The Location - Routing Problem: an innovative approach.

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Abstract: The logistics system of a firm deals with purchased materials carrying, controls work in process in each production phase and the flow of the products delivered to the customer, defines the backtracking of a discarded, disused or damaged product, to reemploy its parts or materials. The structures displaced within the logistics network must guarantee an opportune level of service and the cutback of logistics costs. Transport system performance are, therefore, of primary importance. In literature they are present accurate mathematical models and effective solution techniques to face location, allocation and distribution problems. The topic is still of extreme scientific interest because of the increasing structural complexity of the models due to the constraints imposed by the "real systems" modelling. In this paper, particularly, a possible approach will be proposed to optimize the routing phase in a Location-Routing Problem (LRP). Results are compared with those obtainable turning to other commonly adopted procedures.

Key-Words: Distributive Logistics*,* Location-Routing Problem, Scheduling, Traveling Salesman Problem, Vehicle Routing Problem

1 Introduction

The logistics system of a firm:

- deals with purchased materials carrying (*acquisition logistics*);
- controls work in process in each production phase (*production logistics system*);
- through the distribution management, it controls the flow of the products delivered to the customer (*distributive logistics*);
- defines the backtracking of a discarded, disused or damaged product, to reemploy its parts or materials (*reverse logistics*).

The structures displaced within the logistics network must guarantee:

- an opportune level of service, with the purpose to locate the product as near as possible to the market (*peripheral warehouses*);
- the cutback of logistics costs. The consignments, related to different products, are gathered so to get meaningful economies of transport (*distribution centres*).

Transport system performance are, therefore, of primary importance: it must guarantee the mobility of the products among the various nodes of the system with high efficiency and punctuality, reducing, at the same time, the transport cost which, in particular cases, can weigh for 50% on the overall logistics costs.

So, the location of the distribution centres (facilities or more simply warehouses) and connected products distribution issues represent some crucial questions [7, 18]. In different productive contexts these two aspects tightly appear interdependent, for such reason they must contemporarily be considered in the development of theoretical models and in the practical planning of the logistics network.

2 Literature review

In literature they are present accurate mathematical models and effective solution techniques to face location, allocation [13] and distribution problems [3, 16], that resort to the concept of integrated logistics systems and whose basis is constituted by a combined *location-routing model* [15, 17]. The main difference between a location-routing problem (*LRP*) and a classical location-allocation problem is that, once the facility is located, in the former it's required that the customers are served along a route, while in the latter every customer is directly connected to the same facility (*radial distribution*) [9, 14]. Considering the first approach, the optimal facility location and the simultaneous construction of the routes leads to a considerable cutback of the overall costs. A LRP, generally speaking, can be assimilate to a vehicular scheduling problem (*Vehicle Routing Problem*, *VRP*) in which the optimal number and location of the facilities

are simultaneously determined with the vehicles scheduling and the circuits (*route*) release [4] so to minimize a particular function (in general, the overall costs: costs of distribution, stocking and transportation).
However, the LRP, considered as “the scheduling of locations taking in account tour scheduling issues”, is, clearly, NP-hard since it is constituted by two NP-hard problems. On the other hand, location and routing problems can be seen as special cases of LRP:

- if each customer has to be directly connected to the facility, the LRP reduces to a classical location problem;
- if the centre location is settled, the LRP can be considered as a VRP.

Solution methodologies can be classified according to the way they create a relationship between the location and routing problems [19, 23].
In *Sequential methods* the location problem is first solved minimizing the distances between facility and consumers (radial distance), then a routing problem is faced. These methods don’t allow a feedback from the routing phase to the location one so a sub-optimal design for the distribution system could be determined.
Clustering solution methods first divide and group the customers, then:

- for each cluster a facility is located and a VRP (or TSP) is executed [21];
- a TSP for each cluster is executed and then the facilities are located.

Iterative heuristics decompose the problem in two sub-problems which are iteratively solved moving the data from a phase to the other.
Although iterative methods are an improvement of sequential methods, when the location algorithm ends, it starts again receiving as input the new information coming from the routing algorithm. From a designing point of view, iterative heuristics give the same importance to these sub-problems. *Hierarchical heuristics* consider, instead, the location as the principal problem and the routing as a subordinate problem.
To solve a LRP it is possible to use multi-phase based procedures, which, breaking-up the problem, reduce its complexity. These ones include the combination of four algorithms:

- *location-allocation first, route second;*
- *route first, location-allocation second;*
- *saving / insertion;*
- *routes improvement /exchange.*

Among these, the last two are often used to solve vehicle routing planning problems within the LRP context [9].

In Min [14, 15] a problem concerning some terminals location (consolidation terminals) is considered. The products coming from different supply centres are first collected in a terminal and then dispatched to the consumers. This issue is somehow more complex than a LRP as there’s a certain number of supply centres and both the centres and the consumers must be assigned to the terminals. The consumers are clustered according to vehicles capacity and the “centroid” of each cluster is used in terminals location.
Barreto et al [1] used a cluster analysis procedure in a LRP heuristic approach (*route first, location second*). The consumers are clustered, a TSP for each cluster is executed and, finally, the facilities are located. Capacity constraints both for the vehicles and the distribution centres are considered (Capacitated Location-Routing Problem, *CLRP*).
Tuzun and Burke [22] employed a *tabu search algorithm* in both location and routing phases, allowing an efficient strategy from the computational point of view.
Wu et al. [24] faced an extension of the LRP, considering multiple type of facilities and fleet with a limited number of vehicles for each different type of vehicle. The LRP is divided in a location-allocation problem (LAP) and a vehicle routing problem (VRP). To solve the LAP and the VRP, the authors developed some heuristic methods based on the *Simulated Annealing* (SA) technique.
In Lin et al. [11, 12], a problem of location and distribution relative to a telecommunications service in Kowloon peninsula (Hong-Kong) is faced. The authors divide the LRP in three phases: facilities location, routing and loading. Each phase is treated applying heuristic or exact algorithms. An initial number of facilities is determined, then applying a specific algorithm [4] and considering capability (warehouses and vehicles) and routes length constraints, the initial routes are established. The routes are "reprocessed" by an improving algorithm (Travelling Salesman Problem based, *TSP*) so to determine the optimal sequence of the nodes. To cut the routing costs, meta-heuristic techniques (Threshold Accepting, *TA*, and Simulated Annealing, *SA*) are used and, at the end of the phase, every route is improved again through TSP to further reduce the distribution costs. Finally, different routes are allocated to a single vehicle until the overall route time doesn’t exceed the established temporal limit. At the end of the loading, a final solution is gotten for the considered number of facilities. If the recorded lowest cost results smaller than the opening cost for a further facility, the algorithm ends; otherwise, the procedure is repeated increasing by one the facilities number.

3 Proposed approach

In this paragraph an alternative approach will be presented to determine an optimal solution to the routing phase faced in a LRP. After a qualitative definition of the problem, it will be presented its analytical formulation. The CLRP will be faced solving the connected LAP and VRP. The results will be validated through some comparative tests.

3.1 Problem definition

A set of consumers and potential facility is given. If d_i is the demand of a consumer, each consumer with d_i>0 must be allocated to a facility so to completely satisfy d_i. The consignment is delivered through vehicles that depart from a facility and operate on circuits that include more customers. The set-up cost of a centre and the unitary distribution cost have been fixed. The vehicles and the potential centres have limited capacity. Facilities location and vehicles routes have to be determined so to minimize the overall costs (location and distribution costs).
The CLRP is bound by the followings conditions:

- The demand of each customer must be satisfied;
- Each customer must be served by a single vehicle;
- The overall demand on every route must be smaller or, at the most, equal to the capacity of the vehicle allocated to the route;
- Each route begins and ends to the same facility.

It is assumed, moreover, that the vehicle fleet is homogeneous and there's no limit to its dimension.

3.2 Graph representation and objective function

Let $G=(V,A)$ be an oriented graph, where $V=\{v_1,\dots,v_{m+n}\}$ is constituted by the nodes $F=\{v_1,\dots,v_m\}$ (potential facilities locations), and by the nodes $I=\{v_{m+1},\dots,v_{m+n}\}$ (demand centres). Each edge $v_i v_j \in A$ represents the existing link between the pair of nodes that defines it and it is associated with a distance, or cost, c_{ij}>0. If some connections between nodes are forbidden it is still possible to consider a complete graph setting to ∞ the distance between them. It's assumed the graph to be symmetrical, therefore $c_{ij}=c_{ji}$. For each potential service node $v_i \in F$ it is known the maximum service capacity Q; for each demand node $vj \in I$ it is known the service demand d_i. The deliveries are effect by a fleet of k vehicles characterized by a maximum capacity K.
If D is the set of potential facilities, I is the customers set, V the vehicles set, the mathematical formulation of the problem is the following:

$$Min \quad \sum_{i\in D} F_i y_i + \sum_{k\in V}\sum_{i\in D\cup U}\sum_{j\in D\cup U,\, j\neq i} C_{ij} x_{ijk} \tag{1}$$

subject to:

$$\sum_{k\in V}\sum_{i\in D\cup I} x_{ijk} = 1 \quad \forall j\in I \tag{2}$$

$$\sum_{i\in D}\sum_{j\in I} x_{ijk} \le 1 \quad \forall k\in V \tag{3}$$

$$\sum_{j\in D\cup I} x_{ijk} = \sum_{j\in D\cup I} x_{jik} \quad \forall k\in V, \forall i\in I\cup D \tag{4}$$

$$\sum_{j\in I}\sum_{i\in D\cup I} d_j x_{ijk} \le Q_k \quad \forall k\in V \tag{5}$$

$$\sum_{j\in I} d_j z_{ij} \le V_i y_i \quad \forall i\in D \tag{6}$$

$$\sum_{j\in I}\sum_{k\in V} x_{ijk} - y_i \ge 0 \quad \forall i\in D \tag{7}$$

$$\sum_{j\in I} x_{ijk} - y_i \le 0 \quad \forall i\in D, \forall k\in V \tag{8}$$

$$\sum_{i\in I} x_{ijk} = \sum_{i\in I} x_{jki} \quad \forall j\in D, \forall k\in V \tag{9}$$

$$\sum_{k\in D}\sum_{i\in S}\sum_{j\in S} x_{ijk} \le |S| - \left\lceil \sum_{p\in S}\frac{d}{Q} \right\rceil \quad \forall S\in I, |S|\ge 2 \tag{10}$$

$$x_{ijk}\in\{0,1\} \ \forall i,j\in I\cup D \tag{11}$$

$$y_i\in\{0,1\} \ \forall i\in D \tag{12}$$

$$z_{ij}\in\{0,1\} \ \forall j\in I, \forall i\in D \tag{13}$$

where:

- F_i is the set-up cost for facility i, $i\in D$;
- C_{ij} is edge i - j cost, $i,j\in D\cup I$;
- d_j is the demand of the customer j, $j\in I$;
- V_i is the capacity of the facility i, $i\in D$;
- Q_k is the vehicle k capacity, $k\in V$;
- x_{ijk}=1 if vehicle k goes from i node to j node, $i,j\in D\cup I, k\in V$;
- $S=\{D\}\cup\{I\}$ is the set of all possible facility locations and customers;
- y_i=1 if a facility is set-up at node i, $i\in D$;
- z_{ij}=1 if customer j is allocated to facility i, $j\in I$, $i\in D$.

The objective function minimizes the set-up costs of the facilities and the distribution costs. Equation (2) guarantees that each customer has been assigned to a single facility, equation (3) guarantees that each vehicle is sent by a single depository. Equation (4) assures that the very same vehicle enters and exits in each node i, equations (5) and (6) assure that vehicle and facilities capacities are not exceeded. Equations (7) and (8) assure that vehicles only come from opened facilities and equation (9) assures that a vehicle leaves and arrives in the same facility. Equation (10) guarantees the absence of sub-circuits (Sub-Eliminator Constraints, *SEC*).

3.3 CLRP solution
To solve the CLRP a heuristic approach is proposed that divides the problem in:

- Location-allocation (*LAP*);
- Routing (*VRP*).

In the first phase, the solution is a set of selected facilities and a project to allocate the customers to the facilities. In computing the distances each customer is directly connected to the nearest facility (*radial distance*). This solution will be used as input for the VRP, producing a set of admissible routes [5, 8, 10].
Because of the aggregative nature of the demand nodes, the various cost/time components in this problem (route cost/time and stopover cost/time in every node. These last ones proportionally increase with demand node dimension) must be transformed into node-to-node cost parameters. If s_i is the stopover cost/time in v_i node and c_{ij} is the travelling cost/time from node v_i to node v_j, than the transformed travelling cost/time, between node v_i and node is given v_j by:

$$c'_{ij} = c_{ij} + \frac{1}{2}\left(s_i + s_j\right) \quad \forall i \neq j \tag{14}$$

In this way, the problem is converted to the classical LRP with no stopover cost/time considered. The transformed cost/time will simply indicated as c_{ij} .

3.3.1 LAP phase
The procedure in this phase consists in the following steps (Fig. 1):

- *Customers allocation to the potential facilities*. In computing the distances each customer will be directly connected to the nearest facility (*radial distance*) if its capacity constraint is not violated, otherwise the customer will be assigned to another facility minimizing the cost function. The output of the phase is an *incidence customer-facility matrix*. Each item of such matrix will be 1 if the customer i is connected to the facility j;
- *Customers distribution list determination*. A set where each item e_i is the number of customers assigned to facility i. The items will be sort in descending order in a following step.
- *Facilities number determination*. In this step a lower bound on the number of facilities is established (N_f). However, the actual distribution of the demand and supply nodes is not considered. Given N_f, it is possible to compute the combinations $\binom{m}{N_f}$.
- *L matrix definition*. Rows are sorted in descending order considering the number of customers assigned to each facility. In other words, the first line of the matrix contains that set of N_f facilities with the highest number of customers in the closeness. The solution obtained in this phase provides the minimum number of facilities to satisfy the whole demand of the customers and the potential facilities configs; these ones will be the input for the VRP phase.

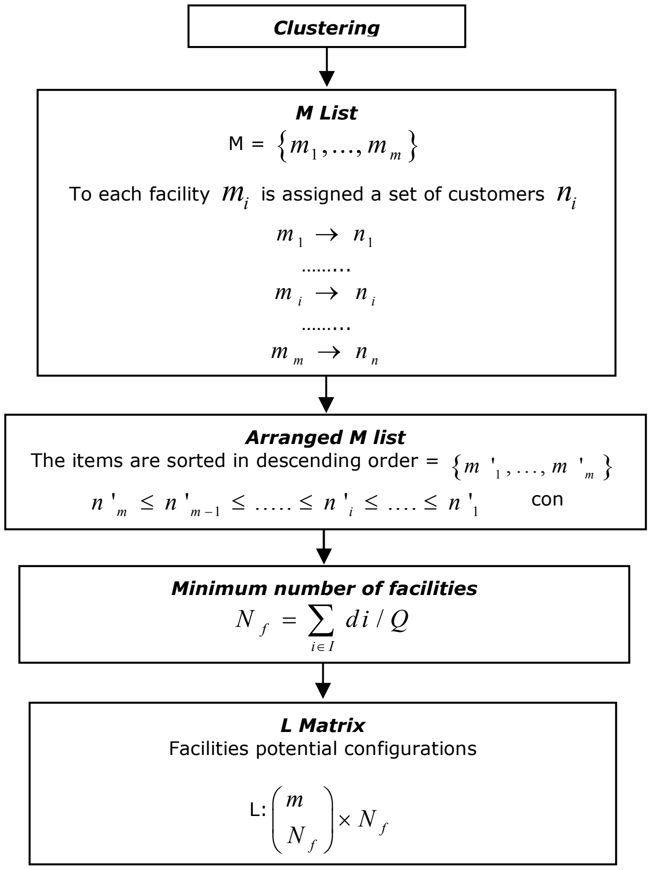

Fig. 1 – LAP phase

3.3.2 VRP phase
Given the *L* matrix the set of the potential configs is constituted by all the sets of facilities previously determined and, therefore, every line of the matrix defines a potential config of facilities to enable. The procedure in this phase consists in the following steps (Fig. 2):

- *Customers allocation to facilities*. In this step the consumers are reassigned to the considered facilities. This step doesn't differ from the analogous one in the LAP procedure: within every cluster a VRP will be performed to determine the necessary routes, satisfying the whole demand of all the customers.
- *Resolution algorithm*. At first a TSP is resolved with no vehicles capacity constraint, then the same TSP is modified to take in account this constraint (*TSP-VRP*). This could be result in an inadmissible solution of TSP for the VRP, and therefore the

initial route is modified, producing a set of routes. The steps to solve the TSP-VRP are:

1. A set of n demand nodes and one facility 0 are given. Requested demand is satisfied by vehicles of capability K;
2. The TSP is performed with no vehicles capacity constraints to determine an initial route containing all the nodes (0-1-2-…-n-0);
3. Considering vehicles capacity constraint the route is modified as it follows:

3.1 Set R → Macro route,
3.2 Initial node = facility;
3.3 Possible travelling directions: 0 → 1; 0 → n;
3.4 Choice of one direction (choosing one direction is due to $c_{ij} = c_{ji}$ hypothesis, symmetrical TSP and VRP);

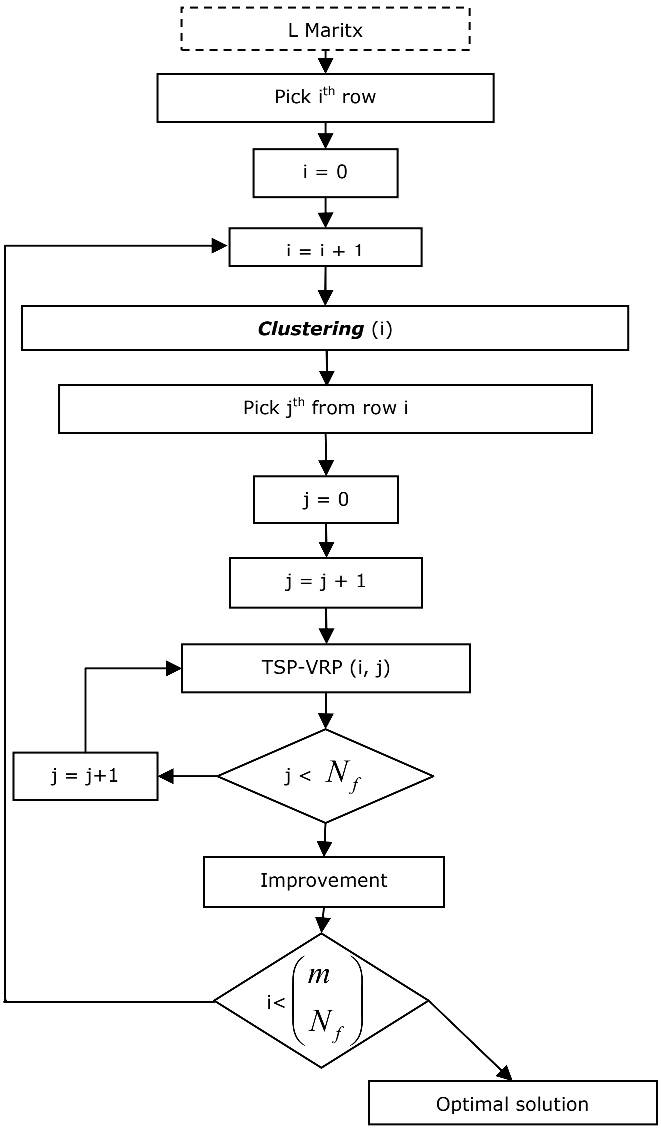

Fig. 2 – VRP phase

3.5 Travel across the edges till the demand of the nodes doesn't exceed K. If $\sum_{i=1}^{n} d_i \le K$ than R is admissible for the VRP, otherwise 3.6;
3.6 Let s and t be two generic consecutive nodes belonging to the circuit (0-1-2-…-s-t-…-n-0) such that $\sum_{i=1}^{s} d_i \le K$ and $\sum_{i=1}^{t} d_i > K$. Since the capacity of the vehicle is overcome, the edge (s, t) is eliminated and the edges ($s,0$) and ($0,t$) are established. In such a way two routes are determined: R' (0-1-…-s-0) and R'' (0-t-t+1-...-n-0). Record R';
3.7 Set R=R'' and go to step (3.1).

The same algorithm (Fig. 3) is performed for both the direction.

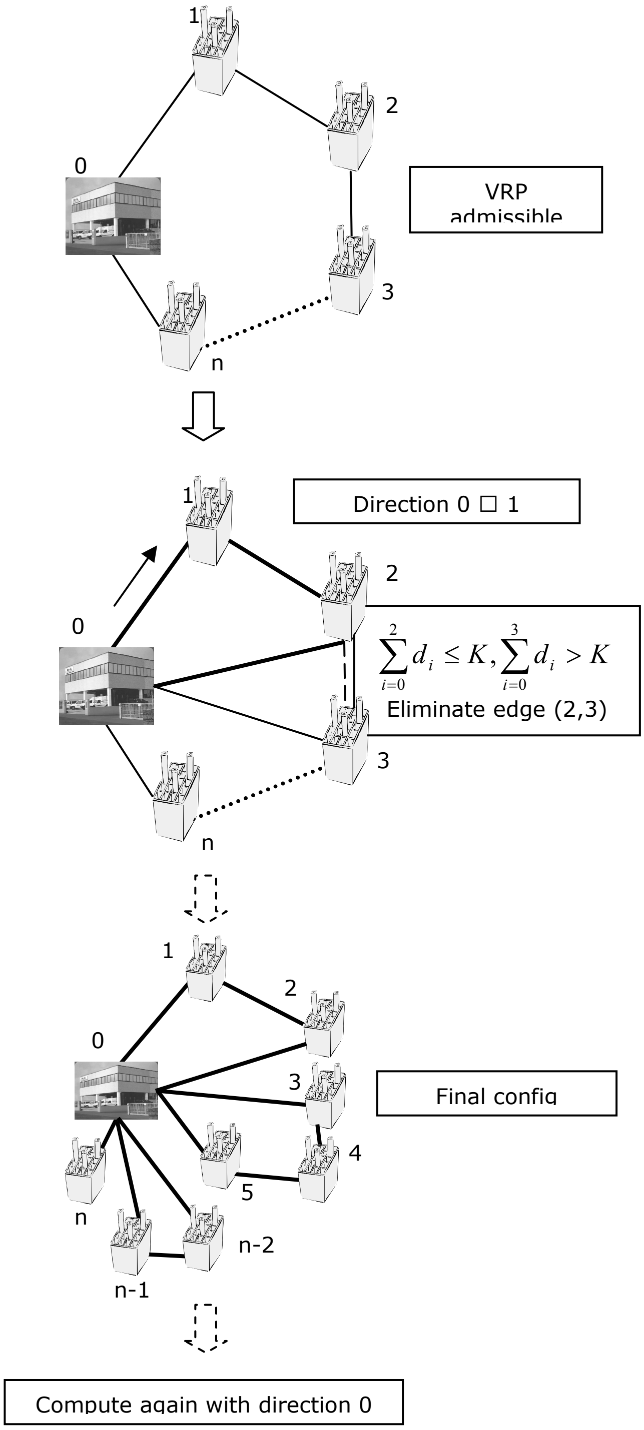

Fig. 3 – Edges elimination

The optimal VRP solution is the one with the lowest cost in the set of found solutions.
The solution for the routing phase, is obtained performing, at first, a TSP and determining a route characterized by the lowest “travelling” cost. The route passes through every node just one time (*Hamiltonian circuit*). In the following phase, when more strength constraint are considered, a set of optimal routes to serve the customers is defined, guaranteeing the optimality of the nodes sequence inside each route. So, the best solution in the whole solutions set is characterized by an optimal number of routes and each route is characterized by the best sequence (in terms of time/costs) of the served demand nodes.
The *TSP-VRP* algorithm starts from a macro-route which is divided, in a second phase, in a certain number of sub-route. The nodes sequence, nevertheless, doesn't change, it’s the optimal one suggested by the resolution of the TSP.

4. Test
Two main group of tests have been conducted.
Initially, obtained results have been examined applying the algorithm to problems characterized by a limited number of nodes. In this phase it have been possible to adopt technique and tools for the exact solution (global optimum) of the problem, both for the conventional VRP procedure and for the *TSP-VRP*. For the latter one, particularly, an exact procedure to determine the Hamiltonian circuit is used. This circuit will be modified to take in account vehicles and facilities constraints. Obtained results allowed a verification of the *TSP-VRP* model.
Subsequently, some problems have been examined characterized by an increased number of nodes. Heuristics or meta-heuristics have been applied, to random instances or to instances proposed in literature, because of the increased computational difficulties. Particularly, tests have been conducted applying Clarke and Wright’s algorithm [4] both in the case of VRP and TSP. The choice of this algorithm is due to its robustness and simplicity. In this phase, obtained results allowed to calibrate the *TSP-VRP* model. Turning to heuristic or meta-heuristic techniques to solve the VRP, if high number of nodes are considered, would result in sub-optimal solutions. Particularly it wouldn’t be able to be respected, within each route, the optimal demand nodes travelling sequence. I order to find an optimal travelling sequence (least route cost and/or minimum route time), a procedure of local improvement applied to each circuit is carried on. The *TSP-VRP* model, nevertheless, allows to bypass such a problem because it stars from the configuration provided by the TSP.

4. 1 Experimental results
An initial set of nodes is assigned. The *VRP* and the *TSP-VRP* have been executed.
To solve the *VRP*, and to find a solution of the *TSP* in the *TSP-VRP*, the Branch and Bound model has been used implemented in LINGO® 10. The number of nodes is equal to 7 (the facility node is included), and the cost matrix of all links is assigned. The *VRP* provided a cost function value equal to 178. Applying the same model to solve the TSP, considering the very same number of nodes, resulted in a *Hamiltonian circuit* whose overall cost is equal to 74. The heuristic “cut” procedure has then determined the number of needed sub-routes to comply with capacity constraints. Obtained result is, as for the *VRP*, equal to 178.
The same problem has been resolved with the heuristic procedures by Clarke and Wright (savings algorithm). The initial star-shaped solution of the VRP provided a cost function value equal to 208. Considering the savings matrix this solution is optimized and a final vale of 178 is obtained. The substantial difference with respect to the TSP-VRP algorithm, nevertheless, is the growth of the iterations number and of the computational speed.
The *TSP-VRP* robustness is proved by the results obtained varying some input factors. Particularly, as concern vehicles capacity variation, obtained results are in Tab. 1 and in Tab. 2.

Node	Demand
1	*Facility*
2	3
3	2
4	4
5	3
6	2
7	2

Tab. 1 - Input data

	Vehicles capacity										
	k	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
Instances	***VRP***	574	471	395	433	433	410	395	395	395	343
	TSP - VRP	574	471	395	433	433	408	395	395	395	343

Tab. 2 - Some experimental results, varying vehicles capacity

In these cases, the TSP almost always provides the optimal initial solution for the VRP. Therefore, vehicles capacity constraint doesn’t generally distort the optimal sequence of the nodes provided by the TSP resolution. The model has then been tested on an increased number of nodes (Tab. 3).

		Nodes number			
	n	***14***	***12***	***10***	***25***
Instances	***VRP***	988	760	670	1356
	TSP-VRP	902	742	670	1325

Tab. 3 - Some experimental results, varying nodes number

It can be noticed that the solution provided by the *TSP-VRP*, is always better than the solution provided by Clarke and Wright's algorithm for the *VRP*. This result shows the better improvement capabilities of the TSP-VRP.

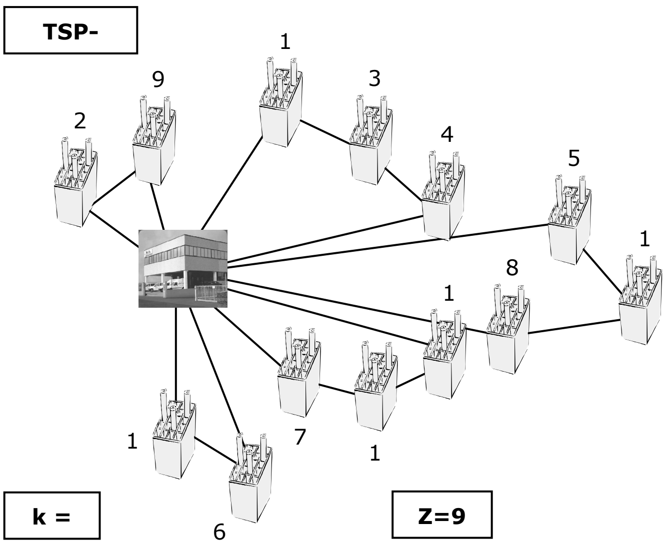

Fig. 4 – *TSP-VRP* final solution

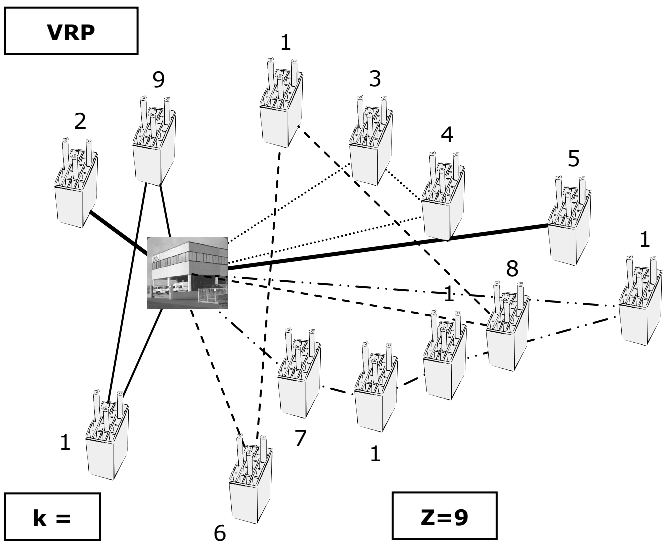

Fig. 5 - Clarke and Wright's algorithm final solution for the *VRP*

5. Conclusions

As known the lower bound for a *VRP* is provided by a *kTSP* [2, 20], where *k* is the minimum number of vehicles to fulfil the overall demand, while an upper bound is generally provided by an heuristic solution of the problem.

Comparing the solutions provided by *TSP* and *VRP* to the same instance, it is noticed that in 95% of cases, the routes provided by the *VRP* derive from the macro-route by *TSP*. The *TSP-VRP* algorithm supplies, therefore, more than satisfactory results. Moreover, the macro-route from which each solution of the VRP derives is always the global optimal one, as the Hamiltonian circuits has been deduced applying exact algorithms. The solutions of the various VRPs are not always optimal, but however they derive from the TSP modifying the macro-route with respect to capacity constrains.

The TSP-VRP model, finally, provides results which depends on:

- the quality of the *TSP* solution;
- the number of nodes considered:
- vehicle capacity.

Using heuristic algorithms to solve the *TSP* and applying the *TSP-VRP*, has shown to provide a 3% decrease in cost function, proving the effectiveness of the model.

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