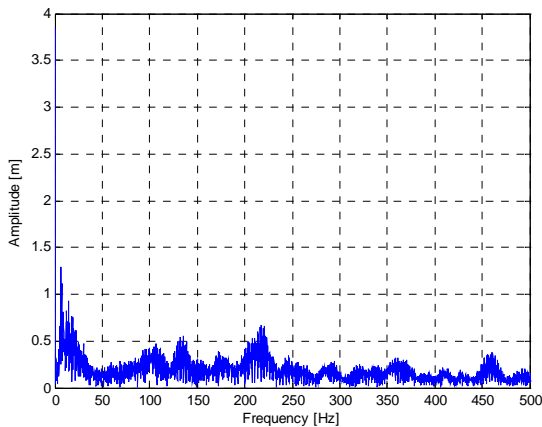


Fig. 20 Frequency response - anvil block

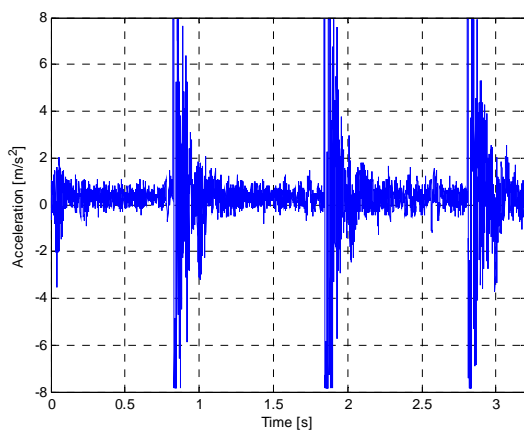


Fig. 21 Wave shape recorded on the foundation vat

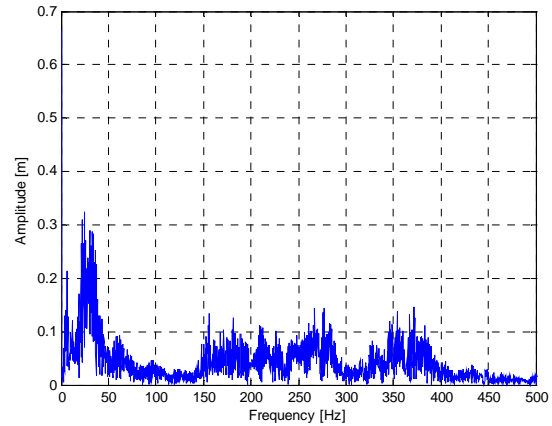


Fig. 22 Frequency response – foundation vat

#### 4 Conclusions

This paper presents a theoretical model to characterize dynamically, a much diversified field of real technological situations in which equipments utilize shocks and vibration in the production process.

It's clearly that in the case of the non-linear elastic characteristic on OZ direction, the dynamical response system is different comparing to linear elastic characteristic on OZ direction case. Theoretically, the presence of nonlinearities characteristic in the viscous-elastic protection system, conducts inevitable to a dynamical response modification (frequency response).

Practically, based on monitoring the frequency response of the technological equipment and detecting its modification, can determine level of elastic characteristic nonlinearities. In the same time, this study represents the beginning of experimentally research development regarding detection of damage in structure of viscous-elastic systems, based on the non-linear vibration technique.

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