Simulation and Design of Nonlinear Controllers based on Distributions Theory

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Abstract: - In this paper, there are presented the discontinuous nonlinear controllers simulation and design based on distributions and their properties. Using the definition of distributions, the mathematical formulas for discontinuous nonlinear elements are determined. This approach allows describing the analog and discrete systems in the same manner. The equations on distributions are simple comparing to the equations with analog and discrete functions. The paper describes the specific properties that can be used in digital and nonlinear controller design. Based on distributions there are modeled, simulated and designed the complex PWM inverter controller and the adaptive DC servo relay controller. This distributions approach implemented as software modules or objects opens new possibilities in software-oriented embedded-controller.

Key-Words: - distributions, nonlinear, PWM, adaptive, relay, controller, simulation, design

1 Introduction

Many process phenomena cannot be represented as a function or as a mathematical formula because of its discontinuities, severe nonlinearities, non-derivation etc. In the functions theory this situations are avoided using the partitioning method, eliminating the discontinuities and never using the derivation operator in critical points.

This approach has many disadvantages. One is that the mathematical model is complicated, consisting of several function segments. Another is the impossibility of derivation use in the discontinuity points. From these reasons the modeling, simulation and design of such systems is quite complicated.

In this paper we propose a more facile and flexible method to eliminate the disadvantages presented above. This method uses the distributions theory and properties.

The distributions extend the functions theory, generalizing the mathematical operators that can thus be applied on the whole domain.

On the other hand, by this method it is possible to define new distributions, which prove to be very useful in controller design.

In this paper, we will use the following distributions definition [2]:

Distributions are real, linear and continuous functional f defined on fundamental functions space $\{\ \varphi_i(x)\ \} = \mathrm{K}^m$ having the properties:

$$\forall\varphi_1(x),\varphi_2(x)\in K^m,\ \alpha,\beta\in R^n: \quad \alpha\cdot\varphi_1(x)+\beta\cdot\varphi_2(x)\in K^m \tag{1}$$

$$\forall\varphi_k(x)\in K^m: \lim_{k\to\infty}\varphi_k(x)=0. \tag{2}$$

$$f: K^m \to R: \quad f[\varphi(x)]\in R. \tag{3}$$

We consider the elementary distributions, which are pulse, step and ramp.

The pulse distribution or Dirac pulse is defined as follows:

$$\forall\varphi(x)\in K^m,\ \delta: K^m\to R;$$
$$\delta[\varphi(x)]=\begin{cases}0; & \varphi(x)\neq 0\\ \infty; & \varphi(x)=0\end{cases} \qquad \int_R \delta[\varphi(x)]dx=1 \tag{4}$$

The step distribution or Heaviside distribution is defined bellow:

$$\forall\varphi(x)\in K^m,\ \theta: K^m\to R;$$
$$\theta[\varphi(x)]=\begin{cases}1; & \varphi(x)\geq 0\\ 0; & \varphi(x)<0\end{cases} \tag{5}$$

The ramp distribution is defined as follows:

$$\forall\varphi(x)\in K^m,\ r: K^m\to R$$
$$r[\varphi(x)]=\begin{cases}x; & \varphi(x)\geq 0\\ 0; & \varphi(x)<0\end{cases} \tag{6}$$

In fig.1 are presented these distributions for the simple case $\varphi(x)=x$.

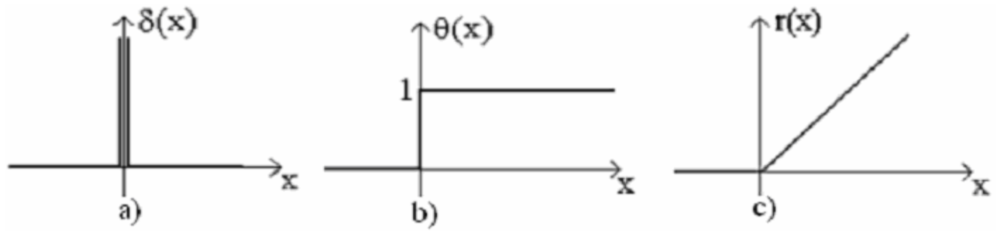

Fig.1. Elementary distributions: a) Pulse; b) Step; c) Ramp

The most important properties of elementary distributions are:

$$\delta(x-x_0)=\varphi(x_0) \tag{7}$$

$$\delta[f(x)]=\sum_{i=1}^{p}\frac{1}{\left|f^{'}(x_i)\right|}\cdot\delta(x-x_i) \tag{8}$$

Where x_i are simple solutions of $f(x)=0$.

$$\theta(-x)=1-\theta(x) \tag{9}$$

$$\frac{d[\theta(x)]}{dx}=\delta(x) \tag{10}$$

$$\frac{d[r(x)]}{dx}=\theta(x) \tag{11}$$

$$\frac{d[f(x)]}{dx}=f(x)^{'}+\sum_{i=1}^{p}s_i\cdot\delta(x-x_i);$$

$$s_i=f(x_i+0)-f(x_i-0) \tag{12}$$

$$\theta(x)-\theta(-x)=sign(x) \tag{13}$$

In fig.2.a, b there are presented the simulation model and results for elementary distributions applied on $f(x)=2\cdot\sin(2\cdot\pi\cdot x)$ function. In order to achieve this elementary distributions model there were used the properties (7) – (13). In fig.2.c is presented the software implementation of the above model.

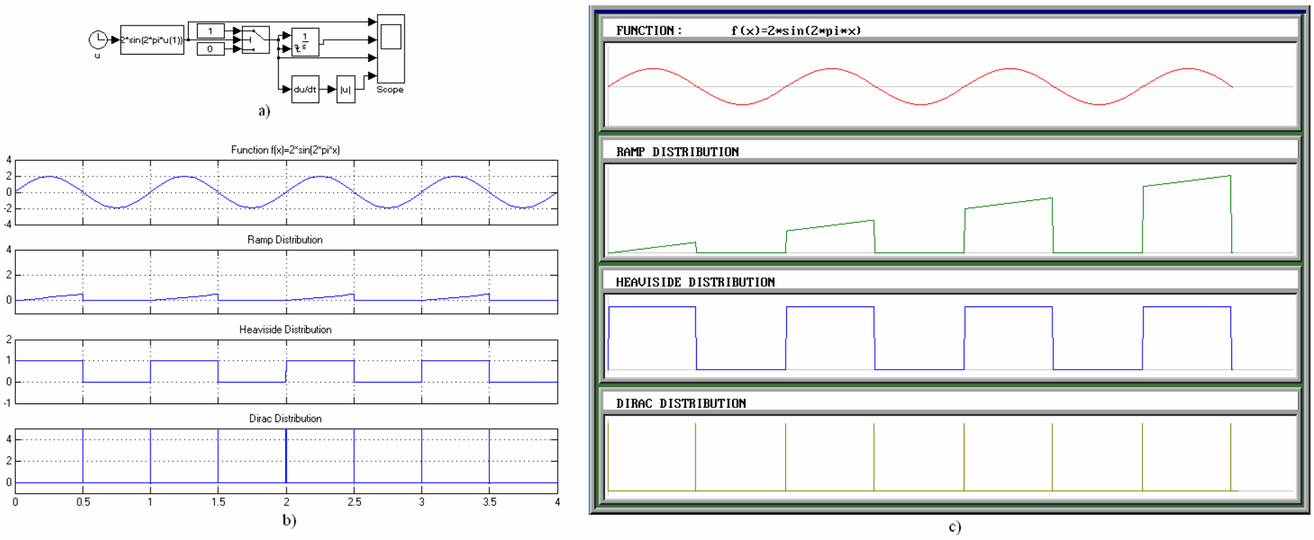

Fig.2. Elementary distributions: a) Model; b) Simulation results; c) Software implementation

2 Problem Formulation

In this section, we present two examples of nonlinear controller simulation and design, as follows: induction motor PWM controller and DC servo adaptive relay controller.

2.1 Induction Motor PWM Controller

The PWM pulses generation is a complicated process consisting of sine wave modulation with a triangular carrier. The pulses are generated when the sine wave is above the triangular carrier. Therefore, it is necessary to calculate the intersection points between the two waves, using the nonlinear equation:

$$R\cdot\sin\left(\alpha+\frac{2\cdot(i-1)\cdot\pi}{3}\right)=$$
$$=(-1)^{k-1}\cdot\frac{2\cdot p}{\pi}\cdot\left(\alpha+\frac{2\cdot(i-1)\cdot\pi}{3}\right)+\frac{(k-1)\cdot\pi}{p} \tag{14}$$

Where: R is the modulation ratio, p is the modulation index, i is the phase number (i=1,2,3) and k is the pulse number (k=1..p). The controller must generate three pulses trains (PWM_i) and other three un-phased by 180° ($\overline{PWM_i}$) to these, as presented in fig.3.

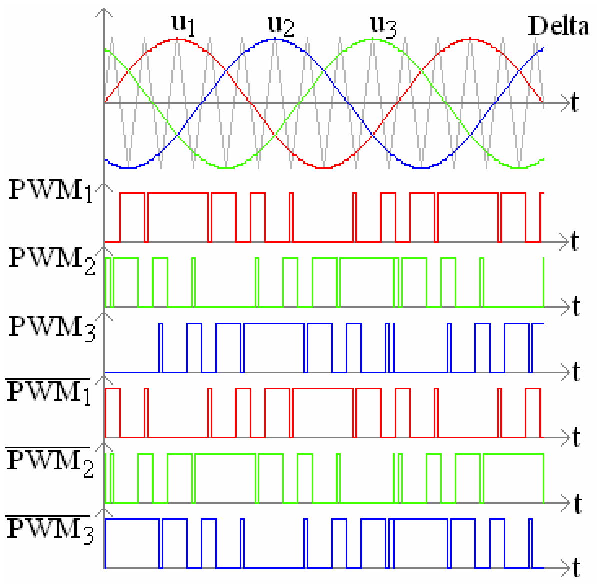

Fig.3. PWM pulses generation principle

In the classical way, there were used approximation methods that lead to programs, which limit drastically the commutation frequency due to processing delays.

Using the distributions method [3], the problem can be solved without approximations and computations, as follows: from a cosine wave with p variable applying the bilateral ramp distribution $r_b(u)=\frac{1}{s}\cdot sign(u)$ there is obtained the triangular wave (delta). Then, applying the step distribution θ(f) on the function $f=R\cdot\sin(\alpha)-r_b(\cos(p\cdot\alpha))$ we get the PWM pulses. Results the general formula for the PWM pulses in distributions:

$$PWM(R,p,f,t)=$$
$$=\theta\left\{R\cdot\sin(2\pi ft)-\frac{1}{s}\cdot sign[\cos(p2\pi ft)]\right\} \quad (15)$$

This formula can be modeled, simulated and software implemented as VLSI embedded controller in a quite simple manner and with faster processing speed then other methods.

2.2 DC Servo Relay Nonlinear Controller

Many processes are controlled using relay type nonlinear controllers that have dead zone, hysteresis and on/off commutation. The right nonlinear controller is chosen according to the process parameters, like delay time constants and gain, by an adaptive strategy.

We will consider a nonlinear control system consisting of a DC servo with angle transducer and an adaptive relay type nonlinear controller. As shown in fig.4 the control system has two loops, one for control and other for adaptation.

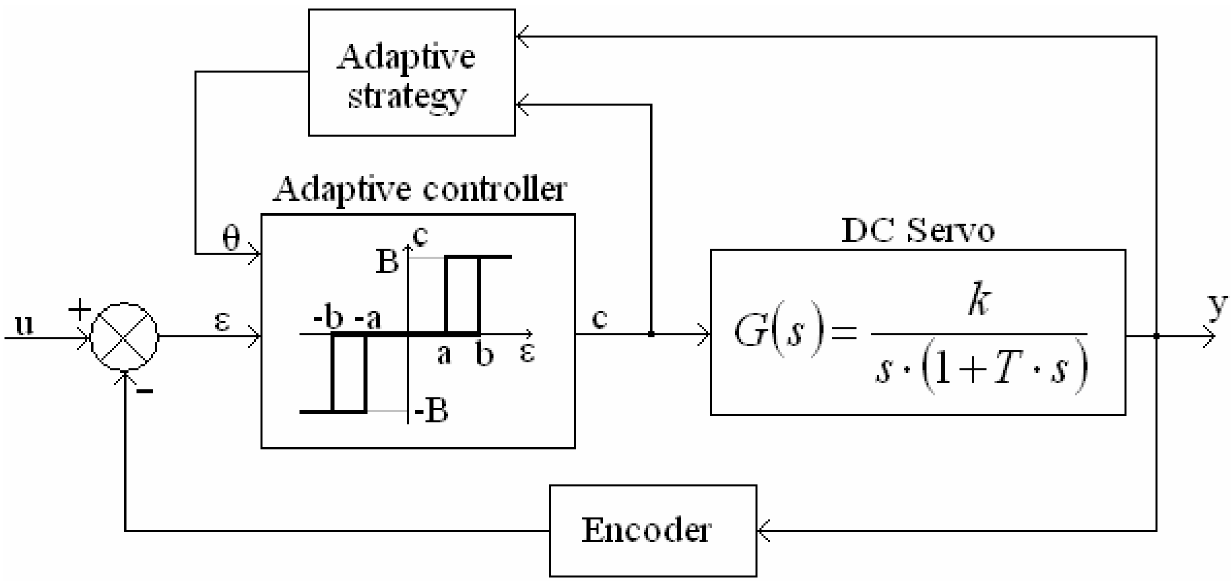

Fig.4. DC servo adaptive control system

In the classical way, it is difficult to write the equations of the control system because of the presence of discontinuity points in the controller characteristic.

In addition, it is very difficult to adapt online the controller structure to the process parameters change.

Using distributions, the controller equation is:

$$c(\varepsilon)=B\cdot\left[\left(\theta(\varepsilon-a)-\theta(-\varepsilon-b)\right)\cdot\theta\left(sign\left(\frac{d\varepsilon}{dt}\right)\right)+\right.$$
$$\left.+\left(\theta(\varepsilon-b)-\theta(-\varepsilon-a)\right)\cdot\left(1-\theta\left(sign\left(\frac{d\varepsilon}{dt}\right)\right)\right)\right] \quad (16)$$

The DC servo input-output equation, based on the transfer function, is:

$$T\cdot\frac{d^2y}{dt^2}+\frac{dy}{dt}=k\cdot c(\varepsilon) \quad (17)$$

We make the notation:

$$\frac{d\varepsilon}{dt}=\dot{\varepsilon} \quad (18)$$

And we get:

$$\frac{d\dot{\varepsilon}}{d\varepsilon}=-\frac{\dot{\varepsilon}+k\cdot B\cdot\left(\theta(\varepsilon-a)-\theta(-\varepsilon-b)\right)\cdot\theta\left(sign\left(\frac{d\varepsilon}{dt}\right)\right)}{T\cdot\dot{\varepsilon}}$$
$$-\frac{k\cdot B\cdot\left(\theta(\varepsilon-b)-\theta(-\varepsilon-a)\right)\cdot\left(1-\theta\left(sign\left(\frac{d\varepsilon}{dt}\right)\right)\right)}{T\cdot\dot{\varepsilon}} \quad (19)$$

These four equations above allow us to model, simulate and design the controller.

3 Problem Solution

3.1 Induction Motor PWM Controller

Using the equation (15) there was determined the model from fig.5.a with the simulation results from fig.5.b.

The software can be easily implemented, based on the algorithm described in the simulation model. It can be written in any programming language, from the PC assembly language to the VLSI hardware description language.

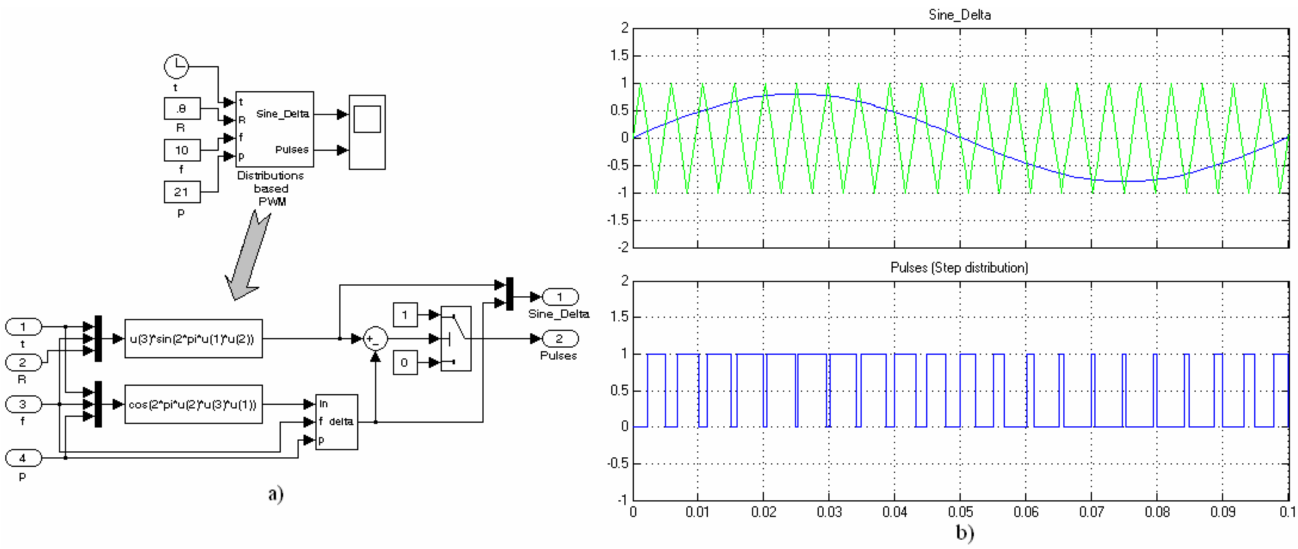

Fig.5. Distributions PWM controller: a) Model; b) Simulation results

3.2 DC Servo Relay Nonlinear Controller

In order to validate the solution there was achieved the model from fig.6.

The adaptive strategy selects the relay parameters a and b according to the estimated values of the gain k and time constant T. Therefore, it is selected one of the four relay controllers from fig.7.a. In fig.7.b are presented the DC servo and controller outputs and error signals. In fig.7.c is shown the phases plane stability of each type of controller.

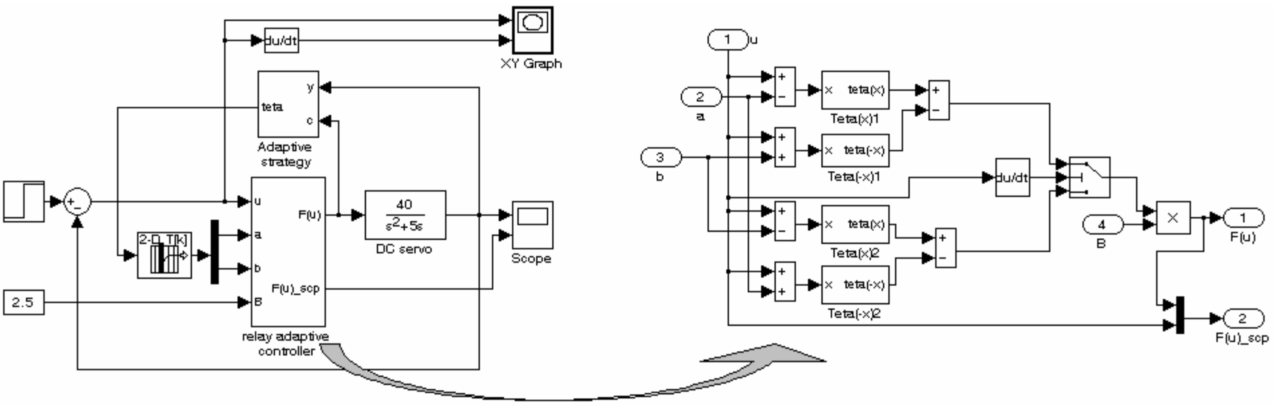

Fig.6. Relay DC servo adaptive controller

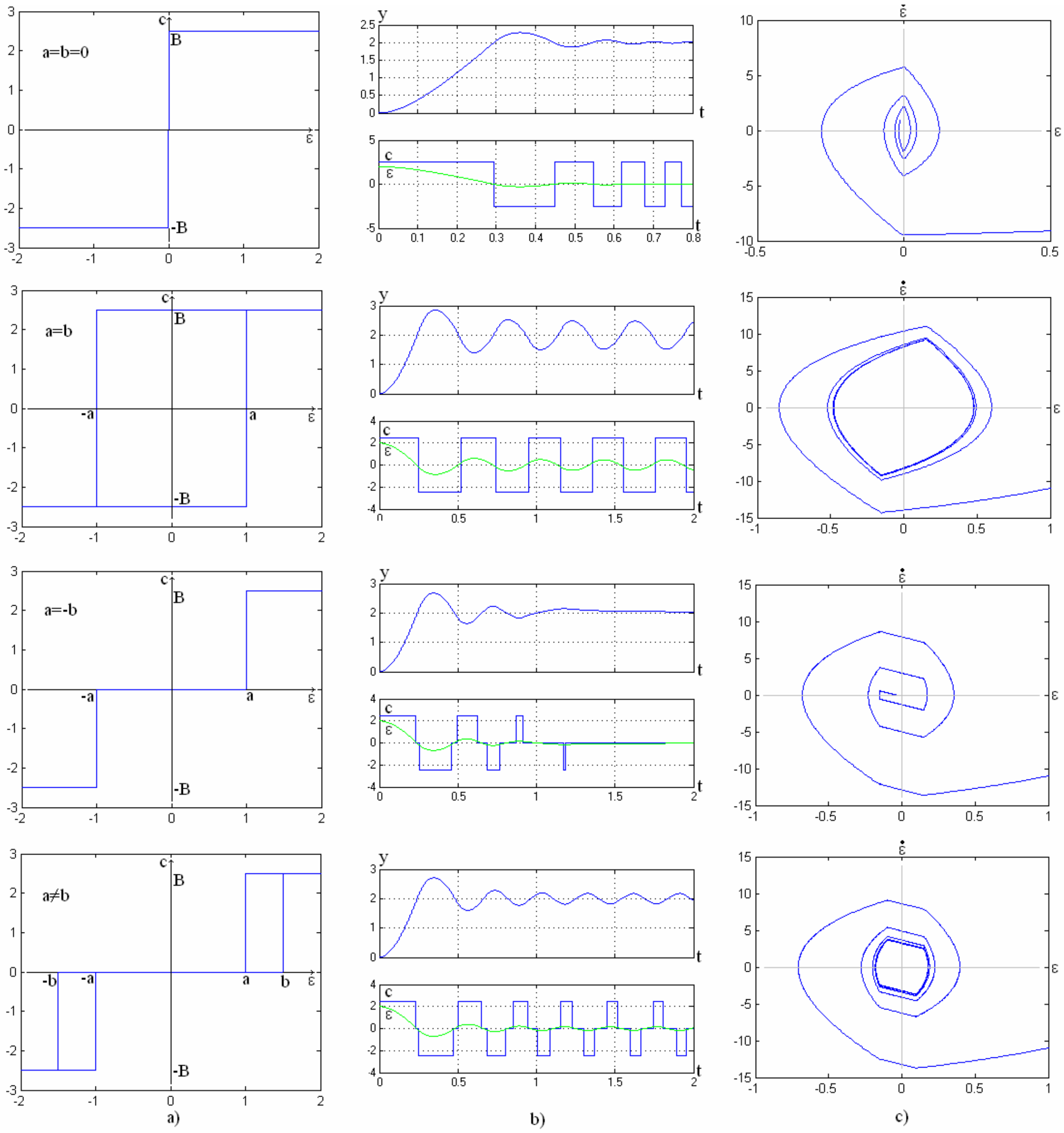

Fig.7. DC servo controller: a) Controller types; b) Time responses; c) Stability

4 Conclusion

The distributions and their properties represent a link between the continuous and digital systems. Using this concept the digital systems can be analyzed and designed using the experience and tools of continuous systems. Using distributions properties many complex problems of digital control can be solved easily and precisely. The distributions are very useful for modeling, simulation and design of discrete and nonlinear controllers. The distributions are also suitable for software implementation and by this offer very good support for embedded software oriented solution of digital control applications.

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