Mathematical Optimization of Solar Thermal Collectors Efficiency Function Using MATLAB

RALUCA LATES[1], ION VISA[1], CIPRIAN LAPUSAN[2]
[1]Product Design and Robotics Department, [2]Mechanism, Fine Mechanic and Mecatronic Department
[1]Transilvania University of Brasov, [2]Technical University of Cluj Napoca
[1]Eroilor, nr.29, Brasov 500036, [2]B-dul Muncii, 103-105, Cluj Napoca
[1]ROMANIA, [2]ROMANIA

Abstract: - Solar thermal collectors are devices which collect solar radiation, they transform it into heat and they deliver it to a heat exchanger through the heat carrier. The aim of the paper is to make an optimization of their efficiency using MATLAB. The efficiency is expressed as a function of geometric and material parameters which are variable. Some of the parameters are fixed and some vary in certain ranges. The maximum value is found for efficiency which corresponds to the optimal combination of values for the variable parameters. In conclusion comparisons between are made with known values for efficiency given by solar collectors producers.

Key-Words: - Solar thermal collectors, Efficiency, Optimization, MATLAB, Function, Non-linear minimization

1 Introduction

The available solar energy can be used to obtain electrical power by using photovoltaic panels or solar thermal collectors which operates at very high temperatures and to obtain heat. Solar thermal collectors are the devices which collect solar radiation, they transform it into heat and they deliver to a heat exchanger through the heat carrier. The paper refers to flat plate solar collectors with liquid heat carrier.

Their construction can vary much and correspond to different efficiencies, so it is necessary to find the optimal combination of the values for parameters which correspond to the maximum value of the efficiency function.

In order to understand what the parameters refer to, the drawing of a typical solar flat plate collector with its components must be explained, Fig.1.

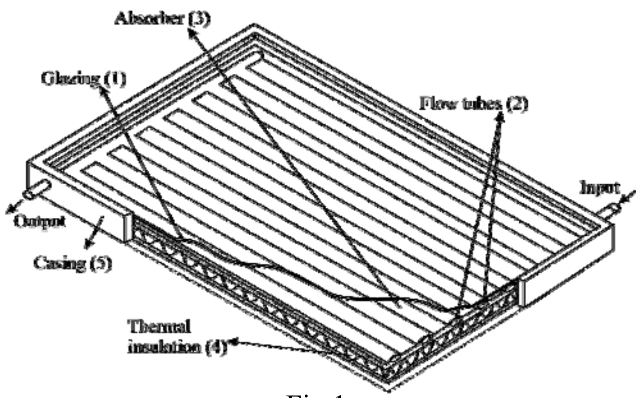

Fig.1

The main components of a typical solar collector, shown in Figure 1, are: the glazing surfaces (*1*) (one, two or three pieces can be used), the flow tubes (*2*) where inside the heat carrier flows, the absorber plate (*3*), thermal insulation (*4*) and the casing (*5*) which protects the components [3, 5, 7].

The literature gives information about the geometry of the collectors' components, about dimensions for them (e.g. diameters of flow tubes, tubes spacing, absorbers thickness etc.). All of these parameters influence the efficiency of solar collectors, so in order to obtain high (maximum) efficiencies the right combination of parameter values has to be found.

The aim of the paper is to make this optimization of the efficiency by mathematical means using MATLAB. The following section presents the efficiency function which has to be maximized.

2 The efficiency function

The efficiency can be expressed as in equation (1) [1, 2, 3], where:

η is the collector's efficiency;

$\dot{m}$ - the flow rate;

C_p - the specific heat of the liquid heat carrier;

U_L - heat loss coefficient which will be expressed in equation (2);

A_C - the area of the collector;

F' - collector efficiency factor, expressed in equation (10);
τ - effective solar transmittance of the collector cover(s);
α - solar absorptance of the collector absorber plate surface;
n - refraction index of the cover plate;
T_i - fluid temperature inside the collector;
T_a - ambient temperature;
H_T - the radiation density on the collector surface.

$$\eta = \frac{\dot{m} C_p}{U_L A_c}\left\{1 - \exp\left[\frac{U_L A_c F'}{\dot{m} C_p}\right]\right\}\left[\frac{\tau\alpha}{1-(1-\alpha)(n-1)^2/(n+1)^2} - U_L\frac{T_i - T_a}{H_T}\right]. \quad (1)$$

The heat loss coefficient can be replaced in equation (1) with:

$$U_L = U_t + U_b + U_e, \quad (2)$$

where U_t is the top loss coefficient and its expression is:

$$U_t = \frac{1}{\dfrac{N}{\dfrac{C}{T_p}\left(\dfrac{T_p - T_a}{N+f}\right)^e} + \dfrac{1}{h_w}} + \frac{\sigma\left(T_p^2 + T_a^2\right)\left(T_p + T_a\right)}{\dfrac{1}{\varepsilon_p + 0{,}00591 N h_w} + \dfrac{2N + f - 1 + 0{,}133\varepsilon_p}{\varepsilon_g} - N}, \quad (3)$$

where:
N is the number of cover plates;
C is defined as:

$$C = 520\left(1 - 0{,}000051\beta^2\right), \quad (4)$$

where β is collector inclination angle;
T_p - absorber plate temperature;
f is defined as:

$$f = \left(1 + 0{,}089 h_w - 0{,}1166 h_w \varepsilon_p\right)\left(1 + 0{,}07866 N\right); \quad (5)$$

h_w - convective heat transfer fluid coefficient caused by the wind:

$$h_w = 2{,}8 + 3{,}0V \quad (6)$$

and v is the wind speed.
ε_p - the thermal emissivity of the absorber plate;
The equation which define e:

$$e = 0{,}43\left(1 - \frac{100}{T_p}\right); \quad (7)$$

σ - Stefan – Boltzman constant;
ε_g - thermal emissivity of cover plate.
In equation (2), U_b is the bottom loss coefficient:

$$U_b = \frac{k}{L}, \quad (8)$$

where k is the thermal conductance of the insulation and L, the insulation width.
Also in equation (2), U_e is the edge loss coefficient and it is found as:

$$U_e = \frac{U}{L_{iz}}\left(\frac{A_l}{A_c}\right), \quad (9)$$

where:
U is the thermal conductance of the edge insulation;
L_{iz} - the width of the edge insulation;
A_l - the edge area of the collector;
The collector efficiency factor F' is expressed as:

$$F' = \frac{1/U_L}{\left(D + d_0\right)\left(\dfrac{1}{U_L\left(d_0 + f'D\right)} + \dfrac{1}{C_b} + \dfrac{1}{\pi d_i h_f}\right)}, \quad (10)$$

where:
D is the distance between the flow tubes;
d_0 - exterior diameter of flow tubes;
It is defined f' as:

$$f' = \frac{\tanh(y)}{y}, \quad (11)$$

where:

$$y = 0{,}5D\left(U_L / kx\right)^{1/2}; \quad (12)$$

k is the thermal conductivity of the absorber plate and x the absorber plate thickness.
C_b - represents the thermal conductance of the bond between the plate and the riser tube:

$$C_b = \frac{A_b k_b}{t_b}, \quad (13)$$

where A_b is the cross-sectional area of the heat transfer through the bond per unit length of riser, k_b

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- thermal conductivity of bond material, t_b - thickness of bond [4].
In equation (10), d_i is the interior diameter of flow tubes, h_f the heat transfer coefficient.

3 Selecting the parameters

The section presents the selected parameters (six) from the efficiency expression which will be the restrictions in the optimization process. The rest of the parameters are considered fixed and values are given for them.
The aim of the paper is to make the optimization from the construction point of view that is the reason why parameters linked to dimensions of different components are selected. Ranges of values where they vary are established from the literature [3, 5, 6].

$$\eta = \frac{27{,}37 L L_{iz}}{1{,}03 L L_{iz} + 0{,}04 L_{iz} + 0{,}004 L}\left\{1 - \exp\left[(0{,}037 + \frac{0{,}0014}{L} + \frac{0{,}000144}{L_{iz}})F'\right]\right\}\left(0{,}85 + \frac{0{,}0044}{L} + \frac{0{,}00044}{L_{iz}}\right). \quad (14)$$

The following parameters are selected and they vary in some ranges:

- The bottom insulation width, $L \in \lfloor 2 \ , \ 10 \rfloor mm$;
- The width of edge insulation $L_{iz} \in \lfloor 2 \ , \ 10 \rfloor mm$;
- The distance between tubes, $D \in \lfloor 87 \ ; \ 98 \rfloor mm$;
- The interior diameter of the flow tubes, $d_i \in \lfloor 32{,}5 \ ; \ 57 \rfloor mm$;
- The exterior diameter of the flow tubes, $d_0 \in \lfloor 35 \ ; \ 60 \rfloor mm$;
- The absorber plate thickness, $x \in \lfloor 0{,}2 \ ; \ 2 \rfloor mm$.

The rest of the parameters are established for a concrete case. The flow rate is established at $\dot{m} = 70 \frac{kg}{h}$. The specific heat of the liquid heat carrier which is a water-glycol mixture with 44% glycol is $C_p = 3570 \frac{J}{kg \cdot K}$.
The transmittance –absorptance product is considered $\tau\alpha = 0{,}96$ where $\alpha = 0{,}95$; the refraction index is $n = 1{,}52$ [3]; the fluid temperature inside the collector $T_i = 373K$; the density of radiation incident on the collector surface is as in non cloudy day in Brasov town, Romania, $H_T = 800 W/m^2$.For the collector chosen to be optimized there is used one cover plate, $N = 1$. It is placed in town of Brasov (latitude $45{,}65^0$), Romania, the collector's inclination angle is $\beta = 45^0$. The absorber plate temperature is assumed as being $T_p = 373K$; the ambient temperature $T_a = 283K$; the wind speed $v = 2m/s$ in order to have ventilation over the collector's surface (as recommended for testing); he thermal emissivity of the absorber plate is $\varepsilon_p = 0{,}95$; the Stefan – Boltzman constant is $\sigma = 5{,}6 \cdot 10^{-8} W/m^2 K^4$; the emissivity of cover plate, for glass, there is $\varepsilon_g = 0{,}94$.
The conduction coefficient of the thermal insulation is $K = 0{,}04 \frac{W}{m \ K}$; the edge area of the collector $A_l = 0.3 m^2$; the collector area $A_c = 2.53 m^2$.
Thermal conductivity of the absorber plate is $k = 1250 W/mK$.
The thermal conductance of the bond between the plate and the riser tube is considered to be very high, so $1/C_b = 0$. The heat transfer coefficient is $h_f = 1000 W/m^2 C$.
After calculation the efficiency function is as shown in eq. (14), where F' is replaced with equation (15).

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$$F' = \frac{1/(1{,}03 + \frac{0{,}04}{L} + \frac{0{,}004}{L_{iz}})}{\left(D + d_0\right)\left(\frac{1}{(1{,}03 + \frac{0{,}04}{L} + \frac{0{,}004}{L_{iz}})\left(d_0 + \frac{\tanh\left(0{,}014D\left(\frac{1{,}03}{x} + \frac{0{,}04}{xL} + \frac{0{,}004}{xL_{iz}}\right)^{1/2}\right)}{0{,}014\left(\frac{1{,}03}{x} + \frac{0{,}04}{xL} + \frac{0{,}004}{xL_{iz}}\right)^{1/2}}\right)} + \frac{1}{1000\pi d_i}\right)} \quad (15)$$

4 Optimization with MATLAB

In order to make the optimization the function *fmincon* from MATLAB is used, which calculate the minimum of a nonlinear function with multi variables and restrictions.

$$\min_{x} f(x). \quad (16)$$

For this reason, in the presented case, the function will be written:

$$f(x) \rightarrow -f(x). \quad (17)$$

The restrictions:

$$A \cdot x \le b; \quad (18)$$

$$Aeq \cdot x = beq; \quad (19)$$

$$lb \le x \le ub; \quad (20)$$

Where *x*, *b*, *beq*, *lb* and *ub* are vectors, *A* and *Aeq* are matrices, and $f(x)$ the function to be calculated.

The expression

$$x = f\min con(fun, x0, A, b, Aeq, lb, ub), \quad (21)$$

defines a set of values for the variables of *x* so $lb \le x \le ub$ and

$$[x, fval] = f\min con(...) \quad (22)$$

calculates the function value for solution *x*.

In order to define the objective function there will be written in an *M* file:

function f=objfun(x). (23)

The function being expressed in equation (14).

The restrictions are written in equation (24). For MATLAB they are translated as in relation (25).

The vectors become as shown in equations (26) and (27).

The maximum value of function will be calculated with (28).

$$\begin{cases} L = x(1) \\ L_{iz} = x(2) \\ D = x(3) \\ d_0 = x(4) \\ x = x(5) \\ d_i = x(6) \end{cases}; \quad (24)$$

$$\begin{cases} 2 \le x(1) \le 10 \\ 2 \le x(2) \le 10 \\ 80 \le x(3) \le 120 \\ 30 \le x(4) \le 70 \\ 0.1 \le x(5) \le 2 \\ 30 \le x(6) \le 70 \end{cases}; \quad (25)$$

$$lb = \left[2,\ 2,\ 80,\ 30,\ 0.1,\ 30\right]; \quad (26)$$

$$ub = \left[10,\ 10,\ 120,\ 70,\ 2,\ 70\right]. \quad (27)$$

$$[x, fval] = f\min con\ \left(@objfun,\ \ x0,\ \ [\ ], [\ ], [\ ], [\ ],\ \ (lb,\ \ ub\right) \quad (28)$$

The value where each variable is checked for calculations is chosen to be the middle of its range:

$$x_0 = 0.5 \cdot (ub + lb). \quad (29)$$

The calculated value for efficiency is $fval = 0.8391$ and the variables which correspond to this maximum value are given in equation (30).

$$\begin{cases} x(1) = 2 \\ x(2) = 2 \\ x(3) = 120 \\ x(4) = 30 \\ x(5) = 0.1 \\ x(6) = 49.9954 \end{cases} \quad (30)$$

5 Conclusion

The paper presents the mathematical optimization made by using MATLAB of the efficiency of solar thermal collectors. The first section makes an introduction of collectors' components in order to be easier to understand the parameters which appear in the efficiency equation presented in the second section.

The described efficiency represents the objective function used in the optimization process. A number of six parameters are selected, linked to the dimensions of different components, which are the restrictions. The other parameters are adopted for a concrete collector.

The result gives the maximum value of efficiency which is *0,8391* and the variable values which correspond to the maximum function.

On the market the efficiencies for solar thermal collectors are given around the value of 78%, see comparison from Figure 2 (column *1* represents the efficiency given by producers of solar collectors and *2* the optimized efficiency).

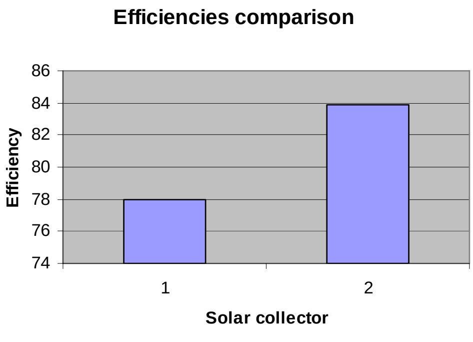

Fig.2

The results show that it is obtained a higher value for efficiency by using the mathematical optimization than those offered on market.

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