

# Advanced Progress in Calculating PFD for Systems with PST

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*Abstract:* - For Safety related systems the indication of the average probability of failure on demand (PFD) taking into consideration of the Proof Test interval is one possibility to compare different systems. In this paper we intend to derive the average PFD for a 1oo1 system taking into consideration as well the Proof Test as a Partial Stroke Test (PST). Thereby we specify a unique mathematic function without a helping probability band. Doing so, we get additional correlations between reduction of PFD and the diagnostic coverage factor and also of the PFD value between a system without PST and a system with PST. Finally we present an approximation in order to calculate the PFD value, if the ratio between the PST interval and the Proof Test interval is very small.

*Key-Words:* - 1oo1-System, Probability of failure on demand, Partial-Stroke-Test, Proof-Test, Relative probability of failure reduction, Diagnostic coverage factor

## 1 Introduction

For each developer of safety related systems it is a challenge to extend the Proof Test interval for a safety related application and to get an identical or even better a smaller Probability of Failure on Demand (PFD) at the same time. In the standard IEC/EN 61508 a Proof Test is defined as a “periodic test performed to detect failures in a safety-related system so that, if necessary, the system can be restored to an “as new” condition or as close as practical to this condition” [1]. One possibility to extend the Proof Test interval is the use of Partial Stroke Test (PST). These tests may be executed between two Proof Tests either manually or automatically, only sometimes or very frequently. In the scientific literature there are only few approaches to describe mathematically the PFD of Safety Related Systems using the PST [2], [3], [4]. The parameters shown in Table 1 are necessary for the equations in this paper.

**Table 1:** Parameters

$PFD_{avg}$	Probability of failure on demand, average
$\lambda_{DU}$	Rate for dangerous, undetectable failures
$DC_{PST}$	Diagnostic coverage factor for a PST
$t_{PST}$	Time of the PST
$t_{PT}$	Time of the proof-test

In [2] the average PFD,  $PFD_{avg1}(t)$ , of a system with PST according to eq. (1) is calculated.

$$PFD_{avg1}(t) = \frac{1}{2} \cdot DC_{PST} \cdot \lambda_{DU} \cdot t_{1.PST} + \frac{1}{2} \cdot (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{PT} \quad (1)$$

In [3] and [4] it is additionally kept in mind that the PFD value after a successful PST only depends on such failures that are not detected by the PST. Therefore, two equations are necessary to mathematically describe this issue, eq. (2) and (3).

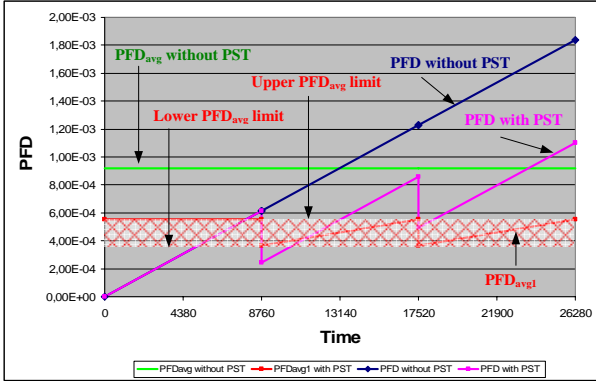
$$PFD_{avg1,w.PST}(t) = PFD_{avg1}(t) \quad (2)$$

$$PFD_{avg1,a.PST}(t) = \frac{1}{2} \cdot (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{PT} \quad (3)$$

So the average PFD can only be described by an average band of probability, Fig. 1.

The calculation of a PFD for a 1oo1 system without a PST is described in e. g. [4] - [8].

For a 1oo1 system with PST in this paper we specify a unique mathematic function without a helping probability band. Doing so, we get additional correlations between reduction of PFD and the diagnostic coverage factor and also of the PFD value between a system without PST and a system with PST.



**Fig. 1:**  $PFD_{avg1}$  with  $\lambda_{DU} = 7 \cdot 10^{-8}$  1/h,  $t_{PT} = 3$  years,  $t_{PST} = 12$  months,  $DC_{PST} = 60\%$

## 2 1001-System with PST, $PFD_{avg3, w. PST}$

With the method of calculation presented here, which as far we know, hasn't been considered yet, one can give a constant value as the average value for the  $PFD$  value. The principle is, that at first an average value will be appraised for each PST interval following the well known method

$$E(t) = \frac{1}{t_1 - t_0} \cdot \int_{t_0}^{t_1} f(t) dt \quad (4)$$

Finally the average value will be generated via all appraised single average value.

The  $PFD$  value between two PSTs will be appraised using the following equation and will occur in sections.

$$PFD = \begin{cases} \lambda_{DU} \cdot t & 0 \leq t < t_{1.PST} \\ \lambda_{DU} \cdot t - DC_{PST} \cdot \lambda_{DU} \cdot t_{1.PST} & t_{1.PST} \leq t < t_{2.PST} \\ \lambda_{DU} \cdot t - DC_{PST} \cdot \lambda_{DU} \cdot t_{2.PST} & t_{2.PST} \leq t < t_{3.PST} \\ \vdots & \vdots \\ \lambda_{DU} \cdot t - DC_{PST} \cdot \lambda_{DU} \cdot t_{last PST} & t_{last PST} \leq t < t_{PT} \end{cases}$$

The average value for the functions defined in sections will be calculated with eq. (4). The result is:

For the interval  $0 \leq t < t_{1.PST}$ :

$$PFD_{avg3} = \frac{1}{2} \cdot \lambda_{DU} \cdot t_{1.PST} \quad (5)$$

For the interval  $t_{1.PST} \leq t < t_{2.PST}$ :

$$\begin{aligned} PFD_{avg3} &= \frac{1}{2} \cdot \lambda_{DU} \cdot \frac{1}{t_{1.PST}} \cdot (t_{2.PST}^2 - t_{1.PST}^2) \\ &\quad - DC_{PST} \cdot \lambda_{DU} \cdot (t_{2.PST} - t_{1.PST}) \\ &= \frac{1}{2} \cdot \lambda_{DU} \cdot \frac{t_{2.PST}^2}{t_{1.PST}} - \lambda_{DU} \cdot t_{1.PST} \cdot \left( \frac{1}{2} + DC_{PST} \right) \end{aligned} \quad (6)$$

For the interval  $t_{2.PST} \leq t < t_{3.PST}$ :

$$\begin{aligned} PFD_{avg3} &= \frac{1}{2} \cdot \lambda_{DU} \cdot \frac{t_{3.PST}^2}{t_{1.PST}} \\ &\quad - \lambda_{DU} \cdot \left( \frac{1}{2} \cdot \frac{t_{2.PST}^2}{t_{1.PST}} + DC_{PST} \cdot t_{2.PST} \right) \end{aligned} \quad (7)$$

For the following PST interval, the corresponding eq. (7) will be used.

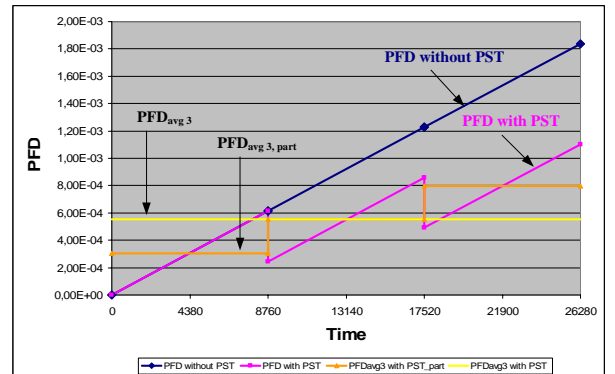
For the last interval  $t_{last PST} \leq t < t_{PT}$ , which ends with the Proof-Test time  $t_{PT}$ :

$$\begin{aligned} PFD_{avg3} &= \frac{1}{2} \cdot \lambda_{DU} \cdot \frac{t_{PT}^2}{t_{1.PST}} \\ &\quad - \lambda_{DU} \cdot \left( \frac{1}{2} \cdot \frac{t_{last PST}^2}{t_{1.PST}} + DC_{PST} \cdot t_{last PST} \right) \end{aligned} \quad (8)$$

Once all  $PFD_{avg3}$  have been appraised in sections, the  $PFD_{avg3, w. PST}$  will be defined via the average value of all  $PFD_{avg3}$ :

$$PFD_{avg3, w. PST} = \frac{1}{\text{Number of PST}} \cdot \sum PFD_{avg3} \quad (9)$$

With the same parameters as used for Fig. 1 we get the  $PFD_{avg3, w. PST}$  value and the trajectory as shown in Fig. 2.



**Fig. 2:**  $PFD_{avg3}$  with  $\lambda_{DU} = 7 \cdot 10^{-8}$  1/h,  $t_{PT} = 3$  years,  $t_{PST} = 12$  months,  $DC_{PST} = 60\%$

## 3 Coherence between $PFD_{w. PST}$ and $PFD_{avg3, w. PST}$

The approach presented up to now to appraise the  $PFD_{avg3, w. PST}$  consists at first to appraise the single  $PFD_{avg}$  value between two PSTs and then to calculate the average value via all  $PFD_{avg}$ .

Underneath, it should be attested, as far as we know for the first time, that the coherence between  $PFD$  und  $PFD_{avg3, w. PST}$  exists. Thereby it will be provided that the intervals between two PSTs are identical, though this is not necessary, as one can easily demonstrate, e. g. with the help of the Riemann's integrable criteria [9]. This

attests that each defined and limited function  $f(x)$  in  $[a, b]$  is than exactly integrated via  $[a, b]$ , if this one has an endless number of discontinuity on  $[a, b]$ . Than the integral will be calculated via the function  $f(x)$  through the separation of the interval  $[a, b]$  into endless small intervals [9].

Generally the following equation counts for the  $PFD_{avg}$ , successively written with  $PFD_{part\ av}$ , in  $n$ -ten PST-interval (generalization of eq. (10))

$$\begin{aligned} PFD_{part\ av} &= \frac{1}{2} \cdot \lambda_{DU} \cdot \frac{t_{n.PST}^2}{t_{1.PST}} \\ &\quad - \lambda_{DU} \cdot \left( \frac{1}{2} \cdot \frac{t_{(n-1).PST}^2}{t_{1.PST}} + DC_{PST} \cdot t_{(n-1).PST} \right) \\ &= \frac{1}{2} \cdot \frac{\lambda_{DU}}{t_{1.PST}} \cdot \left[ t_{n.PST}^2 - t_{(n-1).PST}^2 \right] \\ &\quad - \lambda_{DU} \cdot DC_{PST} \cdot t_{(n-1).PST} \end{aligned} \quad (10)$$

with

$$t_{(n-1).PST} = t_{n.PST} - t_{1.PST} \quad (11)$$

follows

$$\begin{aligned} PFD_{part\ av} &= \frac{1}{2} \cdot \frac{\lambda_{DU}}{t_{1.PST}} \cdot \left[ t_{n.PST}^2 - (t_{n.PST} - t_{1.PST})^2 \right] - \\ &\quad \lambda_{DU} \cdot DC_{PST} \cdot (t_{n.PST} - t_{1.PST}) \\ &= \frac{1}{2} \cdot \frac{\lambda_{DU}}{t_{1.PST}} \cdot \left[ 2 \cdot t_{n.PST} \cdot t_{1.PST} - t_{1.PST}^2 \right] - \\ &\quad \lambda_{DU} \cdot DC_{PST} \cdot (t_{n.PST} - t_{1.PST}) \\ &= \lambda_{DU} \cdot t_{n.PST} - \frac{1}{2} \cdot \lambda_{DU} \cdot t_{1.PST} - \\ &\quad \lambda_{DU} \cdot DC_{PST} \cdot (t_{n.PST} - t_{1.PST}) \\ &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{n.PST} + \\ &\quad \left( DC_{PST} - \frac{1}{2} \right) \cdot \lambda_{DU} \cdot t_{1.PST} \\ &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{n.PST} + A \end{aligned} \quad (12)$$

whereas

$$A = \left( DC_{PST} - \frac{1}{2} \right) \cdot \lambda_{DU} \cdot t_{1.PST} \quad (13)$$

is.

In order to calculate the average value ( $PFD_{avg3, w.PST}$ ) via all  $PFD_{part\ av}$ , it will be provided that  $n$  intervals exist. Thereby the following equation should count, what would otherwise be a limit of the demonstration:  $k = 1 \dots n$

$$t_{n.PST} = t_{PT},$$

It means, that the time of the  $n$ . PSTs coincides with the time of the Proof Test, and

$$t_{0.PST} = 0, \text{ the time of the process to be defined.}$$

Firstly the single  $PFD_{part\ av}$  of all  $n$  intervals will be calculated.

$$\text{With } t_k = k \cdot t_{1.PST} \quad (14)$$

it results

$$\begin{aligned} k = 1: \quad PFD_{part\ av} &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot 1 \cdot t_{1.PST} + A \\ &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot 1 \cdot t_{1.PST} + \\ &\quad \left( DC_{PST} - \frac{1}{2} \right) \cdot \lambda_{DU} \cdot t_{1.PST} \\ &= \frac{1}{2} \cdot \lambda_{DU} \cdot t_{1.PST} \end{aligned}$$

Corresponds to eq. (5)!

$$k = 2: \quad PFD_{part\ av} = (1 - DC_{PST}) \cdot \lambda_{DU} \cdot 2 \cdot t_{1.PST} + A$$

$$k = 3: \quad PFD_{part\ av} = (1 - DC_{PST}) \cdot \lambda_{DU} \cdot 3 \cdot t_{1.PST} + A$$

$$k = 4: \quad PFD_{part\ av} = (1 - DC_{PST}) \cdot \lambda_{DU} \cdot 4 \cdot t_{1.PST} + A$$

etc... up to

$$\begin{aligned} k = n: \quad PFD_{part\ av} &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot n \cdot t_{1.PST} + A \\ &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{n.PST} + A \\ &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{n.PST} \\ &\quad + \left( DC_{PST} - \frac{1}{2} \right) \cdot \lambda_{DU} \cdot t_{1.PST} \\ &= \lambda_{DU} \cdot t_{n.PST} - \frac{1}{2} \cdot \lambda_{DU} \cdot t_{1.PST} \\ &\quad - DC_{PST} \cdot \lambda_{DU} (t_{n.PST} - t_{1.PST}) \end{aligned}$$

This equation is the same as eq. (8) for the PST-interval of  $t_{last\ PST} \leq t < t_{PT}$ , as shown in the following calculation:

$$\begin{aligned} k = n: \quad PFD_{part\ av} &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot n \cdot t_{1.PST} + A \\ &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{n.PST} + \\ &\quad \left( DC_{PST} - \frac{1}{2} \right) \cdot \lambda_{DU} \cdot t_{1.PST} \\ &= \lambda_{DU} \cdot t_{n.PST} - \frac{1}{2} \cdot \lambda_{DU} \cdot t_{1.PST} - \\ &\quad DC_{PST} \cdot \lambda_{DU} (t_{n.PST} - t_{1.PST}) \end{aligned}$$

with eq. (12) counts for

$$\begin{aligned} k = n: \quad PFD_{part\ av} &= \frac{1}{2} \cdot \frac{\lambda_{DU}}{t_{1.PST}} \cdot \left[ t_{n.PST}^2 - (t_{n.PST} - t_{1.PST})^2 \right] - \\ &\quad \lambda_{DU} \cdot DC_{PST} \cdot (t_{n.PST} - t_{1.PST}) \end{aligned}$$

and with  $t_{n.PST} = t_{PT}$  and  $t_{n.PST} - t_{1.PST} = t_{last\ PST}$  results for

$$\begin{aligned} k = n: \quad PFD_{part\ av} &= \frac{1}{2} \cdot \frac{\lambda_{DU}}{t_{1.PST}} \cdot \left[ t_{PT}^2 - t_{last\ PST}^2 \right] \\ &\quad - \lambda_{DU} \cdot DC_{PST} \cdot t_{last\ PST} \end{aligned}$$

which is the same as in eq. (8).

When the sum is made via all  $PFD_{part\ av}$ , i. e from  $k = 1$  up to  $k = n$ , so results:

$$\begin{aligned}
\Sigma PFD_{part\ av} &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot \\
&\quad (1 + 2 + 3 + 4 + \dots + n) \cdot t_{1.PST} + n \cdot A \\
&= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot \frac{n \cdot (n + 1)}{2} \cdot t_{1.PST} \\
&\quad + n \cdot A
\end{aligned} \tag{15}$$

To calculate the average value of the  $PFD_{avg3, w.PST}$  from eq. (15), this equation must still be divided over the amount of PST intervals:

$$\begin{aligned}
PFD_{avg3, w.PST} &= \frac{1}{n} \cdot \Sigma PFD_{part\ av} \\
&= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot \frac{(n + 1)}{2} \cdot t_{1.PST} + A \\
&= \frac{1}{2} \cdot (1 - DC_{PST}) \cdot \lambda_{DU} \cdot (n + 1) \cdot t_{1.PST} + A
\end{aligned} \tag{16}$$

With  $(n + 1) \cdot t_{1.PST} = t_{PT} + t_{1.PST}$  and eq. (13)

$$A = \left( DC_{PST} - \frac{1}{2} \right) \cdot \lambda_{DU} \cdot t_{1.PST}$$

it results

$$\begin{aligned}
PFD_{avg3, w.PST} &= \frac{1}{2} \cdot (1 - DC_{PST}) \cdot \lambda_{DU} \cdot (t_{PT} + t_{1.PST}) + \\
&\quad \left( DC_{PST} - \frac{1}{2} \right) \cdot \lambda_{DU} \cdot t_{1.PST} \\
&= \frac{1}{2} \cdot (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{PT} + \\
&\quad \left[ \frac{1}{2} \cdot (1 - DC_{PST}) + \left( DC_{PST} - \frac{1}{2} \right) \right] \cdot \lambda_{DU} \cdot t_{1.PST} \\
&= \frac{1}{2} \cdot (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{PT} + \frac{1}{2} \cdot DC_{PST} \cdot \lambda_{DU} \cdot t_{1.PST} \\
&= \frac{1}{2} \cdot \left[ (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{PT} + DC_{PST} \cdot \lambda_{DU} \cdot t_{1.PST} \right]
\end{aligned} \tag{17}$$

Compare this equation with

$$\begin{aligned}
PFD_{w.PST}(t) &= DC_{PST} \cdot \lambda_{DU} \cdot t_{1.PST} + (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t \\
&= DC_{PST} \cdot \lambda_D \cdot t_{1.PST} + (1 - DC_{PST}) \cdot \lambda_D \cdot t \\
&= \lambda_D \cdot [DC_{PST} \cdot t_{1.PST} + (1 - DC_{PST}) \cdot t]
\end{aligned} \tag{18}$$

to time  $t = t_{PT}$ :

$$\begin{aligned}
PFD_{w.PST}(t = t_{PT}) &= DC_{PST} \cdot \lambda_{DU} \cdot t_{1.PST} + \\
&\quad (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{PT} \\
&= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{PT} + \\
&\quad DC_{PST} \cdot \lambda_{DU} \cdot t_{1.PST}
\end{aligned}$$

so one finds, that counts:

$$PFD_{avg3, w.PST} = \frac{1}{2} \cdot PFD_{w.PST}(t = t_{PT}) \tag{19}$$

#### 4 Coherence between the relative probability of failure reduction $B_{1rel}$ and the $DC_{PST}$ factor

Assuming that all PST intervals have the same length, i. e.  $t_{n.PST} = t_{1.PST}$ , and so further the inequation  $t_{1.PST} \ll t_{PT}$  counts, a coherence between the relative probability of failure reduction  $B_{1rel}$  and the  $DC_{PST}$  factor should be appraised in the following. For the probability of failure reduction  $B_1$  counts at time  $t_{PT}$  of the Proof Test the equation, see [3] ( $PFD_{wo.PST}$ :  $PFD$  value for a system without a PST;  $PFD_{w.PST}$ :  $PFD$  value for a system with a PST):

$$\begin{aligned}
B_1(t = t_{PT}) &= PFD_{wo.PST}(t = t_{PT}) - PFD_{w.PST}(t = t_{PT}) \\
&= \lambda_{DU} \cdot DC_{PST} \cdot (t_{PT} - t_{1.PST})
\end{aligned} \tag{20}$$

A relative probability of failure reduction  $B_{1rel}$  can therefore at time  $t_{PT}$  be defined as follow:

$$\begin{aligned}
B_{1rel}(t = t_{PT}) &= \frac{B_1(t = t_{PT})}{PFD_{wo.PST}(t = t_{PT})} \\
&= \frac{PFD_{wo.PST}(t = t_{PT}) - PFD_{w.PST}(t = t_{PT})}{PFD_{wo.PST}(t = t_{PT})} \\
&= \frac{\lambda_{DU} \cdot DC_{PST} \cdot (t_{PT} - t_{1.PST})}{\lambda_{DU} \cdot t_{PT}} \\
&= DC_{PST} \cdot \left( 1 - \frac{t_{1.PST}}{t_{PT}} \right)
\end{aligned} \tag{21}$$

Provided that  $t_{1.PST} \ll t_{PT}$  one obtains the approximation

$$B_{1rel}(t = t_{PT}) \approx DC_{PST} \tag{22}$$

If one dissolves eq. (21) according to  $PFD_{w.PST}(t = t_{PT})$  this would mean for practical application, provided that  $t_{1.PST} \ll t_{PT}$  counts, the following:

$$\begin{aligned}
PFD_{w.PST}(t = t_{PT}) &= PFD_{wo.PST}(t = t_{PT}) - \\
&\quad B_{1rel}(t = t_{PT}) \cdot PFD_{wo.PST}(t = t_{PT}) \\
&= PFD_{wo.PST}(t = t_{PT}) \cdot \\
&\quad (1 - B_{1rel}(t = t_{PT})) \\
&= PFD_{wo.PST}(t = t_{PT}) \cdot (1 - DC_{PST})
\end{aligned} \tag{23}$$

It means that the probability of failure of a system with PST at time  $t = t_{PT}$  depends only, provided that,  $t_{1.PST} \ll t_{PT}$  counts, on the probability of failure of a system without PST and the  $DC_{PST}$  factor!

To underline the validity of this statement, the results achieved in this paper will be compared with each other. In a first work following parameters are given.

$$t_{PT} = 3 \text{ Jahre} = 26280 \text{ h and}$$

$$t_{1,PST} = 4380 h = 0,5 \text{ years}$$

$$// = 8760 h = 1 \text{ year}$$

$$// = 13140 h = 1,5 \text{ years}$$

The chosen failure rate is

$$\lambda_D = \lambda_{DU,PT} = 7 \cdot 10^{-8} \frac{1}{h}$$

The results for the relative probability of failure reduction  $B_{1rel}$  according to the eq. (21) are shown in Table 2.

**Table 2: Table of value for the rel. probability of failure reduction  $B_{1rel}$  with  $t_{PT} = 3 \text{ years}$**

$DC_{PST}$ $t_{1,PST}$	60%	70%	80%	90%
4380	50,00%	58,33%	66,67%	75,00%
8760	40,00%	46,67%	53,33%	60,00%
13140	30,00%	35,00%	40,00%	45,00%

To check the validity of eq. (23), the  $PF D$  value for the exact value for  $PF D_{w.PST}(t = t_{PT})$  according to eq. (18) as well as the one from eq. (23) calculated value and the relative difference of both values are given in Table 3. The ratio between  $t_{1,PST}$  and  $t_{PT}$  averages there

$$\text{for } t_{1,PST} = 4380 h: \frac{t_{1,PST}}{t_{PT}} = \frac{4380 h}{26280 h} = 0,1666$$

$$\text{for } t_{1,PST} = 8760 h: \frac{t_{1,PST}}{t_{PT}} = \frac{8760 h}{26280 h} = 0,3333$$

$$\text{for } t_{1,PST} = 13140 h: \frac{t_{1,PST}}{t_{PT}} = \frac{13140 h}{26280 h} = 0,5$$

**Table 3: Table of value for the rel. probability of failure reduction  $B_{1rel}$  with  $t_{PT} = 3 \text{ years}$**

$DC_{PST}$ $t_{1,PST}$	60%			70%		
	exac. PFD	app. PFD	rel. $\Delta$ in %	exac. PFD	app. PFD	rel. $\Delta$ in %
4380	9,20E-04	7,36E-04	20,00	7,67E-04	5,52E-04	28,00
8760	1,10E-03	7,36E-04	33,33	9,81E-04	5,52E-04	43,75
13140	1,29E-03	7,36E-04	42,86	1,20E-03	5,52E-04	53,85
$DC_{PST}$ $t_{1,PST}$	80%			90%		
	exac. PFD	app. PFD	rel. $\Delta$ in %	exac. PFD	app. PFD	rel. $\Delta$ in %
4380	6,13E-04	3,68E-04	40,00	4,60E-04	1,84E-04	60,00
8760	8,58E-04	3,68E-04	57,14	7,63E-04	1,84E-04	75,00
13140	1,10E-03	3,68E-04	66,67	1,01E-03	1,84E-04	81,82

As one can see on the values in Table 3 a ratio  $\frac{t_{1,PST}}{t_{PT}} = \frac{4380 h}{26280 h} = 0,1666$  is not satisfying to become an adequate small difference between the exact  $PF D$  and the approximated  $PF D$ . In this case it means that the  $PF D$  value must be calculated with the exact formula for a system with PST.

The result would be different if one changes the parameters as shown here:

$$t_{PT} = 1 \text{ Jahr} = 8760 h$$

$$t_{1,PST} = 24 h = \frac{1}{365} \text{ years}$$

$$// = 48 h = \frac{2}{365} \text{ years}$$

$$// = 168 h = \frac{7}{365} \text{ years}$$

The chosen failure rate is

$$\lambda_D = \lambda_{DU,PT} = 3,8 \cdot 10^{-7} \frac{1}{h}$$

In Table 4 we can see the result for the relative probability of failure reduction  $B_{1rel}$  with these parameter values. It may be assessed that the values of  $B_{1rel}$  is nearly equal to the chosen  $DC_{PST}$  factor.

**Table 4: Table of value for the rel. Failure probability reduction  $B_{1rel}$  with  $t_{PT} = 1 \text{ year}$**

$DC_{PST}$ $t_{1,PST}$	50%	65%	75%	85%
24	49,86%	64,82%	74,79%	84,77%
48	49,73%	64,64%	74,59%	84,54%
168	49,04%	63,75%	73,56%	83,37%

To check the validity of eq. (23), the  $PF D$  value for the exact value of  $PF D_{w.PST}(t = t_{PT})$  according to eq. (18) and the ones from eq. (23) calculated values as well as the relative difference of both values are given in Table 5. The ratio between  $t_{1,PST}$  and  $t_{PT}$  averages there:

$$\text{for } t_{1,PST} = 24 h: \frac{t_{1,PST}}{t_{PT}} = \frac{24 h}{8760 h} = 0,00274$$

$$\text{for } t_{1,PST} = 48 h: \frac{t_{1,PST}}{t_{PT}} = \frac{48 h}{8760 h} = 0,00548$$

$$\text{for } t_{1,PST} = 168 h: \frac{t_{1,PST}}{t_{PT}} = \frac{168 h}{8760 h} = 0,01918$$

As one can see on the values in Table 5, a ratio  $\frac{t_{1,PST}}{t_{PT}} < 0,02$  is satisfying to become an adequate small difference between the exact  $PF D$  and the approximated  $PF D$ .

It should be observed that the approximation is optimally adapted when the  $DC_{PST}$  factor is also small. It means in this case that it is easier to calculate the  $PF D$ -Value for a System with PST using the approximation formula.

**Table 5: Table of value for the rel. failure probability reduction  $B_{1rel}$  with  $t_{PT} = 1 \text{ year}$** 

$DC_{PST}$						
$t_{1,PST}$	50%			65%		
	exac. PFD	app. PFD	rel. $\Delta$ in %	exac. PFD	app. PFD	rel. $\Delta$ in %
24	1,67E-03	1,66E-03	0,27	1,17E-03	1,17E-03	0,51
48	1,68E-03	1,66E-03	0,82	1,18E-03	1,17E-03	1,00
168	1,69E-03	1,66E-03	1,89	1,20E-03	1,17E-03	2,92
$DC_{PST}$						
$t_{1,PST}$	75%			85%		
	exac. PFD	app. PFD	rel. $\Delta$ in %	exac. PFD	app. PFD	rel. $\Delta$ in %
24	8,39E-04	8,32E-04	0,82	5,07E-04	4,99E-04	1,53
48	8,48E-04	8,32E-04	1,88	5,16E-04	4,99E-04	3,27
168	8,78E-04	8,32E-04	5,19	5,52E-04	4,99E-04	9,58

## 5 Coherence between $PFD_{wo.PST}$ and $PFD_{avg3, w.PST}$

From both previous chapters 3 and 4 a light coherence between the  $PFD$  values  $PFD_{wo.PST}$ , i. e. for a system without PST, and the average  $PFD$  value,  $PFD_{avg3, w.PST}$ , i. e. for a System with PST, can be established. Provided that, all PST intervals have the same length, i. e.  $t_{n.PST} = t_{1.PST}$ , and that the inequation  $t_{1.PST} \ll t_{PT}$  counts. It results then from eq. (19) and (23)

$$\begin{aligned}
 PFD_{avg3, w.PST} &= \frac{1}{2} \cdot PFD_{wo.PST}(t = t_{PT}) \\
 &= \frac{1}{2} \cdot PFD_{wo.PST}(t = t_{PT}) \cdot (1 - DC_{PST})
 \end{aligned} \quad (24)$$

To check the validity of this equation the same parameters as mentioned before are used again:

$$t_{PT} = 1 \text{ Jahr} = 8760 \text{ h}$$

$$t_{1,PST} = 24 \text{ h} = \frac{1}{365} \text{ years}$$

$$// = 48 \text{ h} = \frac{2}{365} \text{ years}$$

$$// = 168 \text{ h} = \frac{7}{365} \text{ years}$$

The chosen failure rate is

$$\lambda_D = \lambda_{DU, PT} = 3,8 \cdot 10^{-7} \frac{1}{h}$$

In Table 6 the PFD-values with exact values for  $PFD_{avg3, mittel}$  according to eq. (5) up to eq. (9), in the eq. (25) are presented generally,

$$\begin{aligned}
 PFD_{avg3, mittel} &= \frac{1}{n} \cdot \sum PFD_{part av} \\
 &= \frac{1}{n} \cdot \sum_n \left( \frac{1}{2} \cdot \frac{\lambda_{DU}}{t_{1,PST}} \cdot [t_{n.PST}^2 - t_{(n-1).PST}^2] \right. \\
 &\quad \left. - \lambda_{DU} \cdot DC_{PST} \cdot t_{(n-1).PST} \right)
 \end{aligned} \quad (25)$$

and the ones from eq. (24) calculated values and the relative difference of both values is given.

**Table 6: Table of comparison to the exactly  $PFD$  value and the approximated  $PFD$  values with  $t_{PT} = 1 \text{ year}$** 

$DC_{PST}$						
$t_{1,PST}$	50%			65%		
	exac. $PFD_{avg3, w}$	app. $PFD_{avg3, w}$	rel. $\Delta$ in %	exac. $PFD_{avg3, w}$	app. $PFD_{avg3, w}$	rel. $\Delta$ in %
24	8,34E-04	8,32E-04	0,27	5,86E-04	5,83E-04	0,51
48	8,39E-04	8,32E-04	0,82	5,90E-04	5,83E-04	1,28
168	8,46E-04	8,32E-04	1,62	6,02E-04	5,83E-04	3,18
$DC_{PST}$						
$t_{1,PST}$	75%			85%		
	exac. $PFD_{avg3, w}$	app. $PFD_{avg3, w}$	rel. $\Delta$ in %	exac. $PFD_{avg3, w}$	app. $PFD_{avg3, w}$	rel. $\Delta$ in %
24	4,20E-04	4,16E-04	0,82	2,54E-04	2,50E-04	1,53
48	4,24E-04	4,16E-04	1,88	2,58E-04	2,50E-04	3,27
168	4,39E-04	4,16E-04	5,19	2,76E-04	2,50E-04	9,58

As one can see on the values in Table 6, a ratio of  $\frac{t_{1,PST}}{t_{PT}} < 0,02$  is satisfying to become an adequate small  $t_{PT}$

difference between the exact and the approximate  $PFD_{avg3, w.PST}$  value. It should be observed that the approximation is optimally adapted when the  $DC_{PST}$  factor is also small. It means in this case, that it is very easy to calculate the  $PFD$  value for a system with PST using the approximation formula eq. (24).

## 6 Conclusion

In this paper the mathematical coherence between the  $PFD$  value of a 1oo1 system without PST and the average  $PFD$  value of a 1oo1 system with PST was presented. If the relative probability of failure reduction is near the  $DC_{PST}$  factors or the ratio between the PST interval and the Proof Test interval is sufficiently small, then for this calculation we can use a simple approximation, see eq. (24).

Advanced studies may deal with other architecture models like 1oo2 or 2oo3 systems. We assume similar coherence between the different  $PFD$  parameters as examined in this paper.

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