Linear Combination for Adaptive Distributed Classification in Wireless Sensor Networks

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Abstract: A fault-tolerant classification system in wireless sensor networks combining distributed detection with error-correcting codes have recently been proposed. A codeword is designed for each hypothesis. Each sensor makes a local decision based on the codeword and its observation result. The local decision is then transmitted to a fusion center to make a final decision. An adaptive redetection algorithm and an adaptive retransmission scheme were later developed to reduce the misclassification probability of the system when the observation is highly noisy, and the transmission channel between the sensor and the fusion center is deeply faded, respectively. The observation result at the sensor and the received data at the fusion center are discarded if they are not reliable in the adaptive method. However, they still have useful information about the hypothesis and should be utilized. This work use Linear Combination (LC) techniques to utilize the unreliable data. Little extra complexity is needed. Simulation results show that the new adaptive method with LC outperforms the original one.

Key–Words: Equal-gain combination, wireless sensor networks, adaptive distributed detection, fading channels, fault-tolerant

1 Introduction

Wireless sensor networks (WSNs) comprise many tiny, low-cost, battery-powered sensors in a small area [1]. The sensors observe environmental variations and then transmit the observation results to other sensors or a base station [2]. The base station or a sensor, serving as a fusion center, collects all observation results, and determines what phenomenon has occurred. The collection is realized using wireless communication technology, and a wireless network is built for multiple accesses. To lower the transmission burden, the observation result is typically denoted by a local decision which is made by the sensor, and which requires fewer bits than the observation result. The local decision is transmitted rather than the observation result. Hence, each sensor must be able to collect, process and communicate data.

The WSN sometimes must be able to function under severe conditions, such as in a battlefield, fire-place or polluted area. The transmission channel, as well as the environmental phenomenon observed by the sensor, is noisy. Furthermore, the observation signal to noise ratio (OSNR) and the channel signal to noise ratio (CSNR) may change quickly. The OSNRs and the CSNRs are thus impossible to estimate accurately. Some sensors may even have unrecogn-ized faults. The traditional distributed classification method thus fails due to inaccurate estimates or faulty sensors. Therefore, a fault-tolerant system must be developed to make the received local decisions error-resistant [3].

Wang et al. [4] proposed Distributed Classification Fusion using Error-Correcting Codes (DCFECCE) to solve this problem by combining the distributed detection theory with the concept of error-correcting codes in communication systems. One sample is detected in each of $N$ sensors for a given phenomenon. A codeword consisting of $N$ symbols is designed for each phenomenon. In other words, a one-dimensional code ($1 \times N$) corresponds to a phenomenon. Thus, $M$ phenomena form an $M \times N$ code matrix. Each symbol with one bit is assigned to each phenomenon. DCFECCE has a much lower probability of misclassification when some sensors are faulty than the traditional distributed classification method. DCFECCE outperforms the method even when CSNR is not correctly estimated.

DCFSD (distributed classification fusion using soft-decision decoding) [5] was later developed by
improving DCFECC. The soft-decision decoding, instead of hard-decision decoding, is utilized to increase decoding accuracy. However, the misclassification probability remains high in the extreme case, i.e., very low SNRs (including OSNRs and CSNRs) because of large observation deviation and unreliable transmission channels. Pai et al. have developed an adaptive retransmission mechanism to resolve the low CSNR problem [6, 7] and then proposed an adaptive redetection algorithm to combat the low OSNR problem [8].

In the adaptive retransmission mechanism, the fusion center calculates the channel reliability of each received detection result while making the final decision. When the final decision is not reliable, the received result with the lowest channel reliability is discarded and the sensor which has sent it will be asked to retransmit its detection result by the fusion center. Similarly, if the observation result of the sensor is located in an unreliable range, it is discarded and the sensor makes another observation in the adaptive redetection mechanism. However, the unreliable observation result at the sensor and the unreliable received detection result at the fusion center still contain information about the environment and the local decision, respectively. They should be utilized to increase the performance of the adaptive distributed classification system.

In this work, we apply Linear Combination (LC) techniques [9] for the utilization of the unreliable data. A new observation result at a sensor is equally combined with the combined result of the previous observations. The combined observation result is then employed to decide whether another observation is necessary or not. If another observation is unnecessary, a local decision based on the combination result is made. The adaptive redetection scheme using the LC technique needs a smaller number of observations and has a lower misclassification probability than the original one. Similarly, the channel reliability of the latest received local decision from the same sensor at the fusion center is equally combined with the combined channel reliability of the previous received local decisions. The fusion center then use the combined channel reliability to decide which sensor is selected for retransmission. Moreover, two methods are proposed to decide if the final decision can be made. The new adaptive retransmission algorithms needs less retransmission times and reach a misclassification probability close to the previous one under the same retransmission criteria.

2 Distributed Detection and The Previous Works

Figure 1 depicts a wireless sensor network for distributed detection with \( N \) sensors deployed for collecting environment variation data and a fusion center for making a final decision of detections. This network architecture is similar to the so-called SEnsor with Mobile Access (SENMA) [10]. At the \( j \)-th sensor, one observation \( y_j \) is undertaken for one of phenomena \( H_i \), where \( i = 1, 2, \ldots, M \). The observation is normally a real number represented by many bits. Transmitting the real number to the fusion center would consume too much power, so a local decision, \( u_j \), is made instead.

### 2.1 Old Adaptive Redetection Algorithm

The DCFECC approach [4] designs an \( M \times N \) code matrix \( T \) not only to correct transmission errors, but also to resist faulty sensors. The application of the code matrix is derived from error-correcting codes. Table 1 lists an example of \( T \), which is the optimal code matrix found in [11]. Row \( i \) of the matrix represents a codeword \( c_i = (c_{i,1}, c_{i,2}, \ldots, c_{i,N}) \) corresponding to hypothesis \( H_i \), and \( c_{i,j} \) denotes a 1-bit symbol corresponding to the decision of sensor \( j \).

The decision region at sensor \( j \) can be represented...

| \( H_1 \) | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| \( H_2 \) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| \( H_3 \) | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| \( H_4 \) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 1: The \( 4 \times 10 \) optimal code matrix [11]
by a set of thresholds [8]. Thus, a local decision rule associated with this threshold set can be performed to determine $u_j$ when $y_j$ is observed. Since the observation result around the threshold is not reliable, an unreliable range is defined around the threshold. For example, four hypotheses $H_1, H_2, H_3$, and $H_4$, are detected and classified with $N = 10$ sensors and a fusion center. These hypotheses are assumed to have Gaussian-distributed probability density functions (pdfs) with the same standard deviation $\sigma$ and means 0, 1, 2, and 3, respectively. Table 1 is used as the code matrix. At each sensor, OSNR is defined as $0 \times \log_{10} \sigma^2$. When $\sigma^2 = 0.6$ and channel noise is zero, the threshold, $T_1$, and the unreliable range, $U_j = [T_1 - \tau_j, T_1 + \tau_j]$ of sensor 1 is illustrated in Fig. 2. If the observation result falls in the unreliable range, it is discarded and another observation is taken. The whole process does not stop until the latest observation is not located in the unreliable range. The adaptive redetection scheme outperforms the non-adaptive algorithm by 2 dB.

### 2.2 Old Adaptive Retransmission Algorithm

DCSD approach utilizes soft decoding to improve the reliability of the final decisions [5]. Set $u = (u_1, u_2, \ldots, u_N)$. The local decision $u$ is transmitted for the final decision to the fusion center. When binary antipodal modulation is deployed, the received data at the fusion center are $\tilde{v} = (\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_N)$, where

$$\tilde{v}_j = \alpha_j (-1)^{u_j} \sqrt{\frac{E_s}{L}} + n_j.$$  

(1)

Notice that $\alpha_j$ is the attenuation factor, $E_s$ is the total transmission energy per sensor, and $n_j$ is the additive white Gaussian noise (AWGN) with the two-sided power spectral density $N_0/2$. The received data are decoded as hypothesis $i$ if

$$p(\tilde{v}|c_i) \geq p(\tilde{v}|c_k) \quad \text{for all } c_k, \text{ where } k = 1, \ldots, M.$$  

(2)

Because $c_{i,j}$ and $c_{k,j}$ are binary, the bit logarithm-likelihood ratio of the received data at the fusion center can be defined as

$$\lambda_j = \ln \frac{\sum_{b_u=0}^{1} p(\tilde{v}_j|u_j = b_u) p(u_j = b_u|c_{i,j} = 0)}{\sum_{b_u=0}^{1} p(\tilde{v}_j|u_j = b_u) p(u_j = b_u|c_{k,j} = 1)}.$$  

Equation (2) is then equivalent to

$$\sum_{j=1}^{N} |\lambda_j - (-1)^{c_{i,j}}|^2 \leq \sum_{j=1}^{N} |\lambda_j - (-1)^{c_{k,j}}|^2.$$  

Denote $\delta_i = \sum_{j=1}^{N} |\lambda_j - (-1)^{c_{i,j}}|^2$. The fusion center decodes the received data as hypothesis $i_{\text{min}}$ if $i_{\text{min}} = \arg \min_i \delta_i$. Define $i_{\text{sec}} = \arg \min_{i,j:j \neq i_{\text{min}}} \delta_i$. A smaller difference $\delta = \delta_{i_{\text{sec}}} - \delta_{i_{\text{min}}}$ indicates that the received data are located around the decision boundary, meaning that the decoding result has a higher error probability. Thus, retransmission of the local decision is necessary.

Define the channel reliability of the received local decision $j$ as

$$\gamma_j = \left| \ln \frac{p(\tilde{v}_j|u_j = 0)}{p(\tilde{v}_j|u_j = 1)} \right|.$$  

Because the retransmission should help the fusion center to differentiate $c_{i_{\text{min}}}$ from $c_{i_{\text{sec}}}$, only the sensor, $j'$, with different symbols corresponding to these two codewords should be chosen, i.e., $c_{i_{\text{min}},j'} \neq c_{i_{\text{sec}},j'}$. Therefore, the fusion center discards the received local decision from sensor $j_{\text{min}}$, where $j_{\text{min}} = \arg \min_{j'} \gamma_{j'}$, and ask it to retransmit its local decision. The retransmission process does not stop until $\delta$ is greater than a predefined threshold.

### 3 New Adaptive Scheme Using EGC

#### 3.1 New Adaptive Redetection

Assume that all observations of a sensor are identically independent distributed (i.i.d.) given $H_i$ and have the same OSNR. According to [9], Equal-Gain Combination (EGC) is the optimal method to combining two observations. Denote $y_j^d, d = 1, 2, \ldots$, as the $d$-th observation of sensor $j$ and

$$\tilde{y}_j^d = \begin{cases} \frac{y_j^d}{2} & \text{if } d = 1 \\ \frac{y_j^d}{2} (\tilde{y}_j^{d-1} + \tilde{y}_j^{d-1}) & \text{else} \end{cases}$$
as the combined observation result of sensor \( j \) in \( d \) observations. We propose an adaptive redetection algorithm for each sensor using the concept of EGC as follows:

**Step 1:** Define the allowed maximum number of observations as \( D \) and set the number of observations, \( d \), to 0.

**Step 2:** The sensor makes an observation of the environment and sets \( d = d + 1 \).

**Step 3:** If the combined observation result in \( d \) observations, i.e., \( \tilde{y}^d_j \) falls in the unreliable range and \( d \leq D \), go to Step 2. Otherwise, the sensor makes a local decision according to \( \tilde{y}^d_j \).

**Step 4:** The sensor transmits the local decision to the fusion center.

Notably, all observations at each sensor may not be combine with equal weights. On the other hand, the new observation is equally combined with the combined result of previous observations. This adaptive mechanism is different from the original application of EGC, where all observations are equally combined, no adaptive mechanism is employed, and the number of observations is fixed. Furthermore, only the combined observation result must be saved at the sensor and an average operation for two values is conducted. Therefore, little extra cost over the old adaptive redetection in Section 2.1 is needed.

We further assume that all hypotheses, \( H_i \), are equally likely to occur. Let \( U_j \) be the unreliable range for sensor \( j \). The probability that the \((d+1)\)-th observation, \( d = 1, 2, \ldots, D - 1 \), is necessary for sensor \( j \) after \( d \) observations can be represented by

\[
P^e_j(d) = \Pr \{ \tilde{y}^1_j \in U_j, \tilde{y}^2_j \in U_j, \ldots, \tilde{y}^d_j \in U_j \}.
\]

Therefore, the expected number of observations for sensor \( j \) can be calculated by

\[
O_j = 1 \times (1 - P^e_j(1)) + 2 \times P^e_j(1)(1 - P^e_j(2)) + \cdots + D \times \prod_{d=1}^{D-1} P^e_j(d).
\]

Define \( C_{ij} \) and \( W_{ij} \) as the range which sensor \( j \) will make a correct and wrong local decision given \( H_i \), respectively. Let \( C_{ij} \) and \( W_{ij} \) be the range which sensor \( j \) will make a correct and wrong local decision given \( H_i \), respectively. Similarly, \( C_{ij}, W_{ij} \) and \( W'_{ij} \) for all \( i \) and \( j \) can be defined found. Therefore, \( O_j, P^e_j \), and \( P^e_j \) can be calculated numerically according the pdfs of \( H_i, i = 1, 2, 3, 4 \).

\[
P^e_j(d) = \frac{1}{M} \sum_{i=1}^{M} \Pr \{ \tilde{y}^1_j \in U_j, \ldots, \tilde{y}^d_j \in C_{ij} | H_i \}
\]

and

\[
P^e_j(D) = \frac{1}{M} \sum_{i=1}^{M} \Pr \{ \tilde{y}^1_j \in U_j, \ldots, \tilde{y}^d_j \in C'_{ij} | H_i \}
\]

and

\[
P^e_j(D) = \frac{1}{M} \sum_{i=1}^{M} \Pr \{ \tilde{y}^1_j \in U_j, \ldots, \tilde{y}^d_j \in W'_{ij} | H_i \}
\]

Consequently, the probabilities that the the local decision of sensor \( j \) is correct and wrong can be found by

\[
P^e_j = P^e_j(D) = \frac{1}{M} \sum_{i=1}^{M} \Pr \{ \tilde{y}^1_j \in U_j, \ldots, \tilde{y}^d_j \in W'_{ij} | H_i \}
\]

\[
\prod_{d=1}^{D-1} P^e_j(d)P^e_j(D)
\]

and

\[
P^e_j(D) = \frac{1}{M} \sum_{i=1}^{M} \Pr \{ \tilde{y}^1_j \in U_j, \ldots, \tilde{y}^d_j \in W'_{ij} | H_i \}
\]

In the example of Section 2.1, the ranges, \( C'_{11} \) and \( C'_{11} \), that sensor 1 will make a correct local decision given \( H_1 \) are \([\infty, T_1 - \tau_1]\) and \([-\infty, T_1)\), respectively. On the other hand, the ranges, \( W_{11} \) and \( W_{11} \), that sensor 1 will make a wrong local decision given \( H_1 \) are \([T_1 + \tau_1, \infty)\) and \([-\infty, T_1 - \tau_1]\), respectively. Similarly, \( C_{ij}, C'_{ij}, W_{ij} \) and \( W'_{ij} \) for all \( i \) and \( j \) can be defined found. Therefore, \( O_j, P^e_j \), and \( P^e_j \) can be calculated numerically according the pdfs of \( H_i, i = 1, 2, 3, 4 \).

### 3.2 New Adaptive Retransmission

Denote \( \tilde{v}^r_j, r_j = 1, 2, \ldots, \) as the \( r_j \)-th received local decision from sensor \( j \) at the fusion center and

\[
\tilde{X}^r_j = \frac{\sum_{b_u=0}^{1} p \left( \tilde{v}^1_j, \ldots, \tilde{v}^r_j | u_j = b_u \right) p (u_j = b_u | c_{i,j} = 0)}{\sum_{b_u=0}^{1} p \left( \tilde{v}^1_j, \ldots, \tilde{v}^r_j | u_j = b_u \right) p (u_j = b_u | c_{k,j} = 1)}
\]
as the combined bit logarithm likelihood ratio of the received local decision \( j \) at the fusion center. Assume that all received local decisions from sensor \( j \) at the fusion center are i.i.d. given its local decision, \( u_j \).

That is,

\[ p \left( \tilde{v}_1^j, \ldots, \tilde{v}_r^j | u_j \right) = p \left( \tilde{v}_1^j | u_j \right) \cdots p \left( \tilde{v}_r^j | u_j \right). \tag{3} \]

The combined bit logarithm likelihood ratio can be rewritten as

\[ \bar{\lambda}_j^r = \frac{\sum_{b_u=0}^{1} \prod_{k=1}^{r_j} p \left( \tilde{v}_k^j | u_j = b_u \right) p \left( u_j = b_u | c_{i,j} = 0 \right)}{\sum_{b_u=0}^{1} \prod_{k=1}^{r_j} p \left( \tilde{v}_k^j | u_j = b_u \right) p \left( u_j = b_u | c_{k,j} = 1 \right)} \]

Moreover, let

\[ \bar{\delta}_i = \sum_{j=1}^{N} \left[ \bar{\lambda}_j^r - (-1)^{c_i,j} \right]^2. \]

Thus, the fusion center decodes the received data as hypothesis \( \tilde{i}_{min} \) if

\[ \tilde{i}_{min} = \arg \min_i \bar{\delta}_i. \]

Define \( \tilde{i}_{sec} = \arg \min_{i, i \neq \tilde{i}_{min}} \bar{\delta}_i \) and \( \delta = \bar{\delta}_{i_{sec}} - \bar{\delta}_{i_{min}} \).

Finally, let

\[ \bar{\gamma}_{j}^r = \ln \frac{p \left( \tilde{v}_1^j, \ldots, \tilde{v}_r^j | u_j = 0 \right)}{p \left( \tilde{v}_1^j, \ldots, \tilde{v}_r^j | u_j = 1 \right)} \]

as the combined channel reliability of the received local decision \( j \) at the fusion center. According (3),

\[ \bar{\gamma}_{j}^r = \ln \frac{\prod_{k=1}^{r_j} p \left( \tilde{v}_k^j | u_j = 0 \right)}{\prod_{k=1}^{r_j} p \left( \tilde{v}_k^j | u_j = 1 \right)} = \sum_{k=1}^{r_j} \ln \frac{p \left( \tilde{v}_k^j | u_j = 0 \right)}{p \left( \tilde{v}_k^j | u_j = 1 \right)}. \]

From the above equation, we can find that \( \bar{\gamma}_{j}^r \) is calculated based on the summation of all logarithmic terms with the same weight, which is the concept of the EGC.

According the above derivation, an adaptive retransmission algorithm for the fusion center is developed as follows:

**Step 1:** Define the allowed maximum number of transmission for sensor \( j \) as \( R \) and the acceptable channel reliability as \( \Gamma \). Set \( r_j = 1, \) for \( j = 1, 2, \ldots, N \). Ask all sensors transmit their local decisions. Compute \( \bar{\gamma}_{j}^r \), for \( j = 1, 2, \ldots, N \).

**Step 2:** Compute \( \bar{\delta}_i, i = 1, 2, \ldots, M \).

**Step 3:** Calculate \( \bar{i}_{min}, \bar{i}_{sec} \) and \( \bar{\delta} \).

**Step 4:** If \( \bar{\delta} \) is lower than a threshold \( \Delta \) and some \( \bar{\gamma}_{j}^r \) is less than \( \Gamma \), the fusion center asks sensor \( \bar{j}_{min} \) to retransmit its local decision and set \( r_{\bar{j}_{min}} = r_{\bar{j}_{min}} + 1 \), where

\[ \bar{j}_{min} = \arg \min_{j'} \bar{\gamma}_{j'}^{r_{j'}.} \]

Calculate \( \bar{\gamma}_{j_{min}}^{r_{j_{min}}} \). Go to Step 2. Otherwise, the fusion center decodes the received local decisions as \( H_{i_{min}} \).

Notably, in Step 1, the allowed maximum number of transmissions is set because the sensor has limited power and the power for a local decision cannot be infinite in practice. The acceptable channel reliability is defined for avoiding useless retransmissions due to low OSNRs [8]. Furthermore, the fusion center must have enough storage to save \( R \) received local decisions for each sensor such that \( \bar{\gamma}_{j}^r \) can be calculated accordingly.

## 4 Performance Evaluation

The proposed scheme was evaluated using several simulations, each comprising \( 10^6 \) Monte Carlo tests. Similar to the distributed classification example in Section 2.1, a fusion center and \( N = 10 \) sensors were deployed to detect and classify four hypotheses \( H_1, H_2, H_3, \) and \( H_4 \). We also assumed that these hypotheses have Gaussian-distributed probability density functions with the same standard deviation \( \sigma^2 \) and means 0, 1, 2, and 3, respectively. The attenuation factors \( \alpha_j \) in (1) had identical and independent Rayleigh distributions with \( E \left[ \alpha_j^2 \right] = 1 \). Furthermore, CSNR is \( 10 \times \log_{10} (E_s/N_0) \). The code matrix in Table 1 was used.

In the first set of simulations, Figure 3 shows performance comparison between the old and new adaptive retransmission algorithms when \( \tau = 0.4, D = \infty \), and CSNR = 10 dB. The OSNR is normalized by the average number of observations per sensor for fair comparison in Fig. 3(a). That is,

\[ \text{OSNR} = -10 \times \log_{10} \sigma^2 + 10 \times \log_{10} \bar{O}, \tag{4} \]
Figure 3: In the case of $\tau = 0.4$, $D = \infty$, and CSNR=10 dB, performance comparison between the old and new adaptive redetection algorithms in the misclassification probability

where

$$\bar{O} = \frac{1}{N} \sum_{j=1}^{N} O_j.$$  

The new adaptive redetection mechanism outperforms the old mechanism, especially in low OSNRs.

Figure 4 illustrates performance comparison among the old (denoted by Old ART) and the new adaptive retransmission algorithm (denoted by ART-EGC) when $\Delta = 4$, $R = \infty$, $\Gamma = 5$, and OSNR = 0 dB. The CSNR is also normalized as the OSNR in (4). The new adaptive retransmission algorithms outperform the old mechanism.

References:


